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Perturbation Analysis of Heat and Mass Transfer in MHD Micropolar Fluid in Presence of Thermal Radiation

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Abstract: This paper deals with the perturbation analysis of mass and heat transfer of an electrically conducting micropolar fluid over an infinite moving permeable plate in the presence of transverse magnetic field. Analytical solutions are obtained for the governing equations. The effects of various physical parameters are presented through graph.

Key words: Micropolar fluid, Heat and mass transfer, MHD, Thermal radiation.

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I. INTRODUCTION

The concept micropolar fluid deals with a class of fluids that exhibits certain microscopic effects arising from the micro motions of the fluid elements. These fluids contain dilute suspension of rigid macromolecules with individual motion that support stress body moments are influenced by spin inertia. Micropolar fluids are those which contain micro-constituents that can undergo rotation which can affect the hydrodynamics of the flow so that it can be distinctly non-Newtonian. Physically micropolar fluid is entitled the fluids containing of bar like elements and anisotropic fluids, for example: animal blood, liquid crystals which are made up of dump bell molecules. Eringen [1] was presented the common theory of micropolar fluids. This theory is to be effective an analysing behaviour of non-Newtonian fluids. The theory of micropolar fluids and derived the constitutive law for fluids with microstructure enlarge by Eringen [2] and [3].

The study of heat and mass transfer flow of an electrically conducting micropolar fluid past a heated surface under the impact of a magnetic field has concerned many researchers due to its vast application in many engineering problems such as MHD generators, nuclear reactors, geothermal energy, extractions and the boundary layer control in the field of aero dynamics.

Heat and mass transfer in magnetohydrodynamic flow of micropolar fluid on a circular cylinder with uniform heat and mass flux studied by mansour et al. [4]. They found that the micropolar fluid tends to reduce the friction and heat transfer rate as compared to Newtonian fluids. Ishak et al.[5] examined the MHD flow of a micropolar fluid towards a stagnation point on a vertical surface. The MHD boundary-layer flow of a micropolar fluid past a wedge with constant wall heat flux investigated by Ishak et al.[6].Sparrow and Cess[7] examined the outcome of magnetic field on free convective heat transfer. The effect of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al.[8]. Soundalgekar et al.[9] investigated the MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field.

The effect of radiation are of vital importance recent developments in hypersonic flights, missile re-entry, rocket combustion chambers, power plants for inter planetary flight and gas cooled nuclear reactors, have focusedattention on thermal radiation as a mode of energy transfer and emphasized the need for improved understanding of radiative transfer in these process. Ishak [10] was investigated the thermal boundary layer flow over a stretching sheet in a micropolar fluid with radiation. The effect of suction/injection on a micropolar fluid past a continuously moving plate in the presence of radiation studied by El-Arabawy[11]. Raptis[12] studied the micropolar fluid flow past a continuously moving plate in the presence of radiation. The effect of variable viscosity on magneto-micropolar fluid flow in the presence of radiation discussed by Elbarbary and Elgazery[13]. Ogulu[14] studied the oscillating plate-temperature flow of apolar fluid past a vertical porous plate in the presence of couple stresses and radiation. Heat transfer of a micropolar fluid in presence of radiation presented by Perdikis and raptis[15]. Rahman and Sattar [16] were studied the transient convective heat transfer flow of a micropolar fluid past a continuously moving vertical porous plate with time dependent suction in the presence of radiation.

II. MATHEMATICAL FORMULATION

Mixed convective two-dimensional electrically conducting micropolar fluid over an infinite vertical moving plate in the presence of thermal radiation. Plate is placed in *x* direction and *y*-axis is measured normal to it. Let$u$ and v be the velocity components along x-axisand *y*-axis respectively. An external magnetic field $B_0$is applied normal to the plate. It is assumed that the magnetic field is of small intensity so that the induced magnetic field is negligible in comparison to the applied magnetic field. Under the above assumptions the governing equations are:

Equation of continuity

$$\frac{\partial v^*}{\partial y} = 0 \quad \dots (1)$$

Equation of linear momentum

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{(\mu + k)}{\rho}\frac{\partial^2 u^*}{\partial {y^*}^2} + 2\frac{\kappa}{\rho}\frac{\partial w^*}{\partial y^*} + g\beta_T(T - T_\infty) - \frac{\sigma}{\rho}B_0^2\, u^* \dots (2)$$

Equation of angular momentum

$$\frac{\partial \omega^*}{\partial t^*} + v^* \frac{\partial \omega^*}{\partial y^*} = \frac{\gamma}{\rho j}\frac{\partial^2 \omega^*}{\partial {y^*}^2} \quad \dots (3)$$

Equation of Energy

$$\rho c_p\left(\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*}\right) = \alpha \frac{\partial^2 T}{\partial {y^*}^2} - \frac{\partial q_r^*}{\partial y^*} \quad \dots (4)$$

Equation of Mass Transfer

$$\frac{\partial C}{\partial t^*} + v^* \frac{\partial C}{\partial y^*} = D\frac{\partial^2 C}{\partial {y^*}^2} \quad \dots (5)$$

Where υ is the kinematic viscosity, κ microrotation viscosity, g is acceleration due to gravity, $\beta_T$is the coefficient of thermal expansion, μ is the coefficient of viscosity, B_0 is constant magnetic field applied along y-axis, α is the thermal conductivity, C_p is the specific heat at constant pressure,q_r^* is radiative heat flux,C_∞ is free stream concentration and T_∞ is free stream temperature. $\gamma$is spin gradient viscosity , j is microrotation per unit mass, D is chemical molecular diffusivity.

The boundary conditions are

$$u^* = u_p^*,\ \ T = T_w + \epsilon\,(T_w - T_\infty)e^{n^*t^*}\ ;\ \omega^* = -n_1\frac{\partial u^*}{\partial y^*}\ ;$$
$$C = C_w + \varepsilon(C_w - C_\infty)e^{n^*t^*} \quad \text{at } y^* = 0 \dots (6)$$

$$u^* \to 0\ ; \quad \omega^* \to 0; \quad T \to T_\infty\ ; \quad C \to C_\infty; \quad as \quad y^* \to \infty \dots (7)$$

We note that n_1 is a constant such $0 \le n_1 \le 1$. The case when n_1 = 0, is called strong concentration which indicates that no microrotation near the wall. In case n_1 = 0.5, it indicates that the vanishing of anti-symmetric part of the stress tensor and denote weak concentration and case n_1 =1 is used for the modelling of turbulent boundary layer flows.

$\gamma = (\mu + \frac{k}{2})\, j = \mu(1 + \frac{K}{2})\, j$, where K = $\frac{\kappa}{\mu}$ is the micro polar or material parameter and j =$\frac{\upsilon}{a}$ as reference length.

The total spin ω^* reduces to the angular velocity.

From the Eq. (1) the suction velocity normal to the plate is a function of time only which is in the form:

$$v^* = -(1 + \varepsilon A e^{n^*t^*})V_0 \quad \dots (8)$$

Where A is a real positive constant and V_0 is a scale of suction velocity which has non-zero positive constant.

Using Roseland's approximation for radiation, we obtain $q_r^* = -(\frac{4}{3}\frac{\sigma^*}{K_1^*})\frac{\partial T^4}{\partial y^*}$, where σ^* is the Stefan-boltzmann constant, K_1^* is the absorption coefficient. We assume that the temperature variation within the flow is such that T^4 may be expand in a Taylor's series. Expanding T^4 about T_∞ and neglecting higher order term we get,

$T^4 = 4TT_\infty^3 - 3T_\infty^4$

III. PROBLEM SOLUTION

We use the following dimensionless variables:

$$u^* = U_0 u;\ v^* = V_0 v\ ;\ y^* = \frac{\upsilon}{V_0}y\ ;\ u_p^* = U_0 U_p\ ;\ \omega^* = \frac{U_0 V_0}{\upsilon}\omega\ ;\ t^* = \frac{\upsilon}{V_0^2}t\ ;\ n^* = \frac{V_0^2}{\upsilon}n\ \theta = \frac{T - T_\infty}{T_w - T_\infty}\ ;\ \phi = \frac{C - C_\infty}{C_w - C_\infty}\ ;\ Pr = \frac{\upsilon}{\alpha}\ ; Sc = \frac{\upsilon}{D}\ ;\ M = \frac{\upsilon\sigma B_0^2}{\rho V_0^2}\ ;\ Gr = \frac{\upsilon g\,\beta_T(T_w - T_\infty)}{U_0 V_0^2}\ ;\ \eta = \frac{2}{2+K}\ ;\ N_r = \frac{4\sigma^* T_\infty^3}{k^* K_1^*} \quad \dots (9)$$

The transformed equations are:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial u}{\partial y} = (1 + K)\frac{\partial^2 u}{\partial y^2} + 2K\frac{\partial \omega}{\partial y} + \mathrm{Gr}\theta - Mu \quad \dots (10)$$

$$\frac{\partial w}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial \omega}{\partial y} = \frac{1}{\eta}\frac{\partial^2 \omega}{\partial y^2} \quad \dots (11)$$

$\frac{\partial\theta}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial\theta}{\partial y} = \frac{1}{Pr}\left(1 + \frac{4}{3}N_r\right)\frac{\partial^2\theta}{\partial y^2}$ ... (12)

$\frac{\partial\phi}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial\phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}$ ... (13)

The boundary conditions (6) and (7) becomes

$u = U_p\,,\ \omega = -n_1\frac{\partial u}{\partial y}\,,\ \theta = 1 + \varepsilon e^{nt}, \phi = 1 + \varepsilon e^{nt}$ aty =0 … (14)

$u \to 0,\quad \omega \to 0\,,\quad \theta \to 0,\quad \phi \to 0 as\ \eta \to \infty$… (15)

To solve Eqs. (10)-(13) subject to the boundary conditions Eqs. (14,15) we may use the following linear transformation for small value of ε

u(y,t)=u_0(y)+$\varepsilon e^{nt}\,u_1 + o(\varepsilon^2)$ …(16)

ω(y,t)=ω_0(y)+$\varepsilon e^{nt}\,\omega_1 + o(\varepsilon^2)$ …(17)

θ(y,t)=θ_0(y)+$\varepsilon e^{nt}\,\theta_1 + o(\varepsilon^2)$ …(18)

ϕ(y,t)=ϕ_0(y)+$\varepsilon e^{nt}\,\phi_1 + o(\varepsilon^2)$ …(19)

After substituting Eq. (16)-(19) into Eq. (10) – (13), we have

(1+K)u_0''+u_0'-Mu_0= -2Kω_0'-Grθ_0 …(20)

(1+K)u_1''+u_1'-(M+n)u_1= -2Kω_1'-Grθ_1-Au_0' … (21)

ω_0''+ηω_0'= 0 …(22)

ω_1''+ηω_1'-nηω_1= -Aηω_0' … (23)

(3+4Nr)θ_0'' +3$Pr\theta_0'$= 0 … (24)

(3+4Nr)θ_1'' +3$Pr.\,\theta_1'$-3Pr.n.θ_1= -3PrAθ_0' … (25)

ϕ_0''+$Sc\phi_0'$= 0 …(26)

ϕ_1''+$Sc\phi_1'$-Sc.n.ϕ_1= -A.Sc.ϕ_0' … (27)

With the following boundary conditions:

u_0=U_p, u_1=0,$\omega_0 = -n_1 u_0'$,$\omega_1 = -n_1 u_1'$,$\theta_0 = 1$, θ_1 =1 ,ϕ_0=1 , $\phi_1 = 1$ at y=0

$u_0 = u_1 = \omega_0 = \omega_1 = \theta_0 = \theta_1 = \phi_0 = \phi_1 = 0$ as y→ ∞… (28)

To solve the nonlinear coupled Eqs. (20) – (27) with satisfying boundary condition(28) we find as

$u_0(y)$=$a_6 e^{-m_4 y} + a_4 e^{-\eta y} + a_5 e^{-\frac{Pr.y}{Nc}}$ …(29)

$u_1(y)$=$a_9 e^{-m_3 y} + a_8 e^{-\eta y} + a_{10} e^{-\frac{Pr.y}{Nc}} + a_{12} e^{-m_5 y} + a_7 e^{-m_1 y} + a_{11} e^{-m_4 y}$ … (30)

$\omega_0(y)$=$a_1 e^{-\eta y}$ …(31)

$\omega_1(y)$=$a_2 e^{-m_1 y} - \frac{A\eta}{n} a_1 e^{-\eta y}$ …(32)

θ_0=$e^{-\frac{Pr.y}{Nc}}$ …(33)

$\theta_1(y)$=$a_3 e^{-m_3 y} - \frac{A.Pr}{n.Nc} e^{-\frac{Pr.y}{Nc}}$ …(34)

$\phi_0(y)$=$e^{-Sc.y}$ …(35)

$\phi_1(y)$=$\frac{A.Sc}{n}(e^{-m_2 y} - e^{-Sc.y})$ …(36)

Where exponential indices and coefficients are given in appendix.

IV. RESULTS AND DISCUSSION

Effects of physical parameters on velocity, micro-rotation, temperature and concentration profilesare shown through figure 1 to 11. The effects of the main controlling parameters as they appear in the governing equations are discussed in the current section. In order to verify the numerical result originate, they are found to be in a good arrangement to previously published paper. The velocity profile u(y)is planned in **Figure 1** for different values of the thermalgrashof number Grwhen K = 0.5, Pr=1, A=0.1,Nr=0.03, M=2, t=1 and Sc=2. This figure shows that when we increase the parameter Gr then the velocity profile u(y) is increases. **Figure2** showthat the effect of micropolar parameter K on velocity profile $u(y)$ when other parameter are fixed. This figure show that increasing the values of the micropolar parameter then the velocity profile isincrease. Also the same effect show that the variation of microrotation parameter n_1 by **Figure 4**. **Figure 3** shows that velocity profile decreasing for magnetic field parameter M is increasing. **Figure 5** show that the different value of thermal grashof number Gr then the angular velocity profile is increases. **Figure 6**represents the angular velocity distribution to increase the micropolar parameter K then increase it but reverse effect represents after a special value of y. **Figure 7,8**show that the increasing the values of $n_1$and M then the angular velocity is decreasing.**Figure 9**represents the temperature profile with respect to Nc. **Figure 10** represents temperature profile is decreases when increasing Pr . Concentration profile decreases when increasing Sc in **figure 11.**

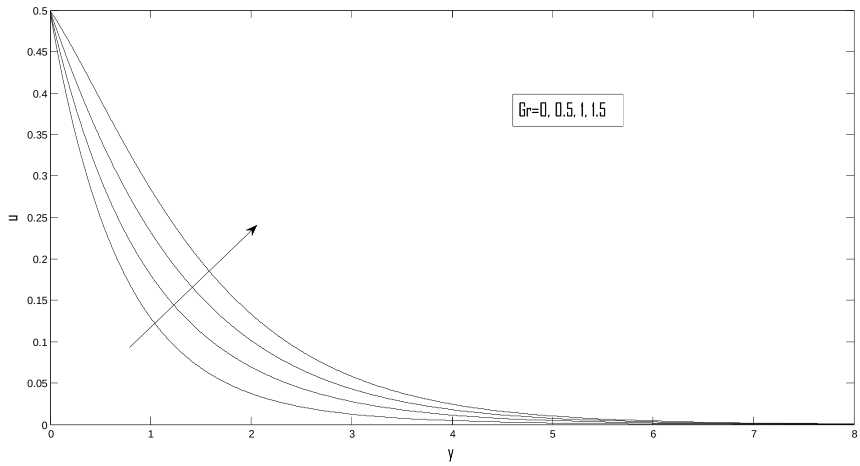

Figure 1 velocity profiles for different values of thermal grashof number Gr whilePr=1, M=2, K=0.5, Sc=2, N_r=0.03 , t=1 and Sc=2

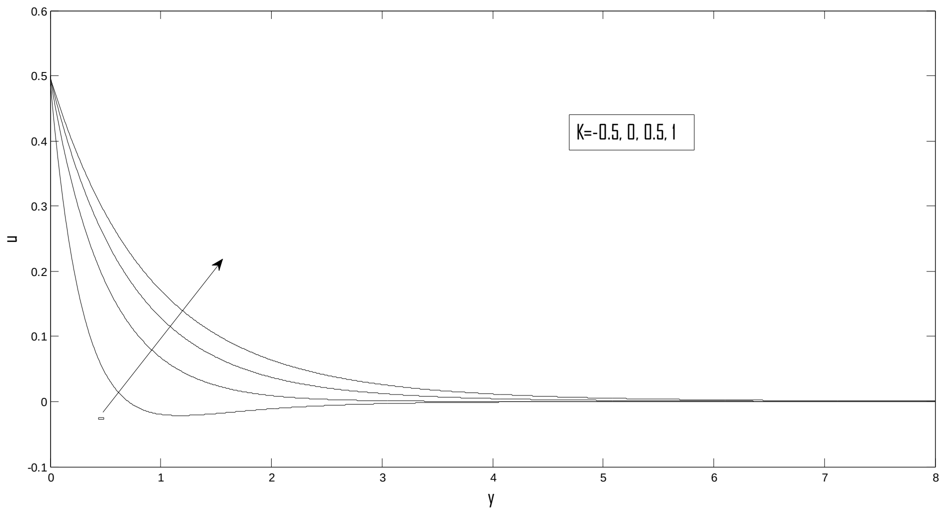

Figure 2 velocity profiles for different values micropolar parameter K whilePr=1, M=2, Sc=2, A=0.1, N_r=0.03 , t=1 and Gr=0

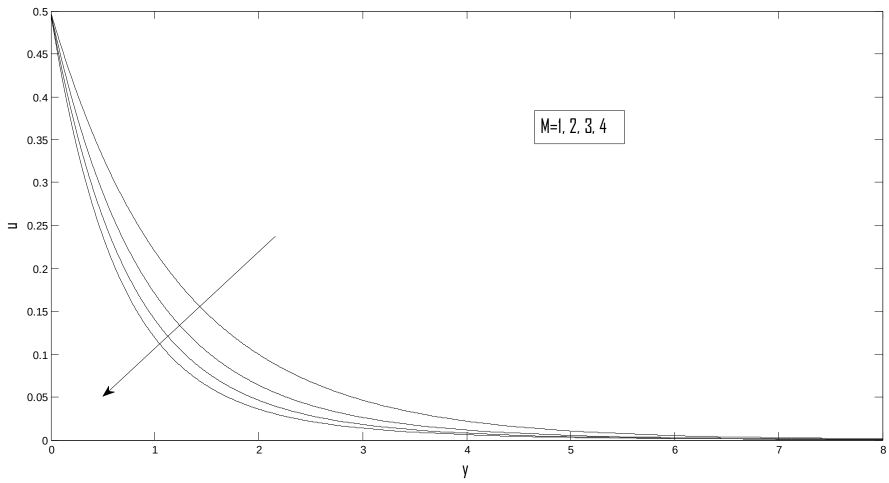

Figure 3 velocity profiles for different values of magnetic parameter M whilePr=1,Sc=2, K=0.5, A=0.1, N_r=0.03 , t=1 and Gr=0

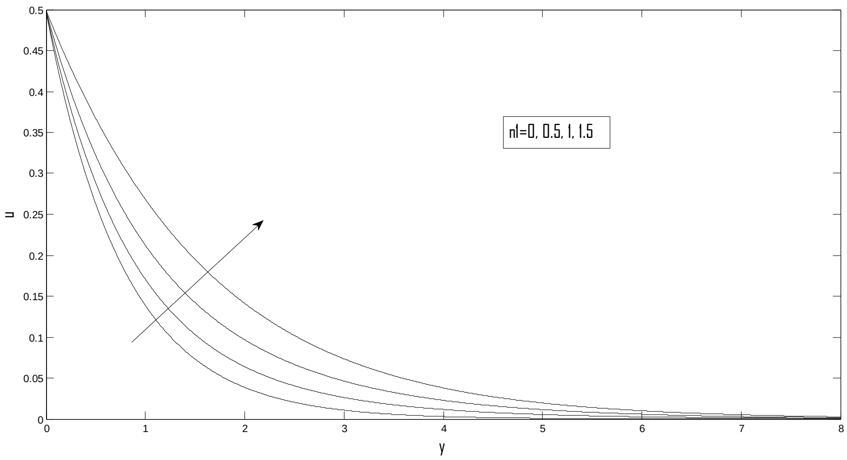

Figure 4 velocity profiles for different values of microrotation parametern_1 whilePr=1, M=2, K=0.5, Sc=2, N_r=0.03, t=1 and Gr=0

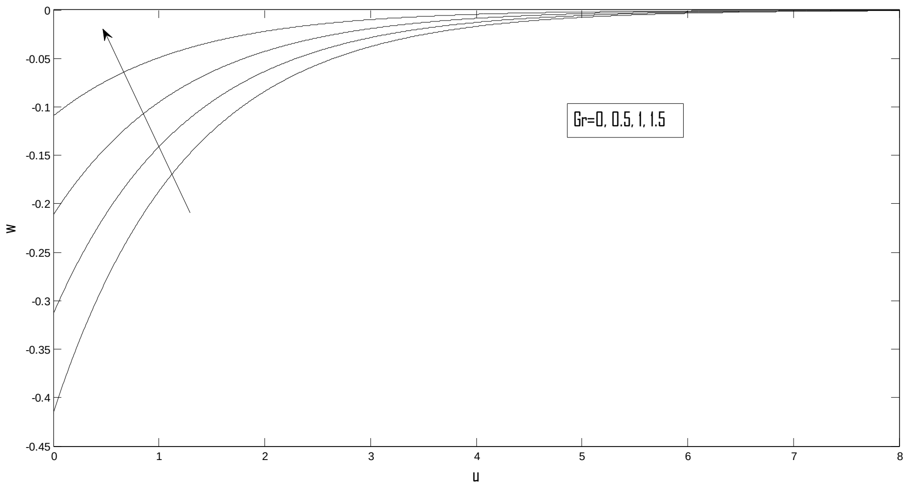

Figure 5 angular velocity profiles for different values of thermal grashof number Gr whilePr=1, M=2, K=0.5, Sc=2, N_r=0.03 , t=1 and A=0.1

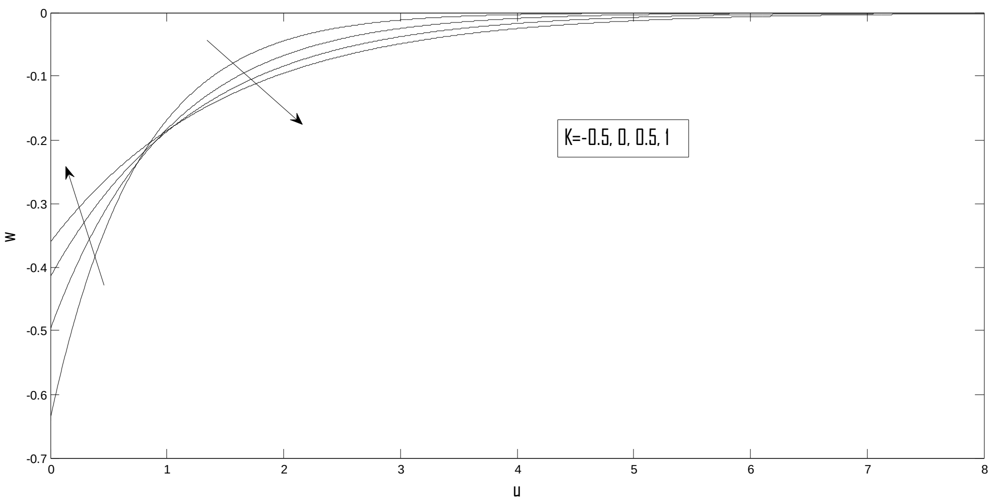

Figure 6 angular velocity profiles for different values of micropolar parameter K whilePr=1, M=2, A=0.1, Sc=2, N_r=0.03 , t=1 and Gr=0

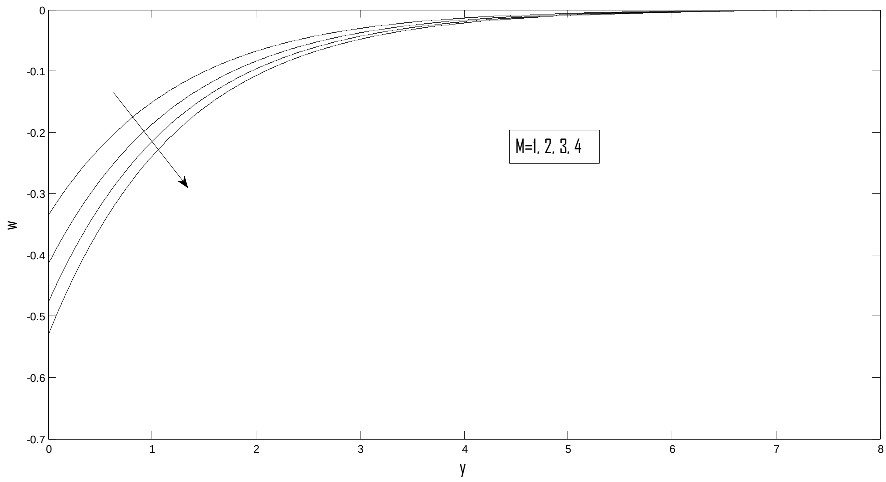

Figure 7 angular velocity profiles for different values of magnetic parameter M whilePr=1, A=0.1, K=0.5, Sc=2, N_r=0.03 , t=1 and Gr=0

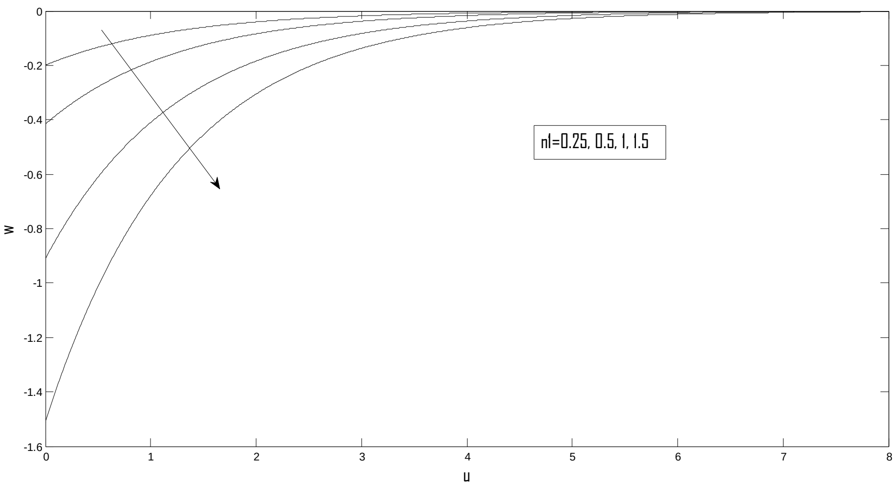

Figure 8 angular velocity profiles for different values of microrotation parameter $n_1$whilePr=1, M=2, K=0.5, Sc=2, N_r=0.03 , t=1 and Gr=0

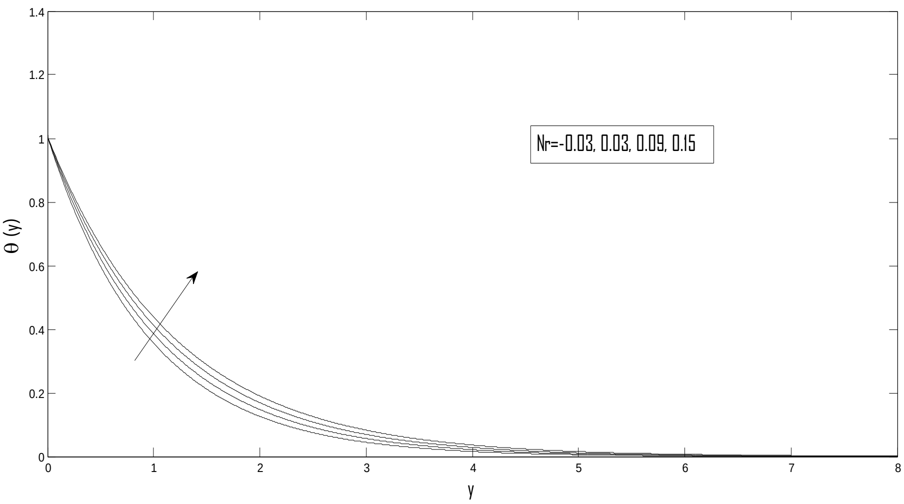

Figure 9 temperature profiles for different values of Nr whilePr=1, M=2, K=0.5, A=0.1, Sc=2, t=1 and Gr=0

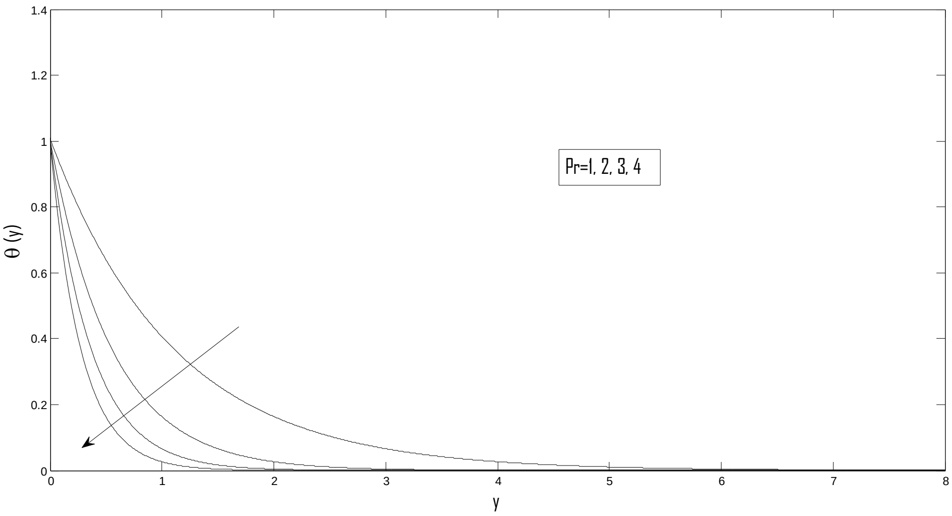

Figure 10 temperature profiles for different values of Pr while M=2, K=0.5, Sc=2, N_r=0.03 , A=0.1, t=1 and Gr=0

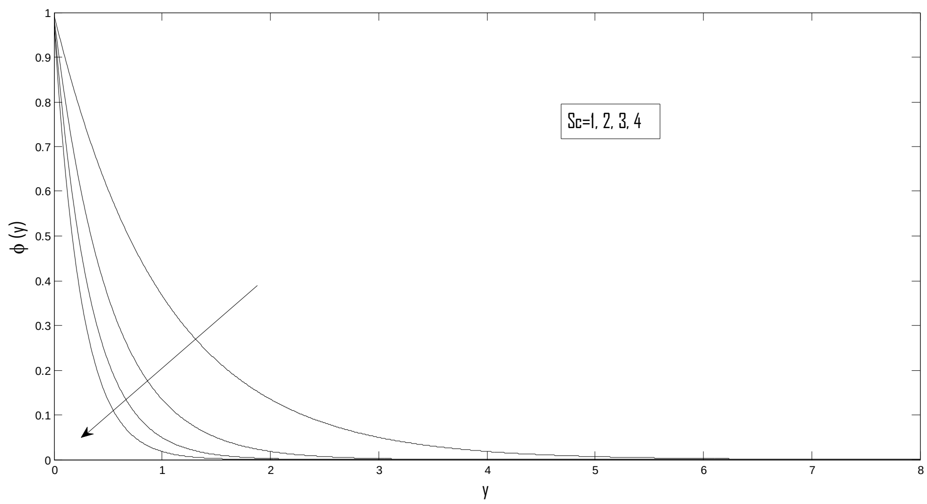

Figure11 concentration profiles for different values of smidt number Sc whilePr=1, M=2,K=0.5, A=0.1, N_r=0.03 , t=1 and Gr=0

Appendix

Nc =1+$\frac{4}{3}N_r$ $\quad m_1 = \frac{\eta+\sqrt{\eta^2+4n\eta}}{2}$

$m_2 = \frac{Sc+\sqrt{Sc^2+4nSc}}{2}$ $\quad m_3 = \frac{Pr+\sqrt{Pr^2+4n.Pr.Nc}}{2.Nc}$

$m_4 = \frac{1+\sqrt{1+4M.(1+K)}}{2(1+K)}$ $\quad m_5 = \frac{1+\sqrt{1+4(1+K)(M+n)}}{2(1+K)}$

$a_1 = -n_1 u_0^{'}$ $\quad a_2 = -n_1 u_1^{'} + \frac{A.\eta.a_1}{n}$

$a_3 = 1 + \frac{A.Pr}{n.Nc}$ $\quad a_4 = \frac{2K.a_1.\eta}{(1+K)\eta^2-\eta-M}$

$a_5 = \frac{-Gr}{(1+K)\frac{Pr^2}{Nc^2}-\frac{Pr}{Nc}-M}$ $\quad a_6 = U_p - a_4 - a_5$

$a_7 = \frac{2K.a_2.m_1}{(1+K)m_1{}^2-m_1-(M+n)}$ $\quad a_8 = \frac{nAa_4\eta-2K.a_1.A.\eta^2}{n[(1+K)\eta^2-\eta-(M+n)]}$

$a_9 = \frac{-Gr a_3}{(1+K)m_3{}^2-m_3-(M+n)}$ $\quad a_{10} = \frac{Gr.A.Pr+nA.Pr.a_5}{n.Nc[(1+K)\frac{Pr^2}{Nc^2}-\frac{Pr}{Nc}-(M+n)]}$

$a_{11} = \frac{A.a_6.m_4}{(1+K)m_4{}^2-m_4-(M+n)}$ $\quad a_{12} = -a_7 - a_8 - a_9 - a_{10} - a_{11}$

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