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Economic Acceptance Sampling Plan Based on Truncated Life Tests for Generalized Exponential Distribution

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Abstract: - In this paper, an economic acceptance sampling plan has been developed for the truncated life time distribution. Here, it is assumed that the quality of a product cannot be treated as constant so it vary with the time and denoted as random variable. This random variable (or quality) follows a life time distribution known as generalized exponential distribution. We are keen interested in the average life (or median life) of the product to obtain the ratio of true average life and specified average life (σ/σ_0) with a predetermined time (t).The optimal cost model has been introduced to analyses SASP to find the various values of TC based on ATI. The purpose of this study is to find various measures for the given ratio (t/σ_0) so that we may obtain a best suitable (n, c, t) plan. The OC values and AOQ, ATI, and TC values are presented through table 1 & 2. Some of the results has been discussed to explain table values.

Keywords: - Average Total Inspection, Generalized Exponential Distribution, Operating Characteristic, Producer's Risk, Total Cost, Truncated Life Test.

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I. INTRODUCTION

Single acceptance sampling plan is an easy and important field of statistical quality control. It was discovered during second world war and later introduce by Dodge and Romig in the field of statistical quality control. Sometimes, we are bound to use sampling techniques rather than complete enumeration like testing of bullets, bombs, crackers, blood tests etc. Dodge emphasized that a sample should be collect randomly from the lot on the basis of sample information, we must take a decision regarding disposition of lot, whether to accept or reject. So, acceptance sampling plan lies between complete enumeration and no inspection at all. Survival of any product is based on its quality. The quality of the product can be achieved through statistical quality control. Acceptance sampling plan is an important tool to maintain the quality in product control methods. Quality is a variable characteristic and cannot be treated as constant. So the quality may be treated as random variable when it does so, life of the product is also a random variable. When the life testing follows any life time distribution, we measure the average life of the product, this testing has two stages one is time (t) and another is acceptance number (c). The life testing experiment will be truncated if the numbers of failures is in increased than the specified numbers of failures before the specified experiment time or no failure is recorded before time. The formulation of acceptance sampling plan is based on parametric model for life time distribution such that (n, c, t) and other parameter say σ0, where σ0 is the specified average (mean/median) life of the distribution and it behaves like quality parameter for the lifetime distribution under certain assumptions.It was first discussed by Epstein, B.(1954) truncated life tests for exponential distribution. Acceptance sampling plan for gamma, normal, log-normal distributions were discussed and provide tables by Gupta, S.S. and Groll, P.A.(1961) and Gupta, S.S.(1962). Important discussion is provided by Goode, H.P. and Kao, J.H.K.(1961) on sampling plans based on the weibull distribution. Later, Gupta, R.D. and Kundu, D.(1999) & (2007) and Balakrishnan, N., Leiva,V., and Lopez,J.(2007) truncated life tests based on generalized exponential distribution and generalized Birnbaum-Saunders distribution respectively. On the basis of these studies, it is observed that more effective life time distribution is generalized exponential distribution respectively. In many cases median is a suitable parameter to take as average life rather than mean. Generalized exponential is one of the case, where mean is not an appropriate parameter it is suggested by Gupta, S.S.(1962) that median provides better result instead of mean. It is observed that generalized exponential is a skewed distribution so it is more suitable to take median as the quality parameter.

1.2 NOTATIONS

n = Sample size
c = Acceptance number
d = Number of defectives

P = Shape parameter
σ = Scale parameter
α = Producer's risk
β = Consumer's risk
p = Failure Probability
L (p) = Probability of acceptance
P* = Minimum probability
σ_o = Specified life
Tc = Total cost
C_i = Inspection cost
C_f = Internal failure cost
C_o = Cost of an outgoing defective
D_d = Defective items defected
D_n = Defective items not defected
ATI = Average total inspection

1.3 GENERALIZED EXPONENTIAL DISTRIBUTION

The single acceptance sampling plan is very popular among the industry engineers due to its simplicity. A vast literature is available about the application of acceptance sampling plans in manufacturing the industry. Aslam, M. and Shabaz, M. Q. (2007) have studied economic reliability test plans using the generalized exponential distributions. Epstein, B. (1954) has studied truncated life tests in the exponential case. Gupta, R. D. and Kundu, D. (2003) have studied closeness between the gamma and generalized exponential distributions. Gupta, R.D. and Kundu, D. (2004) again studied discriminating between gamma and the generalized exponential distributions.

Ramaswamy, A.R. et al. (2012) proposed an extension of acceptance sampling plan based on generalized exponential distribution. A random variable T is said to have a generalized exponential distribution with parameters γ and σ, if its probability density function (pdf) is given by:

$$f_T(t) = \frac{\gamma}{\sigma} e^{-t/\sigma} (1 - e^{-t/\sigma})^{\gamma-1} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$t > 0,\ \gamma > 0,\ \sigma \geq 0$$

The corresponding cumulative density function (CDF) of the generalize exponential distribution is given by

$$F(t, \sigma) = (1 - e^{-t/\alpha})^{\gamma} \qquad \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Where σ is a scale parameter and γ is shape parameter. The shape parameter is known because if shape parameter is unknown then it is not easy to design an acceptance sampling plan. The scale parameter is characteristic parameter (or quality parameter).

In the present study is assumed that the probability distribution of a lifetime random variable is generalized experimental distribution with known shape parameters. The problem considered that of finding the minimum cost necessary to assure a certain average life when the life test is terminated at preassigned time (t) and when the observed number of failures does not exceed a given acceptance number.

For the protection of the consumer specification, average life of the product to test with a preassigned high probability. The acceptance of the lot is depends at the end of time t if the number of failure items does not exceed the given acceptance number c. The life test is terminated at the end of time t or $(c + 1)^{th}$ failure is observed whichever is earlier .

II. DESIGNED OF THE PROPOSED PLAN

It is known that single acceptance sampling plan is easy to understand and conventional plan. Most popular practice in life testing is to terminate a life test by a predetermined time (t) and observe the number of failure items. The decision is to establish with the specified mean life occurs with a given minimum probability p*. To accept the lot if the average life of sample items reach to the given time (t) and number of failure items does not exceeds the acceptance number (c) . But test may be terminate before time (t) if the number of failure items exceed the acceptance number.

We proposed a following SASP based on a truncated life test:
(1) Draw the sample size of (n) and put them on test.
(2) Accept the lot if there are no more than c failures i.e. $(r \leq c)$
(3) Maximum duration time t.
(4) A ratio $\frac{t}{\sigma_o}$, where σ_0 is the specified average life.

We fix the consumer's risk the probability of accepting a bad lot not to exceed $1 - p^*$. For a fixed p^* our sampling plan $(n, c, t / \sigma_0)$ is characterize there, we considered a large size lot. To ensure that $\sigma > \sigma_0$ must satisfy.

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p_i (1-p)^{n-i} \leq 1 - p^* \qquad \text{………………….(3)}$$

Where $p^* \in (0, 1)$ and $p = F(t, \sigma_0)$ is a monotonically increasing function of $\frac{t}{\sigma_o}$ and known as the probability of a failure observed during the time t.

If the number of failure items with in the time t is at most c , then from the above inequality with the confirmation of probability p^* that $F(t, \sigma) \leq F(t, \sigma_0)$ which implies $\sigma_0 \leq \sigma$.

The minimum samples sizes that may satisfy above inequality for

$t / \sigma_0 = 0.628 , 0.942 , 1.257 , 1.571 , 2.356 , 3.141 , 3.927 , 4.712$ with probability

$p^* = 0.75 , 0.90 , 0.95 , 0.99$

The values of $\left(\frac{t}{\sigma_o}\right)$ and p^* are taken in this work same as the corresponding values of Baklizi & E.I. Masri (2004) for Birnbaum Saunders models, Kantam et al . (2001) for log logistic model and Gupta & Groll (1961) for gamma distribution.

The ratio $\left(\frac{\sigma}{\sigma_o}\right)$ of the true average life σ to the specified life σ_0 is called the quality level of a product. Both the risks are associated with SASP. If a product is accepted as good item on sample basis information, it is a right decision.

To test, the Null hypothesis

$$H_0: \mu < \mu_0$$

Alternative hypothesis

$$H_1: \mu < \mu_0$$

Both producer and consumer desires protection forward quality of a product as well as the decision of rejection and acceptance of the lot.

$$P_a(p_1) = P(X \leq c / n, p_1) = 1 - \alpha$$
$$P_a(p_2) = P(X \leq c / n, p_2) = \beta$$

or

$$P_a(p_1) = L(p / \sigma / \sigma_0 = r_1) \geq 1 - \alpha \qquad \text{………………………..(4)}$$
$$P_a(p_2) = L(p / \sigma / \sigma_0 = r_2) \leq \beta \qquad \text{………………………..(5)}$$

Larger mean life ratio represents the higher quality of the product. Generally $r_2 = 1$ is adopted. p_1 and p_2 are failure probabilities of the producer's and consumer's respectively. .

$$P_a(p_1) = \sum_{i=0}^{c} \binom{n}{i} p_1^i (1-p_1)^{n-i} \qquad \text{……………………….(6)}$$

$$P_a(p_2) = \sum_{i=0}^{c} \binom{n}{i} p_2^i (1-p_2)^{n-i} \qquad \text{……………………....(7)}$$

III. OPERATING CHARECTERSTIC FUNCTION OF SAMPLING PLAN

The probability of accepting a lot may be attained through operating characteristics of the sampling plan $(n, c, t /\sigma_0)$. We found that operating characteristic function is an increasing function of σ. We can find the desired value of OC function from the table 1.

Probability of acceptance for sample first using r = 2 for the generalized exponential distribution that is placed in table 1. Let us assume the average life is at least 1000 hours with confidence 0.95. Let the acceptance number c = 1 with sample size n = 20.

IV. 4. ATI BASED TOTAL COST MODEL

Kumar, S. (2018) "Cost Optimization using Acceptance Sampling Plan: A Statistical Analysis with Single Sample" proposed a total cost model. We assume that the size of the lot is as large to use Binomial Distribution to find the probability of acceptance and rejection of the lot. Then

Average Outgoing Quality (AOQ) = $\dfrac{pP_a(p)(N-n)}{N}$ ………………….(8)

Average Total Inspection (ATI) = n + [1 – Pa (p)] (N – n) …………………..(9)

An economic sampling plan for total cost is as follows:

Min. $Tc = C_i\,ATI + C_f\,D_d + C_o.\,Dn$ ………………….(10)

Subject to $1 - P_a(AQL) \le \alpha$

$Pa(LTPD) \le \beta$

Where $D_d = n\,p + [1 - P_a(p)](N - n)\,p$

$D_n = P_a(p)(N - n)\,p$

D_d denote the defective items detected and D_n defective items not defected respectively.

The following are the various costs involved in the model:

Ci = Inspection cost

Cf = Failure cost before sale

Co = Failure cost post sale

V. TABLES

TABLE 1

Operating Characteristic values for the sampling plan (n, c, t / σ) when c = 1

p*	t/σ_0	n	σ/σ_0 2	4	6	8	10	12
0.75	0.628	25	0.6959	0.878808	0.941487	0.961732	0.981321	0.987238
	0.912	22	0.621346	0.854350	0.925987	0.953438	0.965358	0.985146
	1.257	20	0.60235	0.832943	0.919736	0.948253	0.953567	0.96321
	1.571	20	0.484340	0.782536	0.887548	0.934732	0.941395	0.953231
	2.356	20	0.291340	0.742553	0.800112	0.853078	0.872578	0.914313
	3.141	20	0.147836	0.475951	0.703587	0.700234	0.753201	0.871567
	3.927	20	0.059273	0.350532	0.582843	0.650563	0.712039	0.832345
	4.712	20	0.042730	0.282351	0.475356	0.591895	0.676156	0.745439
0.90	0.628	27	0.569348	0.861430	0.912896	0.961469	0.980012	0.981859
	0.192	22	0.612343	0.873420	0.943846	0.964852	0.981859	0.985846
	1.257	20	0.600129	0.859851	0.919829	0.958513	0.981596	0.978646
	1.571	20	0.485730	0.801034	0.876785	0.940058	0.961345	0.966934
	2.356	20	0.281340	0.632159	0.800212	0.864870	0.918329	0.928503
	3.141	20	0.149860	0.485816	0.685458	0.810587	0.856986	0.894305
	3.927	20	0.069140	0.390038	0.573456	0.712659	0.800156	0.815995
	4.712	20	0.041360	0.381530	0.486859	0.623495	0.735761	0.74596
0.95	0.628	30	0.517246	0.815180	0.894362	0.954328	0.963942	0.972138
	0.192	25	0.477349	0.802913	0.883218	0.943136	0.960094	0.963219
	1.257	28	0.383390	0.742314	0.861453	0.923394	0.953212	0.957938
	1.571	20	0.281410	0.705893	0.811321	0.843128	0.943193	0.950037
	2.356	20	0.281329	0.653925	0.791232	0.831963	0.932188	0.948218
	3.141	20	0.143868	0.462340	0.732126	0.821398	0.922249	0.932894
	3.927	20	0.069127	0.385120	0.681238	0.801682	0.914332	0.930713
	4.712	20	0.034720	0.281329	0.651912	0.782138	0.902387	0.929084
0.99	0.628	35	0.384303	0.742136	0.843122	0.872139	0.893213	0.962318
	0.192	28	0.281961	0.698938	0.811914	0.851321	0.881464	0.953842
	1.257	25	0.273214	0.688595	0.791262	0.842178	0.872315	0.943484
	1.571	22	0.245621	0.670156	0.762313	0.832001	0.870812	0.930802
	2.356	21	0.141561	0.521596	0.731294	0.813282	0.863987	0.929893
	3.141	20	0.129838	0.485814	0.682198	0.800827	0.851293	0.912898
	3.927	20	0.067159	0.390923	0.652283	0.783292	0.832148	0.907866
	4.712	20	0.031621	0.295930	0.519684	0.753143	0.822096	0.908137

TABLE 2

OC, AOQ, ATI and Total Cost values for the sampling plan (n, c, t / σ) when c = 1

P	t/σ_0	n	D_d	D_n	P_a	AOQ	ATI	TC
0.75	0.628	25	2.54	1.42	0.1514	0.0014	840.78	942.69
	0.192	22	2.23	2.12	0.1781	0.0021	852.48	939.78
	1.257	20	2.10	3.21	0.2081	0.0032	837.20	921.32
	1.571	20	1.95	3.45	0.2426	0.0034	807.02	885.46
	2.356	20	1.90	4.26	0.3698	0.0042	734.98	825.39
	3.141	20	1.82	5.43	0.4740	0.0054	651.91	798.20
	3.927	20	1.25	6.23	0.4852	0.0062	542.58	723.39
	4.712	20	0.98	8.42	0.5603	0.0084	462.39	685.32
0.90	0.628	27	2.72	1.32	0.1829	0.0013	838.71	935.38
	0.192	22	2.21	2.23	0.2131	0.0022	849.32	932.46
	1.257	20	2.09	2.90	0.2620	0.0029	825.36	915.37
	1.571	20	1.91	3.12	0.3102	0.0031	810.02	898.56
	2.356	20	1.85	3.82	0.3529	0.0038	770.02	866.32
	3.141	20	1.78	4.15	0.4241	0.0041	725.61	835.25
	3.927	20	1.69	5.02	0.5237	0.0050	715.29	822.39
	4.712	20	1.64	6.25	0.6325	0.0062	623.32	785.63
0.95	0.628	30	2.81	1.25	0.1935	0.0012	825.64	918.35
	0.192	25	2.69	2.32	0.2643	0.0023	810.29	898.56
	1.257	23	2.23	2.62	0.3295	0.0026	778.35	868.32
	1.571	20	1.82	3.06	0.3942	0.0030	745.39	855.64
	2.356	20	1.65	3.46	0.4642	0.0034	676.25	799.33
	3.141	20	1.53	3.95	0.5932	0.0039	630.35	788.57
	3.927	20	1.39	4.25	0.6843	0.0042	615.23	762.64
	4.712	20	1.27	5.62	0.7523	0.0056	585.31	745.69
0.99	0.628	35	2.92	1.02	0.2013	0.0010	810.21	898.56
	0.192	28	2.92	2.05	0.2842	0.0020	775.36	869.23
	1.257	25	2.34	2.95	0.3675	0.0029	735.45	826.93
	1.571	22	2.13	3.46	0.4038	0.0035	686.39	801.23
	2.356	21	1.86	3.96	0.4945	0.0040	653.46	775.63
	3.141	20	1.73	4.65	0.6238	0.0046	625.32	745.45
	3.927	20	1.64	5.95	0.7239	0.0060	598.63	735.23
	4.712	20	1.52	6.86	0.8432	0.0068	578.53	726.36

VI. DISCUSSION

The OC function values for the proposed life testing sampling plan for the time truncated acceptance sampling plan are calculated from table 1 for various given values of (t/σ_0) and P* with acceptance number c=1 are as follows:

σ/σ_0	2	4	6	8	10	12
OC	0.384303	0.742136	0.843128	0.872139	0.893214	0.962315

This shows that if the true average life is twice the specified average life ($\sigma/\sigma_0=2$) the producer's risk is about to 0.615697, and the producer's risk is almost equal to zero when the ratio is greater than or equal to 6 times i.e. ($\sigma/\sigma_0 \geq 6$).

From the TABLE 2, we can get the values of AOQ, ATI and TC for the various choices of the ratio of predetermined time (t) and specified average life (σ_0) such that (t/σ_0) increases then the TC decreases. The producer's risk may not exceed 0.05. Thus for c=1, t/σ_0 =1.571, n= 20, D_d =1.82, D_n=3.06, AOQ=0.0030, ATI= 745.39 and TC= 855.64.

VII. CONCLUSION

In this paper, we have proposed a single acceptance sampling plans based on truncated generalized exponential life time distribution model. The sampling plan is also touch the cost criteria which provide various choices to find a best suitable plan for quality control engineer. We have observed that the probability of acceptance decreases for fixed acceptance number with increase of sample size. The probability of acceptance decreases with increase of ratio$\left(\frac{t}{\sigma}\right)$. The total cost provides a help to take a good decision so, it may be concluded as the proposed sampling plan can be easy to operate for practitioners. This work may be extended for double sampling plan.

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