Perturbation Analysis of Heat and Mass Transfer in MHD Micropolar Fluid in Presence of Thermal Radiation

Dr. Mamta Goyal¹, Gyanchand Gurjar²
¹Associate Professor Department of Mathematical Sciences University of Rajasthan, Jaipur, Rajasthan, India
²Department of Mathematical Sciences University of Rajasthan, Jaipur, Rajasthan, India

Abstract: This paper deals with the perturbation analysis of mass and heat transfer of an electrically conducting micropolar fluid over an infinite moving permeable plate in the presence of transverse magnetic field. Analytical solutions are obtained for the governing equations. The effects of various physical parameters are presented through graph.

Key words: Micropolar fluid, Heat and mass transfer, MHD, Thermal radiation.

I. INTRODUCTION

The concept micropolar fluid deals with a class of fluids that exhibits certain microscopic effects arising from the micro motions of the fluid elements. These fluids contain dilute suspension of rigid macromolecules with individual motion that support stress body moments are influenced by spin inertia. Micropolar fluids are those which contain micro-constituents that can undergo rotation which can affect the hydrodynamics of the flow so that it can be distinctly non-Newtonian. Physically micropolar fluid is entitled the fluids containing of bar like elements and anisotropic fluids, for example: animal blood, liquid crystals which are made up of dump bell molecules. Eringen [1] was presented the common theory of micropolar fluids. This theory is to be effective an analysing behaviour of non-Newtonian fluids. The theory of micropolar fluids and derived the constitutive law for fluids with microstructure enlarge by Eringen [2] and [3].

The study of heat and mass transfer flow of an electrically conducting micropolar fluid past a heated surface under the impact of a magnetic field has concerned many researchers due to its vast application in many engineering problems such as MHD generators, nuclear reactors, geothermal energy, extractions and the boundary layer control in the field of aero dynamics.

Heat and mass transfer in magnetohydrodynamic flow of micropolar fluid on a circular cylinder with uniform heat and mass flux studied by mansour et al. [4]. They found that the micropolar fluid tends to reduce the friction and heat transfer rate as compared to Newtonian fluids. Ishak et al. [5] examined the MHD flow of a micropolar fluid towards a stagnation point on a vertical surface. The MHD boundary-layer flow of a micropolar fluid past a wedge with constant wall heat flux investigated by Ishak et al. [6], Sparrow and Cess [7] examined the outcome of magnetic field on free convective heat transfer. The effect of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al. [8]. Soundalgekar et al. [9] investigated the MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field.

The effect of radiation are of vital importance recent developments in hypersonic flights, missile re-entry, rocket combustion chambers, power plants for inter planetary flight and gas cooled nuclear reactors, have focused attention on thermal radiation as a mode of energy transfer and emphasized the need for improved understanding of radiative transfer in these process. Ishak [10] was investigated the thermal boundary layer flow over a stretching sheet in a micropolar fluid with radiation. The effect of suction/injection on a micropolar fluid past a continuously moving plate in the presence of radiation studied by El-Arabawy [11]. Raptis [12] studied the micropolar fluid flow past a continuously moving plate in the presence of radiation. The effect of variable viscosity on magneto-micropolar fluid flow in the presence of radiation discussed by Elbarbary and Elgazery [13]. Ogulu [14] studied the oscillating plate-temperature flow of apolar fluid past a vertical porous plate in the presence of couple stresses and radiation. Heat transfer of a micropolar fluid in presence of radiation presented by Perdikis and raptis [15]. Rahman and Sattar [16] were studied the transient convective heat transfer flow of a micropolar fluid past a continuously moving vertical porous plate with time dependent suction in the presence of radiation.
II. MATHEMATICAL FORMULATION

Mixed convective two-dimensional electrically conducting micropolar fluid over an infinite vertical moving plate in the presence of thermal radiation. Plate is placed in \( x \) direction and \( y \)-axis is measured normal to it. Let \( u \) and \( v \) be the velocity components along \( x \)-axis and \( y \)-axis respectively. An external magnetic field \( B_0 \) is applied normal to the plate. It is assumed that the magnetic field is of small intensity so that the induced magnetic field is negligible in comparison to the applied magnetic field. Under the above assumptions the governing equations are:

\[
\begin{align*}
\frac{\partial u^*}{\partial t} + v^* \frac{\partial u^*}{\partial y} &= \frac{(\mu + k)}{\rho} \frac{\partial^2 u^*}{\partial y^2} + 2 \frac{k}{\rho} \frac{\partial w^*}{\partial y} + gB_0 (T - T_w) - 2 \frac{\partial w^*}{\partial y} u^* + g \beta \gamma (T - T_w) - \frac{2}{\rho} \frac{\partial^2 u^*}{\partial y^2}; \quad \text{Equation of momentum} \quad (2)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial w^*}{\partial t} + v^* \frac{\partial w^*}{\partial y} &= \frac{\gamma}{\rho j} \frac{\partial^2 u^*}{\partial y^2}; \quad \text{Equation of angular momentum} \quad (3)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial q^*}{\partial t} + v^* \frac{\partial q^*}{\partial y} &= \frac{\alpha}{\rho j} \frac{\partial^2 T^*}{\partial y^2} - \frac{\partial q^*}{\partial y}; \quad \text{Equation of Energy} \quad (4)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial c^*}{\partial t} + v^* \frac{\partial c^*}{\partial y} &= D \frac{\partial^2 c^*}{\partial y^2}; \quad \text{Equation of Mass Transfer} \quad (5)
\end{align*}
\]

Where \( v \) is the kinematic viscosity, \( \kappa \) microrotation viscosity, \( g \) is acceleration due to gravity, \( \beta \gamma \) is the coefficient of thermal expansion, \( \mu \) is the coefficient of viscosity, \( B_0 \) is constant magnetic field applied along \( y \)-axis, \( \alpha \) is the thermal conductivity, \( C_p \) is the specific heat at constant pressure, \( q^* \) is radiative heat flux, \( C_m \) is free stream concentration and \( T_m \) is free stream temperature. \( \gamma \) is spin gradient viscosity, \( j \) is microrotation per unit mass, \( D \) is chemical molecular diffusivity.

The boundary conditions are:

\[
\begin{align*}
&u^* = u^*_y, \quad T = T_w + \epsilon (T_m - T_w) e^{\gamma^* y}; \quad \omega^* = -n_1 \frac{\partial u^*}{\partial y}; \quad C = C_m + \epsilon (C_m - C_0) e^{\gamma^* y}; \quad \text{at} \quad y^* = 0...(6)
\end{align*}
\]

\[
\begin{align*}
u^* \rightarrow 0; \quad \omega^* \rightarrow 0; \quad T \rightarrow T_m; \quad C \rightarrow C_m; \quad \text{as} \quad y^* \rightarrow \infty \quad (7)
\end{align*}
\]

We note that \( n_1 \) is a constant such \( 0 \leq n_1 \leq 1 \). The case when \( n_1 = 0 \), is called strong concentration which indicates that no microrotation near the wall. In case \( n_1 = 0.5 \), it indicates that the vanishing of anti-symmetric part of the stress tensor and denote weak concentration and case \( n_1 = 1 \) is used for the modelling of turbulent boundary layer flows.

\[
\gamma = (\mu + \frac{k}{2}) j = (1 + \frac{K}{2}) j, \quad \text{where} \quad K = \frac{k}{2} \text{is the micro polar or material parameter and} \quad j = \frac{u}{a} \text{as reference length.}
\]

The total spin \( \omega^* \) reduces to the angular velocity.

From the Eq. (1) the suction velocity normal to the plate is a function of time only which is in the form:

\[
\nu^* = -(1 + \epsilon A e^{\gamma^* y}) V_0 \quad (8)
\]

Where \( A \) is a real positive constant and \( V_0 \) is a scale of suction velocity which has non-zero positive constant.

Using Roseland’s approximation for radiation, we obtain \( q^* = -\left( \frac{4}{3} \frac{\sigma}{K_2} \frac{\partial T}{\partial y} \right) \), where \( \sigma^* \) is the Stefan-boltzmann constant, \( K_2^* \) is the absorption coefficient. We assume that the temperature variation within the flow is such that \( T^* \) may be expand in a Taylor’s series. Expanding \( T^* \) about \( T_m \) and neglecting higher order term we get,

\[
T^* = 4T_{m}^{-3} \frac{3}{7} T^4
\]

III. PROBLEM SOLUTION

We use the following dimensionless variables:

\[
\begin{align*}
u^* &= U_0 u; \quad v^* = V_0 v; \quad y^* = \frac{y}{V_0}; \quad u^*_y = U_0 U_y; \quad \omega^* = \frac{U_0 V_0}{v} \omega; \quad t^* = \frac{v}{V_0^2} t; \quad n^* = \frac{V_0^2}{v} n \theta = \frac{T - T_m}{T_w - T_m}; \quad \phi = \frac{C - C_0}{C_m - C_0}; \quad \text{Pr} = \frac{V_0}{u}; \quad Sc = \frac{V_0}{D}; \quad M = \frac{U_0 B_0^2}{V_0^2} ; \quad Gr = \frac{V_0^2 \beta (T_m - T_w)}{U_0 V_0^2}; \quad \eta = \frac{2}{2 + \kappa}; \quad N_t = \frac{4 \sigma^* T_{m}^{3}}{\kappa K_2^*}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial u^*}{\partial t} + (1 + \epsilon A e^{\gamma^* y}) \frac{\partial u^*}{\partial y} = (1 + K) \frac{\partial^2 u^*}{\partial y^2} + 2K \frac{\partial w^*}{\partial y} + \text{Gr} \theta - Mu \quad \ldots (10)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial w^*}{\partial t} + (1 + \epsilon A e^{\gamma^* y}) \frac{\partial w^*}{\partial y} - \frac{1}{n} \frac{\partial^2 u^*}{\partial y^2} \ldots (11)
\end{align*}
\]
The boundary conditions (6) and (7) becomes
\[ u = U_p, \quad \omega = -n_i \frac{du}{dy}, \quad \theta = 1 + e^{-\alpha t}, \quad \phi = 1 + e^{-\alpha t} \quad \text{at} \quad y = 0 \]
\[ u \rightarrow 0, \quad \omega \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \]
To solve Eqs. (10)-(13) subject to the boundary conditions Eqs. (14,15) we may use the following linear transformation for small value of \( \epsilon \)
\[ u(y,t) = u_0(y) + \epsilon u_1(y,t) + o(\epsilon^2) \quad \text{... (16)} \]
\[ \omega(y,t) = \omega_0(y) + \epsilon \omega_1(y,t) + o(\epsilon^2) \quad \text{... (17)} \]
\[ \theta(y,t) = \theta_0(y) + \epsilon \theta_1(y,t) + o(\epsilon^2) \quad \text{... (18)} \]
\[ \phi(y,t) = \phi_0(y) + \epsilon \phi_1(y,t) + o(\epsilon^2) \quad \text{... (19)} \]
After substituting Eq. (16)-(19) into Eq. (10) – (13), we have
\[ (1+K)u_0 + \epsilon u_1 = -2Ku_0 - Pr \theta_0 \quad \text{... (20)} \]
\[ (1+K)u_1 + \epsilon u_2 = -2Ku_1 - Gr \theta_1 - Au \quad \text{... (21)} \]
\[ \omega_0 + \epsilon \omega_1 = 0 \quad \text{... (22)} \]
\[ \omega_1 + \epsilon \omega_2 = 0 \quad \text{... (23)} \]
\[ (3+4Nr) \phi_0 + \epsilon \phi_1 = 0 \quad \text{... (24)} \]
\[ (3+4Nr) \phi_1 + 3Pr \phi_2 = 0 \quad \text{... (25)} \]
\[ \phi_0 + \epsilon \phi_1 = 0 \quad \text{... (26)} \]
\[ \phi_1 + \epsilon \phi_2 = 0 \quad \text{... (27)} \]
With the following boundary conditions:
\[ u_0 = U_p, u_1 = 0, \omega_0 = -n_i u_0, \omega_1 = -n_i u_1, \theta_0 = 1, \quad \theta_1 = 1, \quad \phi_0 = 1, \quad \phi_1 = 0 \quad \text{as} \quad y \rightarrow \infty \quad \text{(28)} \]
To solve the nonlinear coupled Eqs. (20) – (27) with satisfying boundary condition (28) we find as
\[ u_0(y) = a_0 e^{-2\alpha y} + a_1 e^{-\alpha y} + a_2 e^{-\alpha y} \quad \text{... (29)} \]
\[ u_1(y) = a_0 e^{-2\alpha y} + a_1 e^{-2\alpha y} + a_2 e^{-\alpha y} + a_3 e^{-2\alpha y} \quad \text{... (30)} \]
\[ \omega_0(y) = a_4 e^{-\alpha y} \quad \text{... (31)} \]
\[ \omega_1(y) = a_5 e^{-\alpha y} \quad \text{... (32)} \]
\[ \theta_0(y) = a_6 e^{-\alpha y} \quad \text{... (33)} \]
\[ \theta_1(y) = a_7 e^{-\alpha y} \quad \text{... (34)} \]
\[ \phi_0(y) = a_8 e^{-\alpha y} \quad \text{... (35)} \]
\[ \phi_1(y) = a_9 e^{-\alpha y} \quad \text{... (36)} \]
Where exponential indices and coefficients are given in appendix.

**IV. RESULTS AND DISCUSSION**

Effects of physical parameters on velocity, micro-rotation, temperature and concentration profiles are shown through figure 1 to 11. The effects of the main controlling parameters as they appear in the governing equations are discussed in the current section. In order to verify the numerical result originate, they are found to be in a good arrangement to previously published paper. The velocity profile \( u(y) \) is planned in **Figure 1** for different values of the thermal Grashof number Gr when \( K = 0.5, Pr = 1, \lambda = 0.1, Nr = 0.03, M = 2, t = 1 \) and \( Sc = 2 \). This figure shows that when we increase the parameter Gr then the velocity profile \( u(y) \) is increases. **Figure 2** show that the effect of micropolar parameter \( K \) on velocity profile \( u(y) \) when other parameter are fixed. This figure show that increasing the values of the micropolar parameter then the velocity profile is increase. Also the same effect show that the variation of microrotation parameter \( n_i \) by **Figure 4**. **Figure 3** shows that velocity profile decreasing for magnetic field parameter \( M \) is increasing. **Figure 5** show that the different value of thermal grashof number Gr then the angular velocity profile is increases. **Figure 6** represents the angular velocity distribution to increase the micropolar parameter \( K \) then increase it but reverse effect represents after a special value of \( y \). **Figure 7** shows that the increasing the values of \( n_i \) and \( M \) then the angular velocity is decreasing. **Figure 9** represents the temperature profile with respect to \( Nc \). **Figure 10** represents temperature profile is decreases when increasing \( Pr \). Concentration profile decreases when increasing \( Sc \) in **figure 11**.
Figure 1 velocity profiles for different values of thermal Grashof number $Gr$ while $Pr=1$, $M=2$, $K=0.5$, $Sc=2$, $N_r=0.03$, $t=1$ and $Sc=2$

Figure 2 velocity profiles for different values of micropolar parameter $K$ while $Pr=1$, $M=2$, $Sc=2$, $A=0.1$, $N_r=0.03$, $t=1$ and $Gr=0$

Figure 3 velocity profiles for different values of magnetic parameter $M$ while $Pr=1$, $Sc=2$, $K=0.5$, $A=0.1$, $N_r=0.03$, $t=1$ and $Gr=0$
Perturbation Analysis of Heat and Mass Transfer in MHD Micropolar Fluid in Presence of Thermal...

**Figure 4** velocity profiles for different values of microrotation parameter $m_1$ while $Pr=1$, $M=2$, $K=0.5$, $Sc=2$, $N_r=0.03$, $t=1$ and $Gr=0$

**Figure 5** angular velocity profiles for different values of thermal grashof number $Gr$ while $Pr=1$, $M=2$, $K=0.5$, $Sc=2$, $N_r=0.03$, $t=1$ and $A=0.1$

**Figure 6** angular velocity profiles for different values of micropolar parameter $K$ while $Pr=1$, $M=2$, $A=0.1$, $Sc=2$, $N_r=0.03$, $t=1$ and $Gr=0$
Perturbation Analysis of Heat and Mass Transfer in MHD Micropolar Fluid in Presence of Thermal...

**Figure 7** angular velocity profiles for different values of magnetic parameter \( M \) while \( \text{Pr}=1, \ A=0.1, \ K=0.5, \ Sc=2, \ \text{Nr}=0.03, \ t=1 \) and \( \text{Gr}=0 \)

**Figure 8** angular velocity profiles for different values of microrotation parameter \( n_1 \) while \( \text{Pr}=1, \ M=2, \ K=0.5, \ Sc=2, \ \text{Nr}=0.03, \ t=1 \) and \( \text{Gr}=0 \)

**Figure 9** temperature profiles for different values of \( \text{Nr} \) while \( \text{Pr}=1, \ M=2, \ K=0.5, \ A=0.1, \ Sc=2, \ t=1 \) and \( \text{Gr}=0 \)
Perturbation Analysis of Heat and Mass Transfer in MHD Micropolar Fluid in Presence of Thermal ...

Figure 10 temperature profiles for different values of Pr while M=2, K=0.5, Sc=2, Nr=0.03, A=0.1, t=1 and Gr=0

Figure 11 concentration profiles for different values of smidt number Sc while Pr=1, M=2, K=0.5, A=0.1, Nr=0.03, t=1 and Gr=0

Appendix

\[ Nc = 1 + \frac{4}{3} N_r \]

\[ m_2 = \frac{2}{1 + \sqrt{1 + 4M(1+K)}} \]

\[ m_4 = \frac{2}{1 + \sqrt{1 + 4M(1+K)}} \]

\[ a_1 = -n_1 \eta_1 \]

\[ a_2 = -n_1 \eta_1 + \frac{A \eta_1 a_1}{\eta} \]

\[ a_3 = 1 + \frac{A \eta_1^2}{\eta Nc} \]

\[ a_4 = \frac{2K a_1 \eta}{(1+K) \eta^2 - \eta - M} \]

\[ m_3 = \frac{Pr + \sqrt{Pr^2 + 4nPr Nc}}{2 Nc} \]

\[ a_5 = \frac{(1+K) \eta^2 - \eta - M}{Pr + \frac{Pr Nc}{2}} \]

\[ a_6 = U_p - a_4 - a_5 \]

\[ a_7 = \frac{2K a_3 + m_1}{(1+K)(m_1 - m_1)(M+n)} \]

\[ a_8 = \frac{n A a_4 \eta^2 - 2K a_5 A \eta^2}{n[1 + (1+K) \eta^2 - \eta - (M+n)]} \]

\[ a_9 = \frac{m_3 - m_3}{(1+K)(m_2 - m_2)(M+n)} \]

\[ a_{10} = \frac{n A a_6 \eta^2 - 2K a_5 A \eta^2}{n[1 + (1+K) \eta^2 - \eta - (M+n)]} \]

\[ a_{11} = \frac{A \eta_1^2 m_4}{(1+K) \eta_1^2 - m_4 - m_4(M+n)} \]

\[ a_{12} = -a_7 - a_8 - a_9 - a_{10} - a_{11} \]
REFERENCES
