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The Solution of a Problem on Stationary Axial Vibrations of a Finite Length Hereditary Elastic Annular Cylindrical Shell

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Abstract: The stationary longitudinal vibrations of a finite length visco-elastic cylindrical shell with regard to singularity of the shell creeping nucleus are investigated. A formula for axial displacement of the shell points are obtained in explicit analytic form. A formula of amplitude-frequency dependence on the base of which an appropriate curve is constructed for the given values of input parameters, is derived.

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I. INTRODUCTION

The issues of vibrations of inelastic thin-walled structural elements are important as before, their results are highly demanded in engineering and building. This is connected with the fact that a great majority of structural elements work at periodic loading conditions. The used materials have explicitly inelastic features of rheological character. The main goal of the investigations is to define the eigen frequencies the knowledge about of which in practical plan allows to carry out preventive measures in order to avoid initiation of resonance.

In the paper, periodic stationary vibrations of a finite length annular cylindrical shell whose material is a hereditary type visco-elastic material are investigated.

II. PROBLEM STATEMENT

Consider a thin-walled annular cylindrical shell of radius R and finite length l. Direct the axis z along the axis of the cylindrical shell, θ is an annular angular coordinate, r is radial coordinate. Accept the denotation:

$\{u_r \,;\, u_\theta \,;\, u_z\}$ is the displacement of the shell points.

Let from the left side the thin-walled cylindrical shell be subjected to longitudinal vibrations of the given amplitude and frequency, and the right end side of the shell be free from forces. If we locate the origin of coordinates z on the left end side of the shell, then the boundary conditions will take the form:

$$u_z = u_0 \cos \omega t; \qquad for \quad z = 0\,; \tag{1}$$

$$\sigma_z = 0; \qquad for \quad z = l. \tag{2}$$

On account of availability of the axial symmetry, the tangential stresses $\sigma_{z\theta}$ and $\sigma_{r\theta}$ in the shell and tangential displacements u_θ equal zero. The remaining displacements and stresses are independent of the tangential angular coordinate θ.

Considering the mean stresses along the shell thickness based on the membrane theory [1] it is easy to see that the displacements $u_r(z;t)$, $u_z(z;t)$ and stresses $\sigma_{zz}(z;t)$, $\sigma_{\theta\theta}(z;t)$ differ from zero. The equations of motion of a thin-walled elastic annular cylindrical shell based on the membrane theory [1] have the form:

$$\begin{cases} \dfrac{\partial^2 u_r}{\partial t^2} = -\dfrac{1}{\rho}\dfrac{E}{1-\nu^2}\left(\dfrac{u_r}{R^2} + \dfrac{\nu}{R}\dfrac{\partial u_z}{\partial z}\right) \\ \dfrac{\partial^2 u_z}{\partial t^2} = \dfrac{1}{\rho}\dfrac{E}{1-\nu^2}\left(\dfrac{\partial^2 u_z}{\partial z^2} + \dfrac{\nu}{R}\dfrac{\partial u_r}{\partial z}\right) \end{cases} \tag{3}$$

where ρ is density of the material, E is Young's modulus, ν is the Poisson ratio.

In order to get an appropriate system of equations of motion for a hereditary elastic thin-walled annular cylindrical shell by the membrane theory, apply the Volterra-Rabotnov correspondence method where it is suggested to change in the equations the modulus of elasticity E and the Poisson ratio ν by the appropriate operators $\tilde{E}$ and $\tilde{\nu}$ [2] :

$$\begin{cases} \tilde{E} = E\left(1-\Gamma^*\right) = \dfrac{E}{1+K^*} \\ \tilde{\nu} = \nu\left(1+\dfrac{1-2\nu}{2\nu}\Gamma^*\right) \end{cases} \tag{4}$$

where K^* is the creeping operator, Γ^* is its resolvent operator called a relaxation operator. These operators have the following representation:

$$K^*\cdot f = \int_{-\infty}^{t} K(t-\tau)\, f(\tau)\,d\tau; \qquad \Gamma^*\cdot f = \int_{-\infty}^{t} \Gamma(t-\tau)\, f(\tau)\,d\tau \tag{5}$$

where $K(t-\tau)$ is a creeping nucleus, $\Gamma(t-\tau)$ is a relaxation nucleus.

The equations of motion for visco-elastic case take the form:

$$\begin{cases} \dfrac{\partial^2 u_r}{\partial t^2} = -\dfrac{1}{\rho}\dfrac{\tilde{E}}{1-\tilde{\nu}^2}\left(\dfrac{u_r}{R^2}+\dfrac{\tilde{\nu}}{R}\dfrac{\partial u_z}{\partial z}\right) \\ \dfrac{\partial^2 u_z}{\partial t^2} = \dfrac{1}{\rho}\dfrac{\tilde{E}}{1-\tilde{\nu}^2}\left(\dfrac{\partial^2 u_z}{\partial z^2}+\dfrac{\tilde{\nu}}{R}\dfrac{\partial u_r}{\partial z}\right) \end{cases}. \tag{6}$$

As a creeping nucleus take the Abel weaksingular nucleus:

$$K(t) = J_\alpha = \frac{\phi}{\Gamma_1(1-\alpha)}t^{-\alpha}; \qquad 0 \le \alpha < 1 \tag{7}$$

where $\Gamma_1(1-\alpha)$ is Euler's gamma function.

Then the relaxation operator will be determined by the operator whose nucleus is Rabotnov's fractional-exponential function [2]:

$$\Gamma^* = \phi \exists_\alpha^*(-\phi). \tag{8}$$

This operator is from the class of resolvent operators, and then using the algebra of Rabotnov's resolvent operators, we get:

$$\frac{\tilde{E}}{1-\tilde{\nu}^2} = \frac{E}{1-\nu^2}\left(1-\frac{\phi}{4}\frac{1+\nu}{1-\nu}\exists_\alpha^*\left(-\frac{\phi}{2(1-\nu)}\right)-\frac{3\phi}{4}\frac{1-\nu}{1+\nu}\exists_\alpha^*\left(-\frac{3\phi}{2(1+\nu)}\right)\right). \tag{9}$$

Boundary condition (2) will take the form:

$$\frac{\partial u_z}{\partial z}+\tilde{\nu}\frac{u_r}{R} = 0; \qquad for \quad z = l. \tag{10}$$

III. PROBLEM SOLUTION

Equations of motion (6) of a thin-walled hereditary-elastic annular cylindrical shell is a system of integro - differential equations whose solutions will be sought in the form of the following Fourier series:

$$\begin{cases} u_r = \sum_{k=1}^{\infty} U_{rk}(z;\gamma_k)e^{-i\gamma_k t} \\ u_z = \sum_{k=1}^{\infty} U_{zk}(z;\gamma_k)e^{-i\gamma_k t} \end{cases} \tag{11}$$

where γ_k are the parameters defined in the course of problem solution, and in the general case they are complex expressions, i is an imaginary unit, $t^2 = -1$.

Substituting representation (11) in the system of equations (6), allowing for (4); (7); (8); (9), and also

$$\exists_{\alpha}^{*}\left(-\beta\right)e^{-i\gamma_k t}=\sum_{n=0}^{\infty}\frac{\left(-1\right)^n\beta^n i^{(n+1)(1-\alpha)}}{\gamma_k^{(n+1)(1-\alpha)}}e^{-\gamma_k t}\ , \tag{12}$$

we get the following system of differential equations with respect to the functions $U_{rk}\left(z;\gamma_k\right)$ and $U_{zk}\left(z;\gamma_k\right)$ of series (11):

$$\begin{cases}\gamma_k^2 U_{rk}\left(z;\gamma_k\right)-\dfrac{E}{\rho R\left(1-\nu^2\right)}\left(\dfrac{1}{R}\left(1-\delta_k^{(1)}\right)U_{rk}\left(z;\gamma_k\right)+\right.\\ \qquad\left.+\left(\nu+\delta_k^{(2)}\right)\dfrac{dU_{zk}\left(z;\gamma_k\right)}{dz}\right)=0\\ \gamma_k^2 U_{zk}\left(z;\gamma_k\right)+\dfrac{E}{\rho\left(1-\nu^2\right)}\left(\left(1+\delta_k^{(1)}\right)\dfrac{d^2U_{zk}\left(z;\gamma_k\right)}{dz^2}+\right.\\ \qquad\left.+\dfrac{1}{\rho}\left(\nu+\delta_k^{(2)}\right)\dfrac{dU_{rk}\left(z;\gamma_k\right)}{dz}\right)=0\end{cases} \tag{13}$$

where the following denotation are accepted:

$$\delta_k^{(j)}=\sum_{n=0}^{\infty}\frac{\left(-1\right)^{n+1}\cdot\phi^{n+1}\cdot\mu_n^{(j)}\cdot i^{(n+1)(1-\alpha)}}{2^{n+2}\cdot\gamma_k^{(n+1)(1-\alpha)}}\ ;$$

$$\mu_n^{(1)}=\frac{1+\nu}{\left(1-\nu\right)^{n+1}}+\frac{3^{n+1}\left(1-\nu\right)}{\left(1+\nu\right)^{n+1}}\ ;$$

$$\mu_n^{(2)}=\frac{\nu}{\left(1-\nu\right)^n}-\frac{3^{n+1}\cdot\nu\left(1-\nu\right)}{\left(1+\nu\right)^{n+1}}+\frac{1-2\nu}{2^{n-1}}-\left(1+\nu\right)\left(\frac{1}{\left(1-\nu\right)^n}-\frac{1}{2^n}\right)+$$
$$+3\left(1-\nu\right)\left(\frac{3^n}{\left(1+\nu\right)^n}-\frac{1}{2^n}\right).\quad .$$

From the first equation of system (13) the function $U_{rk}\left(z;\gamma_k\right)$ may be expressed by the function $U_{zk}\left(z;\gamma_k\right)$:

$$U_{rk}\left(z;\gamma_k\right)=\frac{E\left(\nu+\delta_k^{(2)}\right)}{\rho R\left(1-\nu^2\right)\gamma_k^2-E\left(1+\delta_k^{(1)}\right)}\cdot\frac{dU_{zk}\left(z;\gamma_k\right)}{dz}. \tag{14}$$

Taking into account this expression in the second equation of system (13), we get the following ordinary differential equation of second order with respect to the functions $U_{zk}\left(z;\gamma_k\right)$:

$$\frac{dU_{zk}\left(z;\gamma_k\right)}{dz}+g\left(z;\gamma_k\right)U_{zk}\left(z;\gamma_k\right)=0. \tag{15}$$

Here we accept the following denotation:

$$\begin{cases}g\left(z;\gamma_k\right)=\dfrac{\gamma_k^2\rho\left(1-\nu^2\right)}{E\left(1+\delta_k^{(1)}+\mu_k\right)}\\ \mu_k=\dfrac{E\left(\nu+\delta_k^{(2)}\right)}{\rho R^2\left(1-\nu^2\right)\gamma_k^2-E\left(1+\delta_k^{(1)}\right)}\ .\end{cases} \tag{16}$$

The solution of equation (15) is representable in the form:

$$U_{zk}=c_{k1}\cdot ch\lambda_k z+c_{k2}\cdot sh\lambda_k z \tag{17}$$

where λ_k are the roots of the characteristic equation:

$$\lambda_k^2 + g\left(z;\, \gamma_k\right) = 0. \tag{18}$$

Note that λ_k is a complex expression and its values for real $\gamma_k > 0$ and $\gamma_k < 0$ are complexly conjugated. Then it suffices to give formulas for the parameter λ_k for $\gamma_k > 0$, that are of the form:

$$\begin{cases} \lambda_k = a_k + ib_k \\ a_k = \dfrac{-p\gamma_k}{m^2+p^2}\sqrt{\dfrac{\rho\left(1-\nu^2\right)}{E}}; \quad b_k = -\dfrac{m\gamma_k}{m^2+p^2}\sqrt{\dfrac{\rho\left(1-\nu^2\right)}{E}} \end{cases}. \tag{19}$$

In its turn, the parameters m and p are expressed in a complicated way by the two main values:

$$A^{(j)} = \sum_{n=0}^{\infty}\frac{(-1)^n \varphi^{n+1}\delta_n^{(j)}}{2^{n+2}\gamma_k^{(n+1)(1-\alpha)}}\cos\frac{\pi(n+1)(1-\alpha)}{2}$$

$$B^{(j)} = \sum_{n=0}^{\infty}\frac{(-1)^{n+1} \varphi^{n+1}\delta_n^{(j)}}{2^{n+2}\gamma_k^{(n+1)(1-\alpha)}}\sin\frac{\pi(n+1)(1-\alpha)}{2}.$$

The constants c_{k1} and c_{k2} of solution (17) are determined from boundary conditions (1) and (10). Satisfying boundary condition (10), we get the following relation the constants c_{k1} and c_{k2}:

$$c_{k2} = \frac{sh\lambda_k l}{ch\lambda_k l}c_{k1}. \tag{20}$$

Substituting this expression in representation of solution (17), we find:

$$U_{zk}\left(z;\, \gamma_k\right) = c_{k1}\cdot\frac{ch\lambda_k(z+l)}{ch\lambda_k l}. \tag{21}$$

Then on the base of the second formula of (11), the longitudinal displacement u_z will take the form:

$$u_z = \sum_{k=1}^{\infty}c_{k1}\cdot\frac{ch\lambda_k(z+l)}{ch\lambda_k l}e^{-i\gamma_k t}. \tag{22}$$

Boundary condition (1) is satisfied assuming in (22)

$$\begin{cases} c_{11} = c_{21} = \dfrac{u_0}{2}; \quad c_{k1} = 0; \quad k \ge 3; \\ \gamma_1 = -\gamma_2 = \omega. \end{cases} \tag{23}$$

In this case in series (22) there remain only the first two summands corresponding to two complexly conjugated values of the parameter λ_k, $k = 1$ and *2*. Transforming the sum of these two complexly conjugated first summands of series (22), we give it the form convenient for practical use:

$$u_z = u_0 A(z;\, \omega)\cdot\cos\left(\omega t - \varphi(z;\, \omega)\right). \tag{24}$$

This solution reflects the desired stationary vibrations with dimensionless amplitude of vibrations $A\left(z;\, \omega\right)$ and shift of vibrations phase $\varphi\left(z;\, \omega\right)$.

So, for the effort free right end of the cylindrical shell we get:

$$\begin{cases} u_z = u_0 A(l;\, \omega)\cdot\cos\left(\omega t - \varphi(l;\, \omega)\right) \\ A(l;\, \omega) = \left(\dfrac{\cos 4b_1 l + cha_1 l}{\cos 2b_1 l + ch2a_1 l}\right)^{\frac{1}{2}} \\ \varphi(l;\, \omega) = arctg\dfrac{sh3a_1 l\cdot\sin b_1 l + sha_1 l\cdot\sin 3b_1 l}{ch3a_1 l\cdot\cos b_1 l + cha_1 l\cdot\cos 3b_1 l} + \\ + \begin{cases} 0; & for \quad ch3a_1 l\cdot\cos b_1 l + cha_1 l\cdot\cos 3b_1 l > 0 \\ \pi; & for \quad ch3a_1 l\cdot\cos b_1 l + cha_1 l\cdot\cos 3b_1 l < 0 \end{cases} \end{cases} \tag{25}$$

In these formulas the parameters a_1 and b_1 are determined from formulas (19) for $\gamma_1 = \omega$.
On the base of formulas for dimensionless amplitude $A(l;\omega)$ the amplitude- frequency characteristics to 5 Htz was calculated for the following dimensional values of the problem parameters: $\rho = 1{,}65 q/cm^3;\ E = 2{,}2\cdot 10^5 kq/cm^2;\ a = 1;\ \alpha = 0{,}5;\ \nu = 0{,}3;\ l = 100 cm;$ $R = 5cm$. The constructed amplitude-frequency curve is given in fig. 1.
As it follows from the cited graph, on the interval under investigation two resonance frequencies are observed, and the value of the resonance amplitude decreases with the growth of the value of amplitude frequency.

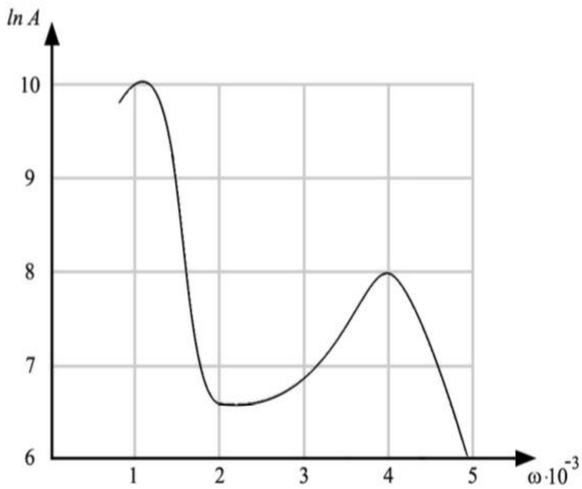

Fig.1 Amplitude-frequency curve for the free end of the shell.

IV. CONCLUSION

A closed analytic representation of the solution of a problem on stationary axial vibrations of a finite length hereditary elastic annular cylindrical shell subjected to the given vibration on one end and the external forces free second end is obtained. A formula was derived for amplitude-frequency dependence on the base of which a graph of an amplitude-frequency dependence is constructed. The character of resonance amplitude decrease with the growth of resonance frequency value is revealed.

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