

Mathematical And Simulation Models Of Lighting Automatic Control System In According With Natural Light

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Abstract: This article presents a mathematical model and a simulation model of lighting automatic control system in according with natural light to achieve standard illuminance and energy saving. The paper presents a method to develop mathematical models of actuators (LED driver and LED), sensor (light dependant resistor and analog-digital converter). PID controller with consideration of integral anti-windup to improve regulation quality of the system was introduced. Simulation model was constructed on Matlab-Simulink to verify mathematical model and to evaluate quality of regulation.

Keywords: Mathematical model, simulation model, lighting, automatic control, LED, LDR, matlab, simulink.

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I. INTRODUCTION

Lighting system consumes a large amount of electricity in the total energy demand of buildings. According to statistics, electricity used for lighting in the whole world accounts for about 19-20% of the total global electricity consumption [1,2]. Therefore, economical and efficient use of energy in lighting is of interest in many countries.

In recent years, LEDs (Light-emitting diode) are growing very fast. LED lights save 50-70% more energy than traditional lamps and the price is decreasing. LEDs also make dimming easier than traditional lamps [3]. LEDs open up many new possibilities in smart lighting.

Smart lighting to bring comfort, energy saving by automatic turning light on/off depending on occupation or automatic regulating light brightness depending on natural light. It is necessary to have an automatic control system in smart lighting. In [4], the system automatically counts people in / out of the room and turns on and off the lights. [5] presents an automatic headlight dimmer which uses a Light Dependent Resistor (LDR) sensor to dim the headlight and switch from high beam to low beam to prevent glare for the opposite car.

An lighting automatic control system consists of 3 parts: the controller; sensors; and actuators. Adaptive dimming control is used in [6] to control both lamps and blinds to take advantage of outdoor lighting, saving electricity while reducing heat and glare. In [7] the authors assessed the interaction between illuminated areas, built a MIMO model (multiple inputs, multiple outputs) from experiments. However, detailed mathematical model for each part of the control system has not been studied much.

In this paper, we will present methods to build mathematical models of sensors, actuators in lighting control system; methods to synthesize PID controller with consideration of integral anti-windup; a simulation model and evaluate results.

II. MATHEMATICAL MODELS

A typical lighting automatic control system is shown in Figure 1. The lighting environment considered in this system is consisted of a LED lamp, a working plane and natural light. A LED lamp is supplied from a LED driver, which is controlled by a microcontroller through PWM. The light from LED is mixed with natural light at a given point on the working plane. From automatic control point of view, natural light is load disturbance. Illuminance measured by an illuminance sensor (In this system, it is a light dependant resistor) is an analog value, which is converted to digital value by an analog-digital converter (ADC). PID controller compares this feedback value with the set point and regulates the LED lamp power through PWM and LED driver.

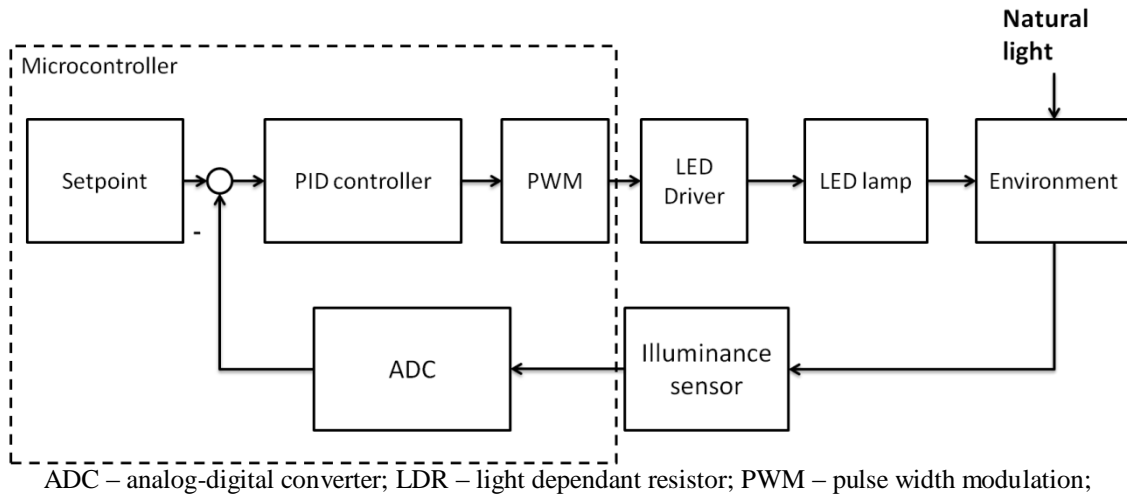


Figure 1. Lighting automatic control schematic in according to natural light

A. Mathematical model of actuator

Actuator consists of pulse width modulation module (PWM), LED driver and LED. LED driver supplies the LED lamp with output current, which is proportional to PWM control signal. The transfer function of PWM, LED driver can be presented as an amplifier with delay. However, these are semiconductor elements with very small delays: PWM about 10ns, LED driver about 200ns.

LED lamps have a typical current to luminous flux characteristic as shown in Figure 2 [8]. The current-luminous flux characteristic is linear, and given that current is proportional to control signal u , thus the luminous flux Φ generated by LED lamp is also proportional to output control signal u .

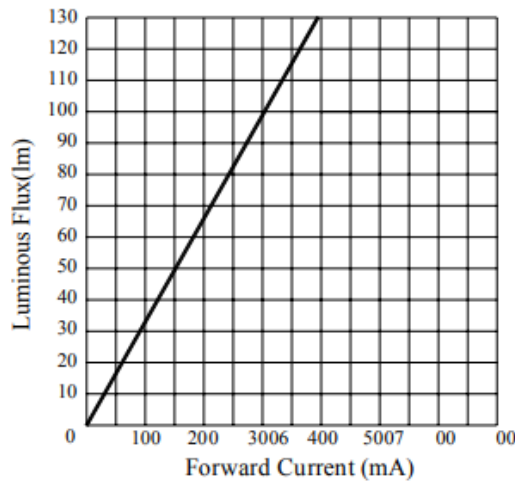


Figure 2. Typical current-luminous flux characteristic of a LED lamp

The lighting environment considered in this system is consisted of a LED lamp, a working plane and natural light.

Let S be a point source of luminous flux and P is a point on the working plane receiving the luminous radiation from S . If Let I be the luminous intensity of the radiation. The illuminance at the point P is:

$$L_a = \frac{I \cos \theta}{d^2} \tag{1}$$

Where θ – angle between normal to the surface and the luminous ray from S to P ; d – distance between S and P . Formula (1) can not be used directly since LED lamp can not be considered as a point source of light. However, we can consider a LED lamp as a set of point sources of light. The resulting illuminance in a given point of the given surface is the sum of illuminances due to each point source of light:

$$L_a = I \sum \frac{\cos \theta_i}{d_i^2} \tag{2}$$

Where θ_i – angle between normal to the surface and the luminous ray from light source number i ; d_i – distance between light source number i and point P .

Note that (1) shows a linear relationship between L_a and I . However, from the modelling perspective, relationship between illuminance and luminous flux Φ is of interest. From the definition of luminous intensity, which is equal to $d\Phi$ in a solid angle $d\omega$, it means luminous intensity I is proportional to overall luminous flux Φ . It means that L_a is proportional to overall luminous flux Φ .

To summarize, group of PWM, LED driver and LED in the considered lighting environment can be modelled as a proportional K1, of which input is control signal PWM u and output is illuminance L_a at point P on the given working plane.

B. Mathematical model of LDR and ADC

The LDR has a resistance-illuminance characteristic as shown in Figure 3 [8]. This relation is linear at the logarithmic scale, the equation reflecting the relationship between the resistance R of the LDR and the illuminance L is:

$$\frac{lgR - lgR_1}{lgL - lgL_1} = \frac{lgR - lgR_2}{lgL - lgL_2} \tag{3}$$

Where R – resistance of the LDR; L – illuminance on the surface of LDR; R_1 – resistance of LDR corresponding to illuminance L_1 ; R_2 – resistance of LDR corresponding to illuminance L_2 ;

The values of R_1, R_2, L_1, L_2 can be retrieved from the datasheet and be corrected with experiments.

Transform (3), move lgR to the left:

$$lgR = \frac{lgR_1 - lgR_2}{lgL_1 - lgL_2} lgL + \frac{lgR_2 lgL_1 - lgR_1 lgL_2}{lgL_1 - lgL_2} \tag{4}$$

Name the parameters:

$$k = \frac{lgR_1 - lgR_2}{lgL_1 - lgL_2}, m = \frac{lgR_2 lgL_1 - lgR_1 lgL_2}{lgL_1 - lgL_2}$$

Raise both side to 10^{th} power, we have the function of R of lgL :

$$R = 10^{k \cdot lgL + m} \tag{5}$$

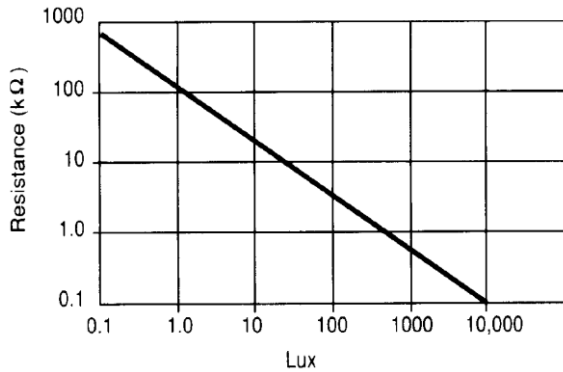


Figure 3. Resistance-illuminance characteristic of LDR

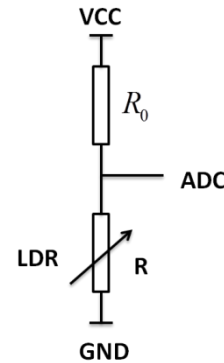


Figure 4. LDR in electrical circuit

Figure 4 shows circuit schematic of LDR. Voltage input to ADC pin of microcontroller depends on resistance of LDR as following:

$$V_{ADC} = V_{CC} \frac{R}{R_0 + R} \tag{6}$$

Where V_{CC} – voltage of supply power of the microcontroller; R_0 – resistor connected in series with LDR to divide voltage.

If ADC has a resolution of n-bit, the digital value that ADC converted from input voltage V_{ADC} is:

$$a = \frac{V_{ADC}}{V_{CC}} \cdot 2^n \tag{7}$$

Combine (6) and (7), we get:

$$\frac{a}{2^n} = \frac{R(L)}{R_0 + R(L)} \tag{8}$$

Instead of using direct value a read from ADC, we use $y = 1 - \frac{a}{2^n}$ as feedback signal to the PID. Substitute (8) to the expression of y:

$$y = 1 - \frac{R}{R_0 + R} = \frac{R_0}{R_0 + R} \tag{9}$$

Substitute (5) to (9), we get:

$$y = \frac{R_0}{R_0 + 10^{k \cdot \lg L + m}} \quad (10)$$

Use the parameters from datasheet of LDR: $L_1 = 1000 \text{ lx}$, $L_2 = 10 \text{ lx}$, $R_1 = 400\Omega$, $R_2 = 9k\Omega$; try the pullup resistor R_0 (in the circuit in Figure 4) with 4 different values: $R_0 = 1k\Omega$, $R_0 = 2,2k\Omega$, $R_0 = 3,3k\Omega$, $R_0 = 6,8k\Omega$; we get a curves $y(\lg L)$ as shown on Figure 5.

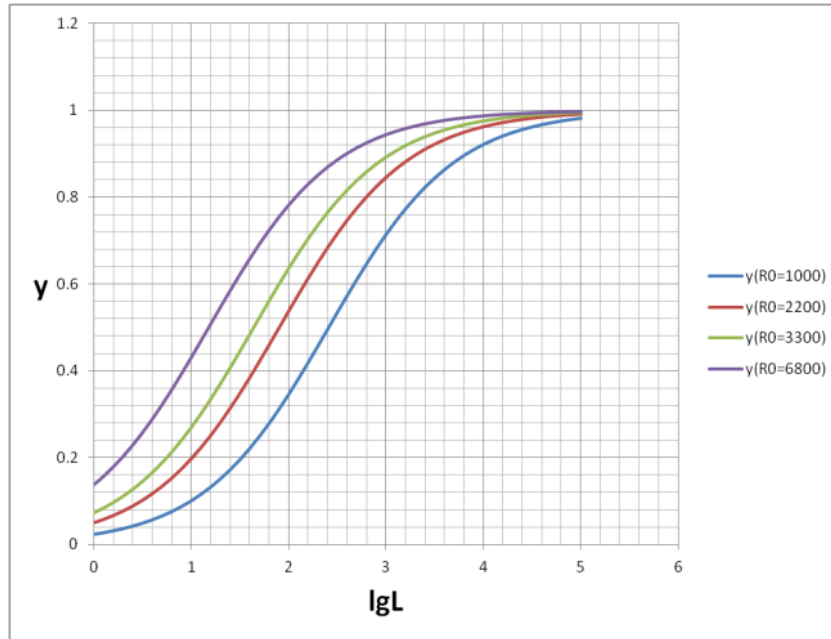


Figure 5. Relationship between y and $\lg L$ in cases $R_0 = 1k\Omega$, $R_0 = 2,2k\Omega$, $R_0 = 3,3k\Omega$, $R_0 = 6,8k\Omega$

According to the lighting standards for work place (TCVN 7114 - 1 : 2008 và ISO 8995 - 1 : 2002), minimum required illuminance is 300 lux, therefore we can consider only the range $10 \text{ lx} < L < 1000 \text{ lux}$, or $1 < \lg L < 3$. Figure 6 presents the linearization of curves $y(\lg L)$ with the equations of lines corresponding to each curve (Note: $x = \lg L$).

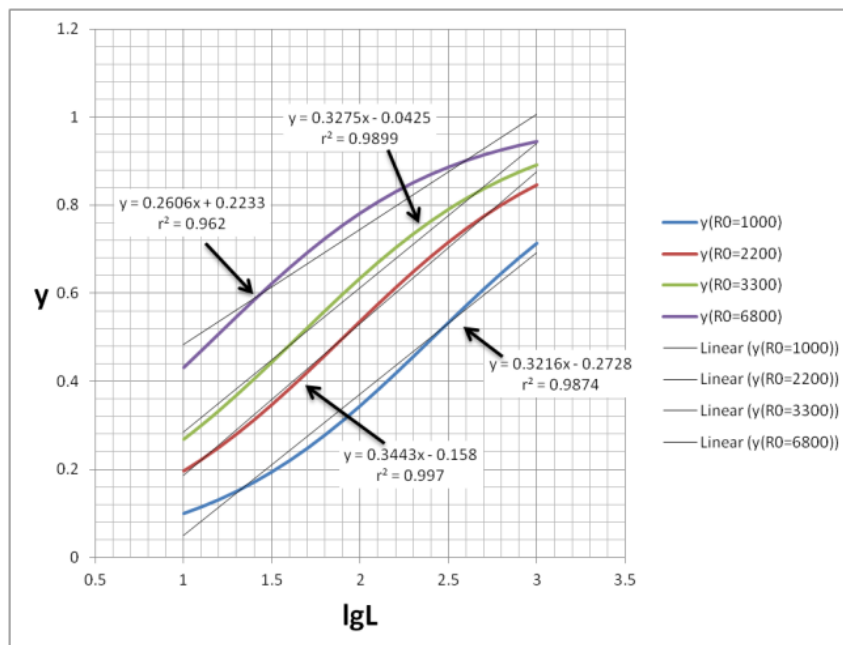


Figure 6. Curves y of $\lg L$ on the range $1 < \lg L < 3$ and linearized lines in 4 cases: $R_0 = 1k\Omega$, $R_0 = 2,2k\Omega$, $R_0 = 3,3k\Omega$, $R_0 = 6,8k\Omega$

Value r^2 on Figure 6 is coefficient of determination, which is a statistical measure of how well the regression predictions approximate the real data points. We can use this index to choose the most appropriate value of resistor R_0 , so that linearization yields the best result. With $R_0 = 2,2k\Omega$, r^2 is the biggest value of all, equals 0,997, and equation of linearized line is:

$$y = 0,3443lgL - 0,158 \quad (11)$$

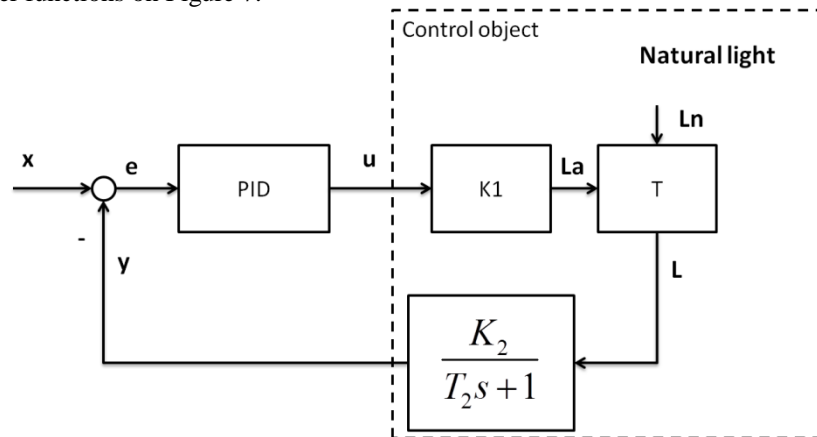
Thus, LDR and ADC can be modelled as a first-order lag

$$G_{sensor}(s) = \frac{K_2}{T_2s + 1} \quad (12)$$

Where $K_2 = 0,3443$ from linearized line; $T_2 = 0,2s$ is time constant, modelling the rise time/fall time of LDR and the conversion time of ADC.

C. PID controller

With the mathematical models of the sensor and actuator, the diagram on Figure 1 now can be presented as diagram of transfer functions on Figure 7.



K_1 – proportional part corresponding to a group of PWM, LED driver and LED; K_2, T_2 – parameters of first-order lag, modelling a group of LDR and ADC; T – process mixing natural light and artificial light; x – set point; y – feedback signal; u – control signal; La – illuminance by artificial light; Ln – illuminance by natural light; L – sum of illuminances;

Figure 7. Light automatic control system in form of transfer functions

The process T of mixing natural light and artificial light is a complicated process, which is depends on how the sensor is placed, the direction of natural light, the color of the ceiling surface, walls, floor, etc. However in our considered lighting environment, this process can be modelled as a simple sum. Natural light is considered to be a load disturbance.

Blocks K_1, K_2, T are grouped as control object with transfer function::

$$G_o(s) = K_1K_2 \quad (13)$$

PID controller's transfer function is as following:

$$G_{PID}(s) = K_p + \frac{K_I}{s} + K_Ds \quad (14)$$

Where K_p – proportional term; K_I – integral term; K_D – derivative term; s – Laplace operator.

In practice, derivative term is rarely used because it is very sensitive to noise and can worsen stability of the system. Thus, only PI controller is used, meaning $K_D = 0$:

$$G_{PID}(s) = K_p + \frac{K_I}{s} \quad (15)$$

Due to the ideal output being physically impossible (actuator's limitations), the output of the controller must being limited with a saturation block:

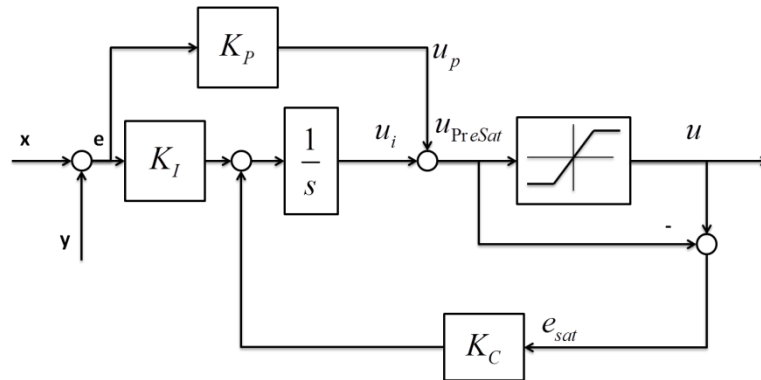
$$u_{sat} = \begin{cases} u & \text{n\u0119u } u_{min} \leq u \leq u_{max} \\ u_{min} & \text{n\u0119u } u < u_{min} \\ u_{max} & \text{n\u0119u } u > u_{max} \end{cases} \quad (16)$$

Where u_{min} – lower limit of the controller; u_{max} – upper limit of the controller.

Integral term in the controller helps to eliminate the static error of closed loop, but also brings integral windup when output saturation is used. The integral term accumulates a significant error, thus overshooting. When output saturation is used, overshoot can be smaller but the transient time is longer.

Therefore, it is necessary to have an integral anti-windup component in the PI controller. This component calculates the difference between the output before and after saturation block, multiply with a coefficient K_C and

add the result to the integral term output in the controller. The diagram of PI controller with saturation block and integral anti-windup component is presented on Figure 8.



u_p - proportional term; u_i – integral term; u_{preSat} – output signal before saturation block; u – output signal after saturation block; e_{sat} – error between signal before and after the saturation block; K_c – coefficient of integral anti-windup.

Figure 8. Diagram of PI controller with saturation block and integral anti-windup component

III. SIMULATION MODELS

Simulation model, which was implemented in Simulink, is shown on Figure 9.

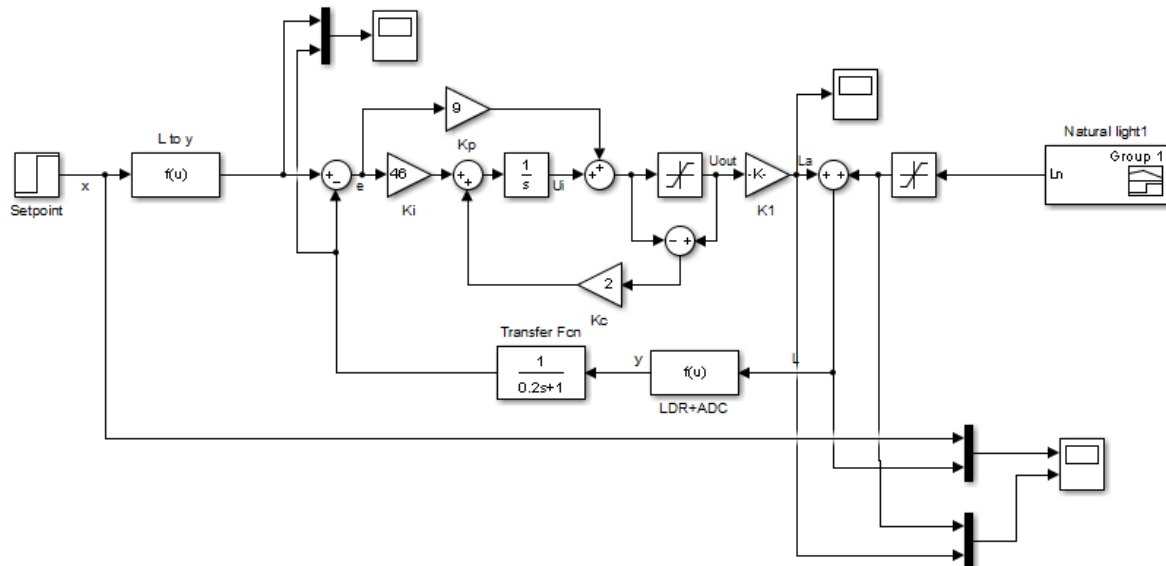


Figure 9. Simulation model of lighting automatic control system in according with natural light

Symbols on Figure 9 are as following:

- LDR+ADC: Light dependant resistor and analog-digital converter with static characteristic described by (10); followed by a first-order lag transfer function to simulate of rise/fall time of LDR;
- K1: PWM, LED driver and LED in considered lighting environment create illuminance L_a
- Natural light: a signal generator to simulate varying natural light with illuminance L_n .

Simulation script is as following:

- Setpoint is 300lx at $t=0s$ then increases to 800lx at $t=20s$;
- Natural light is 1lx at $t=0s$, increases to 400lx at $t=10s$, to 1000lx at $t=25s$, down to 200lx at $t=40s$, and to 1lx at $t=50s$

Simulation results are shown on Figure 10 (without integral anti-windup) and Figure 11 (with integral anti-windup). The performance indices of lighting automatic control system are shown on Table 1. In this system, there isn't an actuator that can do negative effect on the output (darken the environment) since only light is used. Darkening may only be done with equipment that can block light like blinds, curtains. When natural light is brighter than the setpoint, light is turned off, but measured value can not reach the setpoint. Thus, the performance indices are considered only in the conditions that natural light brightness is lower than the setpoint.

Simulation results show that the system with integral anti-windup react better to the changes in setpoint and natural light. Control system without anti-windup is saturated in many points, and doesn't react to the change of setpoint (at $t=20s$, Figure 10) or change of natural light (at $t=40s$, Figure 10).

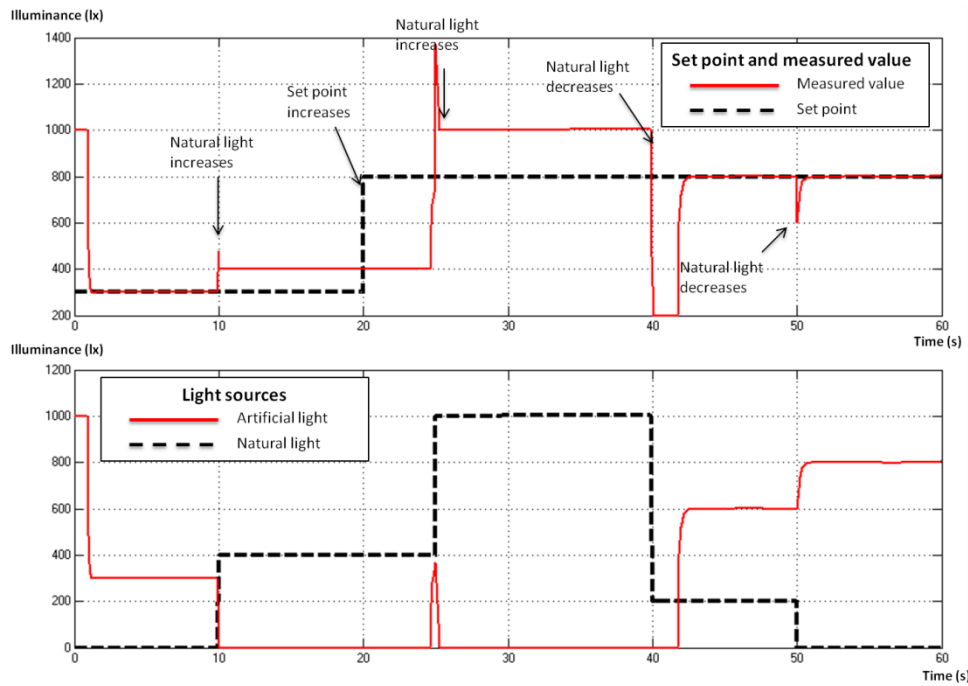


Figure 10. Simulation results of lighting automatic control system WITHOUT integral anti-windup

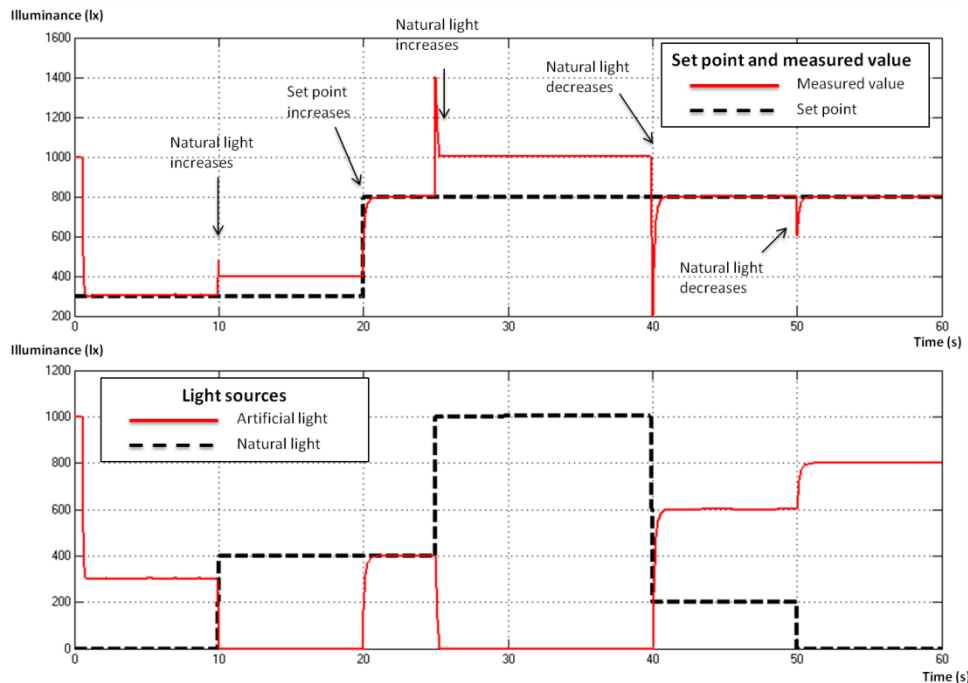


Figure 11. Simulation results of lighting automatic control system WITH integral anti-windup

Table 1. Comparison between control systems with and without integral anti-windup

Performance index	Without integral anti-windup	With integral anti-windup
Transient time	5s	0.5s
Overshoot	0%	0%
Static error	0%	0%

IV. CONCLUSION

The article described the construction of mathematical models of each equipment in the lighting automatic control system in according with natural light. The models were built based on the characteristics of actuator and sensor (LED lamp, LDR) and peripherals of microcontroller (PWM, ADC). The paper has proposed: (1) a formula to transform measured value from ADC before feeding to the controller; (2) a method to choose pullup resistor, which connected in serie with LDR, so that linearity of the measurement can be as high as possible.

A simulation model was implemented in Simulink. Simulation results show that the system with integral anti-windup react better to the changes in setpoint and natural light. The transient time is less than 0,5s, which is good enough for a lighting system.

This is the first part of a research project about smart lighting for classroom in Viet Nam. The project aims to develop a network of intelligent LED lamps, which allows to separately control each LED lamp in the classroom with consideration of natural light and the interaction with each other. The next step of the project is to develop mathematical and simulation models of this system and to implement the control algorithm as embedded software in microcontroller to assess the mathematical and simulation models.

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