Electroelastic modeling and experimental validations of piezoelectric energy harvesters for wireless sensing networks powered by non-harmonic motion

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Abstract: A wireless sensor network (WSNs) is a solution to many environmental, security, and process monitoring problems. Although the fast developments of small dimension, low cost and low power circuit and sensor, energy supply is becoming a big barrier to wireless sensor networks. Piezoelectric energy harvesting techniques are still in a development stage. In this paper, electroelastic modeling, analytical and numerical solutions, and experimental validations for piezoelectric energy harvesters in wireless sensing networks powered by non-harmonic motion have been proposed. Future research needs and challenges of electroelastic modeling of piezoelectric energy harvesters in wireless sensing network powered by non-harmonic motion have been also discussed.

Keyword: Wireless sensor network, Energy harvesting, mathematical models, non-harmonic motion

I. INTRODUCTION:

In recent years, wireless sensing networks powered by non-harmonic motion has been vigorously explored [1-5]. Wireless sensor networks are result of combination of sensing nodes which commutate with each other. Cost efficiency of sensor nodes can be increased by lowering the cost of manufacturing and less maintenance requirement. This creates challenges for sensor calibration, making of package for survival in difficult environment and specially the efficient supply and utilization of power. In past decade, battery technology is progressively improving with the development of less power consuming electronics. But these inventions are not sufficient with respect to increasing demand of many WSN applications. This creates necessity to take more interest in the development of alternative systems which are capable of extracting electronic energy from existing environmental sources like ambient light, vibration, thermal gradient and motion etc. Alternative sources of power can help in establishment of wireless miniature devices with long and maintenance-free life-times. This is the main reason behind increasing popularity of energy harvesting from different ambient sources and piezoelectric energy harvesting due to inertial energy from ambient motion. Piezoelectric energy harvesting can be done with random low frequencies, motion sources like body motions [6-7]. Body motion is a non-harmonic motion with a wide range of amplitudes and used as a discontinuous energy provider for the energy harvester. Main advantage of piezoelectric energy harvesters is that they are designed in such a way the energy is available for power a device. Electroelastic modeling for non-harmonic motion-based energy harvesting are based on the assumption that the resonant frequencies of the piezoelectric beams are several times higher than frequencies of the motion sources [8]. Models are used for examine the validation of the device as well as analyze its performance of the device. There are several approaches to construct mathematical models for piezoelectric energy harvesting. A comprehensive literature on electroelastic models on vibration-based piezoelectric energy harvesting has been introduced by A. Erturk and D. J. Inman [9-10]. This paper aims at building a suitable electroelastic model that can be used in non-harmonic motion piezoelectric energy harvesters for wireless sensing networks powered by non-harmonic motion.

II. PIEZOELECTRIC CONVERSION

In last few decades, piezoelectric transduction has attracted researchers due to high power density and ease of application of piezoelectric materials. Piezoelectric materials have a special property that they are mechanically strained. When strained exist they generate electric filed or when electric field is applied the material undergoes strain [12]. There are several materials which possess piezoelectric behaviour. For example: lead zirconatetitanate (PZT) polyyvinylideneflouride (PVDF) and barium titanate (BaTiO3) etc. Among them,
PZT is a brittle piezoceramic which has a high electro-mechanical coupling coefficient, \( k \) (up to 0.75). Due to its great stiffness and high \( k \) value, PZT is widely used material of piezoelectric harvester. Since, piezoceramics show high stiffness due to which they preferred for piezoelectric energy harvesters for wireless sensing networks powered by non-harmonic motion. Frequency of non-harmonic motion is slightly more than resonant frequency found in most situations. Therefore, a more suitable configuration will be required for cantilever arrangement. In next section, we have discussed briefly. When two piezoceramic layers are bounded together in a configuration is known as bimorph while if only one layer is used then structure is known as unimorph. There are two ways to obtain a bimorph: Piezo layers can be connected either in series or parallel.

### III. ANALYTICAL APPROACH: PIEZOELECTRIC BIMORPH CANTILEVER

The structure of symmetrical bimorph piezoelectric energy harvester configurations is presented in Figure 1. The cantilevered beam, whose translational displacement, \( g(t) \) and a superimposed small rational displacement \( h(t) \), consists of three layers. The middle layer is the substructure and its thickness is \( t_s \), the upper and lower layer of the beam. The cantilever is fixed at \( x = 0 \), and a tip mass, \( M_0 \), is mounted on the free end of the beam. The wires are covering the surfaces of the piezoceramic layers are connected with a resistor, \( R_L \). \( R_i \) is connected to the beam as its electrical load and the voltage across it is \( V(t) \).

The governing equations of a bimorph piezoelectric energy harvester in the physical coordinates are given by

\[
\begin{align*}
\frac{\partial}{\partial x} \left[ D \frac{\partial^3 V_{rel}(x,t)}{\partial x^3} + C_s I \frac{\partial^4 V_{rel}(x,t)}{\partial x^4 \partial t} + C_m V_{rel}(x,t) + m_p \frac{\partial V_{rel}(x,t)}{\partial t} \right] \\
= - \left[ m_p - M_s \omega(x - L) \right] \frac{\partial^2 V_b(x,t)}{\partial t^2}
\end{align*}
\]

where \( V_b(x,t) = g(t) + x \) \( h(t) \) is the transverse displacement of base and \( V_{rel}(x,t) \) is the non-harmonic motion response i.e. neutral axis displacement of the beam related to the base in transverse direction. \( C_s \) is the inter damping coefficient of the composite structure. \( D \) is the bending stiffness of the beam, \( m_p \) is the mass per unit length of the beam. Suppose that the base is fixed \( V_b(x,t) \) can be set equal to zero. Putting the value of \( V_b(x,t) \) in equation (1) results

\[
M \left( \frac{\partial}{\partial x} \left[ D \frac{\partial^3 V_{rel}(x,t)}{\partial x^3} + C_s I \frac{\partial^4 V_{rel}(x,t)}{\partial x^4 \partial t} + C_m V_{rel}(x,t) + m_p \frac{\partial V_{rel}(x,t)}{\partial t} \right] \right) = 0
\]

The electrochemical coupling term in physical coordinates is defined as \( \eta = \tilde{S}_{31} w (w_p + w_s) / 2 \) if layers the piezoelectric are connected in series and \( \eta = \tilde{S}_{31} w (w_p + w_s) \) if layers the piezoelectric are connected in parallel. (11-13). Here, \( w \) is the width of layer, \( w_p \) is the thickness of each piezoceramic layer and \( w_s \) is the thickness of substructure layer, \( \tilde{S}_{31} \) is the effective piezoelectric stress constant. \( \tilde{S}_{31} \) is the stress in the \( x \) direction; \( s \) and \( p \) represent substructure and piezoelectric layer respectively. The inertial bending moment for upper layer is defined as \( M_{1}(x,t) = w \int_{t_{s,1/2}}^{t_{p,1/2}} T^p dz \), for middle layer, \( M_{2}(x,t) = w \int_{1/2}^{t_{s,1/2}} T^s dz \) and for lower layer, \( M_{3}(x,t) = w \int_{t_{p,1/2}}^{t_{s,1/2}} T^s dz \). Now stress component using hook’s law and constitutive equation of

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transduction is obtained as \( T_1^s = Y_s S_1^s \) and \( T_1^p = K^p S_1^p - g_{31} E_3 \) where \( Y_s \) is a young’s modulus, \( K^p \) is a piezoelectric modulus at constant electric field and \( g_{31} \) is the effective piezoelectric stress constant.

The strain for each layer is \( S_i(x, y, z) = -z \frac{\partial^2 v(x, t)}{\partial x} \).

There are two cases of connection: Series and Parallel. In series connection, Polarity exists in two layers are opposite in sign and this gives the same electric field in both layers. \( E_r(t) = -\frac{v(t)}{2t_p} \). In case of parallel connection, \( E_p(t) = -\frac{2v(t)}{t_p} \). The differential equation for series connection piezoelectric beam can be obtained as

\[
Y_i \frac{\partial^4 v(x, t)}{\partial x^4} + c_s \frac{\partial^3 v(x, t)}{\partial x^3 \partial t} + c_a \frac{\partial v(x, t)}{\partial t} + m_p \frac{\partial^2 v(x, t)}{\partial t^2} - \beta \frac{v(t)}{2} \left[ \frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] = 0.
\]

The coupling coefficient \( \beta \) is obtained by the dimension of beam and the effective piezoelectric stress constant.

\[
\beta = \frac{E_{31}^s W}{2t_p^2} \left[ L^2 + t_p LS \right]
\]

The expression for \( Y_i \) is determined as \( Y_i = \frac{2w}{3} \left\{ Y_i \left( \frac{t_p}{2} \right)^3 + c_{s1}^p \left[ t_p^3 + \frac{3}{2} t_p t_e \left( t_p + \frac{t_e}{2} \right) \right] \right\} \).

Suppose \( \delta \) is the mass density then mass per unit length is \( m_p = w(\delta t_s + 2\delta t_p t_p) \). The vibrational response related to its non-harmonic motion with proportional damping assumption, in physical co-ordinate is expressed as

\[
v(x, t) = \sum_{r=1}^{\infty} \lambda_r(x) \phi_r(t)
\]

where the mass-normalized eigen function \( \lambda_r(x) \) and model mechanical coordinate \( \phi_r(t) \) are obtained from electrochemically uncoupled problem and are expression of the \( r \)th mode.

Suppose that non-harmonic motion of a vibrating base of the form \( g(t) = W_o + W e^{int} \) and \( h(t) = \theta_o + \theta e^{i\omega t} \) is given where \( W_o \) and \( \theta_o \) are translational and small rotational displacement. The model electromechanical coupling for parallel and series cases is obtained for parallel connection \( \tilde{\theta}_r = \tilde{\varepsilon}_{31}(h_p + h_i) b_\lambda_r(L) \) and \( \tilde{\theta}_r = \tilde{\varepsilon}_{31}(h_p + h_i) b_\lambda_r(L)/2 \) for series connection.

Now, mechanical forcing term due to base excitation is

\[
F_r = \omega^2 W_0 \left( m \sum_0^L \lambda_r(x) dx + M, \lambda_r(L) \right) + \omega^2 \theta_0 \left( m \sum_0^L x \lambda_r(x) dx + M, L \lambda_r(L) \right).
\]

IV. STATIC ANALYSIS FOR COMPARISON BETWEEN THE ANALYTICAL MODEL AND SIMULATED MODEL

Material PZT-5H is used for application of simulated model. The cantilever structure is simulated for both series and parallel connection ignoring tip mass and dimension of simulated beam. Carbon fiber is used for substructure layer. Dimension considered for simulated piezoelectric beam are taken as width=1.4mm, length=25mm, Piezoelectric layer=0.08 and 0.09 mm. The properties of substructure layers are measured as young’s modulus =110 GPa, density= 1.4 Kg/m³, poison’s ratio= 0.09 and relative permittivity =1. Now, simulation of a series connection piezoelectric beam model results the displacement and the open circuit voltage \( r \). This has been done by applying a transverse tip load of 0.040 N to the beam. We found that when a tip load deflects the piezoelectric beam, tip displacement and voltage generation across the beam has been occurred. The magnitude of tip displacement was 1.34 while open circuit voltage was 33.2 V. Thereafter, a parametric sweep on tip force is applied and we observed the tip displacement and the open circuit voltage, herediffent forces are applied to the free end of the beam. So, we found that the two methods match each other well considering the
static analysis of the PZT-5H piezoelectric beam. Again, we repeated this procedure for PZT-5A, PZT-5D and PZT-5J piezoelectric materials with the same dimensions. For all materials we observed that, the two methods match each other well with little difference (less than 10%) with respect to tip displacement and open circuit voltage simulations in terms of the static analysis. Within materials, we found that PZT-5A beam can generate the highest surface voltage whereas the PZT-5H beam is the worst. This study provides an acceptable agreement and shown that the analytical model can be used as an effect tool for modeling the piezoelectric energy harvesters for wireless sensing networks powered by non-harmonic motion.

V. CONCLUSIONS AND FUTURE WORK

This paper presents a study of Electroelastic models of piezoelectric energy harvesters for wireless sensing networks powered by non-harmonic motion. Models show that feasibility of non-harmonic motion based piezoelectric energy harvesters as a promising power for wireless sensor network in monitoring application. Analytical models have been found to be similar to an equivalent circuit model which can also be used when unequal multiple modes of non-harmonic motion are considered. From the performance predicted by the piezoelectric energy harvesting system powered by non-harmonic motion mode analysis, we observed that many harvesters generate much less power. Therefore further work should be focused into reducing the power consumption of the circuit is necessary. By development of a custom integrated chip optimization of all components, it canachieve. For this purpose, new electroelastic model for piezoelectric energy harvesting system needs to develop for further research to achieve above mention goal.

REFERENCES


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