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Thermo-mechanical Analysis of a Crack in an Infinite Functionally Graded Elastic Layer

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Abstract: This paper aims to develop a steady state thermoelastic solution for an infinite functionally graded layer of finite thickness with a crack in it lying in the middle of the layer and parallel to the faces of the layer. The faces of the layer are maintained at constant temperature of different magnitude. The layer surfaces are supposed to be acted on by symmetrically applied concentrated forces of magnitude $\frac{P}{2}$ with respect to the centre of the crack. The applied concentrated force may be compressive or tensile in nature. The problem is solved by using integral transform technique. The solution of the problem has been reduced to the solution of a Cauchy type singular integral equation, which requires numerical treatment. Both normalized thermo-mechanical stress intensity factor (**TMSIF**), thermal stress intensity factor (**TSIF**) and the normalized crack opening displacement are determined. Thermal effect and the effects of non-homogeneity parameters of the graded material on various subjects of physical interest are shown graphically.

Key words and phrases: Fourier integral transform, Singular integral equation, Stress-intensity factor.

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I. INTRODUCTION

Every solid material has its own characteristics in respect of its elastic behavior, density, porosity, thermal and electrical conductivity, magnetic permeability, and so on. In numerous situations, a particular solid does not necessarily fulfill all the criteria for a particular purpose. For example, aerospace industry requires light materials with high strength; the outer part of a space craft body should be a non-conductor of heat such that the heat generated due to friction does not disturb the interior part made of high strength metal. It is difficult to find materials with light weight but high strength or high strength metal with zero heat conductivity. These difficulties are overcome at present using two or more solids at a time to generate a new solid which will fulfill most of our requirements. Composite materials, fibre-reinforced materials and functionally graded materials (FGM) are some of newly manufactured materials which are used at present in various areas of applications. Two solids say A and B with specific properties are used to form a FGM in such a way that the composition gradually vary in space following a definite designed rule. This means that at a particular point in the medium the FGM shows (100 - x) % of A's property and x% of B's property, $(0 \leq x \leq 100)$. The idea was originated in Japan in early part of the eighties in the last century and has been found to be very useful in respect of applications in various areas like resisting corrosiveness, controlling thermal activity, increasing strength and toughness in materials etc. FGMs have also several biomedical applications.

Thermal loading on solids has significant effects in their after load behavior and so should be dealt with utmost care. As such, the study of thermoelastic problems has always been an important branch in solid mechanics[1,2]. In the design of a structure in engineering field , considerable attention on thermal stress is a natural task, because many structural components are subjected to severe thermal loading which might cause significant thermal stresses in the components, especially around any defect present in the solid. Thermal stresses along with the stresses due to mechanical loadings can give rise to stress concentration in an around the defects and can lead to considerable damage in the structure.

In literature, problems related to defects such as cracks in solids have been studied in detail for various kinds of solid medium. Cracks in a solid may be generated due to several reasons such as uncertainties in the loading process, compositional defects in materials, inadequacies in the design, deficiencies in construction or maintenance of environmental conditions, and several others. Consequently, almost all structures contain cracks, either due to manufacturing defects or due to inappropriate thermal or mechanical loading. If proper attention to load condition is not paid, the size of the crack grows, catastrophically leading to a structure failure. A list of work on crack problems by earlier investigators has been provided [3,4,5,6,7,8,9,10,11,12] etc. Among the recent works on crack problems in solids of above mentioned characteristics, notable are the works [13,14,15,16,17,18,19,20,21] etc.

For a solid with a crack in it loaded mechanically or thermally, determination of stress intensity factor (**SIF**) becomes a very important task in fracture mechanics. The **SIF** is a parameter that gives a measure of stress concentration around cracks and defects in a solid. **SIF** needs to be understood if we are to design fracture tolerant materials used in bridges, buildings, aircraft, or even bells. Polishing just won't do if we detect crack. For a thermoelastic crack problem thermal stress intensity factor (**TSIF**) is a very important subject of physical interest. Literature survey shows good number of papers dealing with thermal stress intensity factors. Among them mention may be made of the works [22,23,24,25,26] etc.

The present investigation aims to find the elastostatic solution in an infinite layer with a crack in it and is under steady state thermal loading as well as mechanical loading. Following the integral transform technique the problem has been reduced to a problem of Cauchy type singular integral equation , which has been solved numerically. Finally , the stress-intensity factors and the crack opening displacements are determined for various thermal and mechanical loading conditions and the associated numerical results have been shown graphically.

II. NOMENCLATURE

λ , μ	:	Lame's constants for isotropic elastic material
ν	:	Poisson's ratio of elastic material
α_t	:	Thermal expansion coefficient of the material
σ_x ; τ_{xy} ; σ_y	:	Stress components in cartesian co-ordinate system
κ	:	Thermal diffusivity of the material
P	:	Applied load in isothermal problem
$\delta(:)$	:	Dirac delta function
p_0	:	Internal uniform pressure along crack surface
β	:	Non homogeniety parameter for elastic coefficients
$k(:)$	:	Stress intensity factor in a medium with a crack in it
T	:	Absolute temperature
T_0	:	Reference temperature
T_1 ; T_2	:	Constant temperature supplied to the material in the lower and upper surfaces respectively

III. FORMULATION OF THE PROBLEM

We consider an infinitely long functionally graded layer of thickness $2h$ weakened by the presence of an internal crack. Cartesian system co-ordinates will be used in our analysis. We shall take y-axis along the normal to the layer surface. The layer is infinite in a direction perpendicular to y-axis. A line crack of length $2b$ is assumed to be present in the middle of the layer. We shall take x-axis along the line of the crack with origin at the center of the crack and investigate the problem as a two dimensional problem in the x-y plane. The crack faces are supposed to be acted upon by tensile force p_0 and the layer surfaces are acted upon by concentrated forces at points on the layer surfaces at distance $2a$, symmetrically positioned with respect to the center of the crack. The concentrated forces are either of compressive in nature or tensile in nature. **Fig.1** displays the geometry of the problem.

In this figure two sided arrows actually correspond to two distinct problems: the inward drawn arrows correspond to compressive loading , while outward drawn arrows correspond to tensile loading. The two different types of loading are (i) a pair of concentrated compressive loads each of magnitude $\frac{P}{2}$ applied symmetrically with respect to the center of the crack at a distance $2a$ apart , (ii) a pair of concentrated tensile loads each of magnitude $\frac{P}{2}$ applied symmetrically with respect to the center of the crack at a distance $2a$ apart. The gravitational force has not been taken into consideration. In deriving analytical solution in the present study the elastic parameters λ and μ have been assumed to vary exponentially in the direction perpendicular to the plane of the crack , following the law

$$\lambda = \lambda_0 e^{\beta|y|}, \ \mu = \mu_0 e^{\beta|y|}, \ -h \le y \le h, \tag{1}$$

where λ_0 and μ_0 are the elastic parameters in the homogeneous medium and β is the non-homogeneity parameter controlling the variation of the elastic parameters in the graded medium.

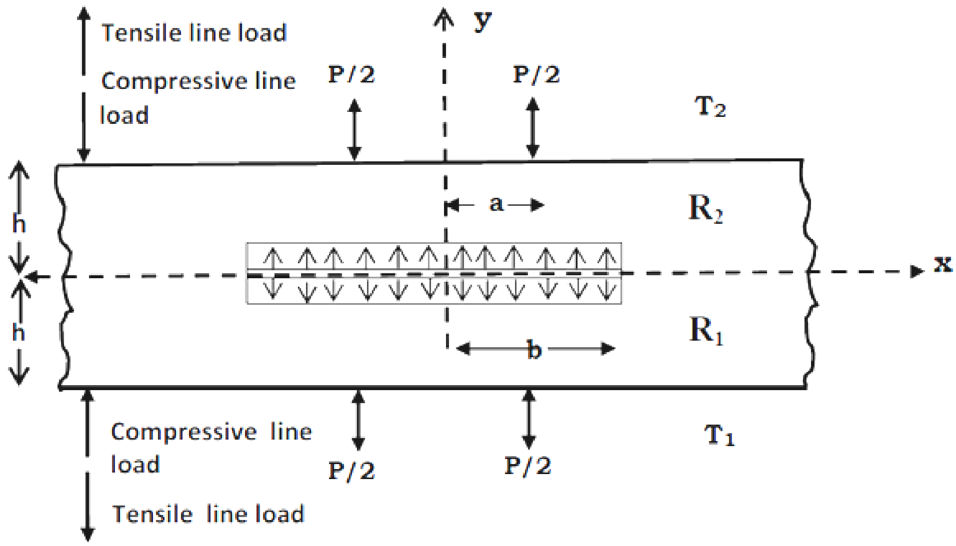


Fig.1: Geometry of the problem.

The strain displacement relations , linear stress-strain relations and equations of equilibrium are , respectively , given by

$$\varepsilon_x = \frac{\partial u}{\partial x}\,,\ \varepsilon_y = \frac{\partial v}{\partial y}\,, \gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{2}$$

$$\sigma_x = \frac{\mu}{\kappa - 1}[\,(1+\kappa)\varepsilon_x + (3-\kappa)\varepsilon_y],$$

$$\sigma_y = \frac{\mu}{\kappa - 1}[\,(3-\kappa)\varepsilon_x + (1+\kappa)\varepsilon_y],$$

$$\tau_{xy} = 2\mu\gamma_{xy}. \tag{3}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \tag{4}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0, \tag{5}$$

where $\kappa = 3 - 4v$ and v is Poisson's ratio. Before further proceeding it will be convenient to adopt non-dimensional variables by rescaling all length variables by the problem's length scale b and the temperature variable by the reference temperature scale T_0 :

$$u' = \frac{u}{b}\,,\ v' = \frac{v}{b}\,, x' = \frac{x}{b}\,, y' = \frac{y}{b}\,, h' = \frac{h}{b},$$

$$T' = \frac{T}{T_0}\,,\ T_1' = \frac{T_1}{T_0}\,, T_2' = \frac{T_2}{T_0}\,, \alpha_t' = \alpha_t T_0. \tag{6}$$

In the analysis below, for notational convenience , we shall use only dimensionless variables and shall ignore the dashes on the transformed non-dimensional variables. Mathematically, the problem under consideration is reduced to the solution of thermoelasticity equations with thermal expansion coefficient α_t and the quantity H , the dimensionless thermal conductivity of the crack surface defined by Carslaw and Jaeger[27].

(i) **Equilibrium equations:**

$$2(1-\nu)\frac{\partial^2 u}{\partial x^2} + (1-2\nu)\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} + \beta(1-2\nu)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 2(1+\nu)\alpha_t\frac{\partial T}{\partial x}, \tag{7}$$

$$(1-2\nu)\frac{\partial^2 v}{\partial x^2} + 2(1-\nu)\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x \partial y} + \beta\left[2(1-\nu)\frac{\partial v}{\partial y} + 2\nu\frac{\partial u}{\partial x}\right] = 2(1+\nu)\alpha_t\frac{\partial T}{\partial y}, \tag{8}$$

(ii) **Steady state heat conduction equation:**

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0,\ \ (0 < x < \infty)\,, \tag{9}$$

and

(iii) **The boundary conditions:**

(a) **Thermal boundary conditions:**

$$T(x\,,-h) = T_1,\ (0 < x < \infty)\,, \tag{10}$$

$$T(x\,,h) = T_2,\ (0 < x < \infty)\,, \tag{11}$$

$$\frac{\partial}{\partial y}T(x,0^+) = \frac{\partial}{\partial y}T(x,0^-) = H[T(x,0^+) - T(x,0^-)],\ (0 < x < 1)\,, \tag{12}$$

$$T(x,0^+) = T(x,0^-),\ (x \geq 1)\,, \tag{13}$$

$$\frac{\partial}{\partial y}T(x,0^+) = \frac{\partial}{\partial y}T(x,0^-),\ (x \geq 1)\,, \tag{14}$$

(b) **Elastic boundary conditions:**

$$\tau_{xy}(x\,,0) = 0,\ (0 < x < \infty)\,, \tag{15}$$

$$\tau_{xy}(x\,,h) = 0,\ (0 < x < \infty)\,, \tag{16}$$

$$\sigma_y(x\,,h) = \mp\frac{P}{2}\delta(x-a),\ (0 < x < \infty)\,, \tag{17}$$

$$\frac{\partial}{\partial x}[v(x,0)] = \begin{cases} f(x), & 0 < x < 1 \\ 0\,, & x > 1 \end{cases}, \tag{18}$$

$$\sigma_y(x,0) = -p_0\,,\ (0 \leq x \leq 1)\,, \tag{19}$$

where u and v are the x and y components of the displacement vector ; $\sigma_x, \sigma_y\,, \tau_{xy}$ are the normal and shearing stress components ; $f(x)$ is an unknown function and $\delta(x)$ is the Dirac delta function. In Eq. (17) positive sign indicates tensile force while negative sign corresponds to compressive force.

IV. METHOD OF SOLUTION

(a) **Thermal part:**

To determine temperature field $T(x,y)$ from Eq. (9) and boundary conditions (10)-(14) we assume

$$T(x\,,y) = U(x,y) + W(y), \tag{20}$$

where $U(x,y)$ and $W(y)$ are two unknown functions satisfying the conditions

$$U(x\,,-h) = U(x,h) = 0, \tag{21}$$

and

$$W(-h) = T_1\,, W(h) = T_2, \tag{22}$$

Under these considerations we get

$$W(y) = \frac{(T_2 - T_1)}{2h}y + \frac{T_1 + T_2}{2}, \tag{23}$$

and

$$U(x,y) = \begin{cases} \left(e^{\eta y} - e^{\eta(2h-y)}\right)A_1\ , & y \geq 0; \\ \left(e^{\eta y} - e^{-\eta(2h+y)}\right)\frac{1+e^{2\eta h}}{1+e^{-2\eta h}}A_1\ , & y \leq 0. \end{cases} \tag{24}$$

for certain constant A_1.

The appropriate temperature field satisfying the boundary conditions and regularity condition can be expressed as:

$$T(x\,,y) = \int_{-\infty}^{\infty}\left[e^{\eta y} - e^{\eta(2h-y)}\right]D(\eta)e^{ix\eta}d\eta + W(y),\ y \geq 0\,, \tag{25}$$

$$T(x\,,y) = \int_{-\infty}^{\infty}\left[e^{\eta y} - e^{-\eta(2h+y)}\right]r_{ab}D(\eta)e^{-ix\eta}d\eta + W(y),\ y \leq 0\,, \tag{26}$$

where $D(\eta)$ is an unknown function to be determined and

$$r_{ab} = \frac{1+e^{2\eta h}}{1+e^{-2\eta h}}, \tag{27}$$

Let us introduce the density function $\Theta(x)$, as

$$\Theta(x) = \frac{\partial}{\partial x}T(x,0^{+}) - \frac{\partial}{\partial x}T(x,0^{-}), \tag{28}$$

It is clear from the boundary conditions (13) and (14) that

$$\int_{-1}^{1}\Theta(s)ds = 0, \tag{29}$$

and

$$\Theta(x) = 0, \ (x \geq 1), \tag{30}$$

Substituting Eqs. (25) and (26) into Eq. (28) and using Fourier inverse transform , we have

$$D(\eta) = \frac{i(1+e^{-2\eta h})}{4\pi\eta\,(e^{-2\eta h}-e^{2\eta h})}\int_{-1}^{1}\Theta(s)e^{is\eta}\,ds, \tag{31}$$

Substituting Eqs. (25) and (26) into Eq. (12) and applying the relation (31) , we get the singular integral equation for $\Theta(x)$ as follows

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{s-x}+k_1(x,s)\right]\Theta(s)ds = \frac{T_1-T_2}{h}, \tag{32}$$

where

$$k_1(x,s) = \int_0^{\infty}\left[1-\frac{2H}{\eta}+\frac{2+e^{2\eta h}+e^{-2\eta h}}{e^{-2\eta h}-e^{2\eta h}}\right]\sin\,\eta(x-s)d\eta, \tag{33}$$

After determining $\Theta(s)$ from the singular integral equation (32) we have the temperature field along the axes as

$$T(x,0) = \begin{cases} \frac{1}{4}\int_{-1}^{1} sign(x-s)\Theta(s)ds + \frac{T_1+T_2}{2}, & y \geq 0; \\ -\frac{1}{4}\int_{-1}^{1} sign(x-s)\Theta(s)ds + \frac{T_1+T_2}{2}, & y \leq 0. \end{cases} \tag{34}$$

$$T(0,y) = \begin{cases} -\frac{1}{4\pi}\int_0^{\infty}\frac{1+e^{-2\eta h}}{e^{-4\eta h}-1}\left[e^{-\eta(2h-y)}-e^{-\eta y}\right]\frac{\sin(s\eta)}{\eta}d\eta\int_{-1}^{1}\Theta(s)ds + \frac{T_2-T_1}{2h} + \frac{T_1+T_2}{2}, & y \geq 0; \\ -\frac{1}{4\pi}\int_0^{\infty}\frac{1+e^{-2\eta h}}{e^{-4\eta h}-1}\left[e^{\eta y}-e^{-\eta(2h+y)}\right]\frac{\sin(s\eta)}{\eta}d\eta\int_{-1}^{1}\Theta(s)ds + \frac{T_2-T_1}{2h} + \frac{T_1+T_2}{2}, & y \leq 0. \end{cases} \tag{35}$$

(b) **Elastic part:**

First of all we observe that due to symmetry of the crack location with respect to the layer and of the applied load with respect to the crack , it is sufficient to consider the solution of the problem in the regions $0 \leq x < \infty$ and $0 \leq y \leq h$. To solve the partial differential equations (7) and (8), Fourier transform is applied to the equations with respect to the variable x.

Utilizing the symmetric condition the displacement components u, v may be written as

$$u(x,y) = \frac{2}{\pi}\int_0^{\infty}\Phi(\xi,y)\sin(\xi x)d\xi, \tag{36}$$

$$v(x,y) = \frac{2}{\pi}\int_0^{\infty}\Psi(\xi,y)\cos(\xi x)d\xi, \tag{37}$$

where $\Phi(\xi,y)$ and $\Psi(\xi,y)$ are Fourier transforms of $u(x,y)$ and $v(x,y)$, respectively with respect to the coordinate x, and ξ is the transformed parameter.

Substituting $u(x,y)$ and $v(x,y)$ from Eqs. (36) and (37) into the equations of equilibrium (7) and (8) we obtain the differential equation for the determination of $\Phi(\xi,y)$,

$$F(D_1)\Phi = \alpha_t\xi\frac{(\kappa-1)(7-\kappa)}{\kappa+1}\frac{\partial^2\overline{T}_c}{\partial y^2}+\frac{1}{2}\alpha_t\xi\left[(7-\kappa)(3-\kappa)+\beta\frac{(7-\kappa)(\kappa-1)^2}{\kappa+1}\right]\frac{\partial\overline{T}_c}{\partial y}$$

$$+\frac{7-\kappa}{4(\kappa+1)}\alpha_t\xi[\beta(3-\kappa)(\kappa+1)(\kappa-1+\beta)+4\xi^2]\overline{T}_c, \tag{38}$$

where

$$F(D_1) = D_1^4+2\beta D_1^3-(2\xi^2-\beta^2)D_1^2-2\xi^2\beta D_1+\xi^2\left\{\xi^2-\frac{(\kappa-3)}{(\kappa+1)}\beta^2\right\}, \tag{39}$$

and $\overline{T}_c(\xi,y)$ is the Fourier Cosine transform of $T(x,y)$ defined by

$$\overline{T}_c(\xi\ ,y) = \int_0^\infty T(x,y)\cos(\xi x)dx\ , \tag{40}$$

The solution of the Eq. (38) is of the form

$$\Phi(\xi,y) = \sum_{i=1}^{4}\left[A_i(\xi)e^{m_iy}+f_i^{(j)}(\xi)\right]\ ,j=1,2 \tag{41}$$

where $A_i(\xi)$, $(i=1,\dots,4)$ are constants to be determined from the boundary conditions , $m_i(i=1,\dots,4)$ are the four complex roots of two biquadratic equation

$$m^4+2\beta m^3-(2\xi^2-\beta^2)m^2-2\xi^2\beta m+\xi^2\left\{\xi^2-\frac{\kappa-3}{\kappa+1}\beta^2\right\}=0, \tag{42}$$

and

$$f_1^{(j)}(\xi) = \frac{1}{F(D_1)}\int_{-\infty}^{\infty}\left\{\frac{(7-\kappa)\alpha_t\xi}{4(\kappa+1)}[\beta(3-\kappa)(\kappa+1)(\kappa-1+\beta)+4\xi^2]\right.$$
$$+\frac{\eta}{2}\left[(7-\kappa)(3-\kappa)\alpha_t\xi+\frac{\alpha_t\beta(\kappa-1)^2(7-\kappa)\xi}{(\kappa+1)}\right]$$
$$\left.+\frac{\eta^2(\kappa-1)(7-\kappa)\alpha_t\xi}{(\kappa+1)}\right\}\frac{i\eta}{\xi^2-\eta^2}D(\eta)(r_{ab})^{(2-j)}e^{\eta y}d\eta\ ,j=1,2 \tag{43}$$

$$f_2^{(j)}(\xi) = \frac{1}{F(D_1)}\int_{-\infty}^{\infty}\left\{-\frac{(7-\kappa)\alpha_t\xi}{4(\kappa+1)}[\beta(3-\kappa)(\kappa+1)(\kappa-1+\beta)+4\xi^2]\right.$$
$$+\frac{\eta}{2}\left[(7-\kappa)(3-\kappa)\alpha_t\xi+\frac{\alpha_t\beta(\kappa-1)^2(7-\kappa)\xi}{(\kappa+1)}\right]$$
$$\left.-\frac{\eta^2(\kappa-1)(7-\kappa)\alpha_t\xi}{(\kappa+1)}\right\}\frac{i\eta}{\xi^2-\eta^2}D(\eta)(r_{ab})^{(2-j)}e^{(-1)^j.\eta(2h-y)}d\eta\ ,j=1,2 \tag{44}$$

$$f_3^{(j)}(\xi) = \frac{1}{F(D_1)}\frac{(7-\kappa)\alpha_t\xi}{4(\kappa+1)}[\beta(3-\kappa)(\kappa+1)(\kappa-1+\beta)+4\xi^2]\frac{(T_2-T_1)}{\pi h}\delta(\xi)y\ ,\ \ j=1,2 \tag{45}$$

$$f_4^{(j)}(\xi) = \frac{1}{F(D_1)}\left\{\frac{1}{2h}\left[(7-\kappa)(3-\kappa)\alpha_t\xi+\frac{\alpha_t\beta(\kappa-1)^2(7-\kappa)\xi}{\kappa+1}\right]\right.$$

$$-\frac{(7-\kappa)\alpha_t\xi}{4(\kappa+1)}[\beta(3-\kappa)(\kappa+1)(\kappa-1+\beta)+4\xi^2]\}\frac{(T_2-T_1)}{\pi}\delta(\xi), j=1,2 \tag{46}$$

where $j = 1, 2$ are for the lower and upper region respectively.
The function $\Psi(\xi, y)$ can then be determined as

$$\Psi(\xi,y)=\sum_{i=1}^{4}\left[M_i(\xi)A_i(\xi)e^{m_i y}+N_i f_i^{(j)}(\xi)\right], j=1,2 \tag{47}$$

where

$$M_i(\xi)=\frac{(\kappa-1)m_i^2+\beta(\kappa-1)m_i-\xi^2(\kappa+1)}{\xi\{2m_i+\beta(\kappa-1)\}}, (i=1,\ldots,4), \tag{48}$$

$$N_i=\frac{(\kappa-1)D_1^2+\beta(\kappa-1)D_1-\xi^2(\kappa+1)}{\xi\{2m_i+\beta(\kappa-1)\}}, (i=1,\ldots,4), \tag{49}$$

It follows from Eq. (42) that $m_3=\overline{m}_1$ and $m_4=\overline{m}_2$ where

$$m_1=-\frac{\beta}{2}+\sqrt{\xi^2+\frac{\beta^2}{4}+i\xi\beta\sqrt{\frac{3-\kappa}{\kappa+1}}}, \tag{50}$$

$$m_2=-\frac{\beta}{2}-\sqrt{\xi^2+\frac{\beta^2}{4}+i\xi\beta\sqrt{\frac{3-\kappa}{\kappa+1}}}, \tag{51}$$

Substituting Eqs. (36) and (37) into Eqs. (2) and (3) and utilizing Eqs. (41) and (47) we obtain

$$\begin{aligned}\frac{1}{2\mu}\sigma_x(x,y)=\frac{2}{\pi}\int_0^{\infty}\frac{1}{2(1-\kappa)}\sum_{i=1}^{4}[\{-(1+\kappa)\xi-(3-\kappa)m_iM_i\}A_ie^{m_iy}\\ +\left\{-\xi(1+\kappa)f_i^{(j)}-(3-\kappa)D_1\left(N_if_i^{(j)}\right)\right\}\Big]cos(\xi x)d\xi, j=1,2\end{aligned} \tag{52}$$

$$\begin{aligned}\frac{1}{2\mu}\sigma_y(x,y)=\frac{2}{\pi}\int_0^{\infty}\frac{1}{2(1-\kappa)}\sum_{i=1}^{4}[\{-(1+\kappa)m_iM_i-(3-\kappa)\xi\}A_ie^{m_iy}\\ +\left\{-(1+\kappa)D_1\left(N_if_i^{(j)}\right)-(3-\kappa)\xi f_i^{(j)}\right\}\Big]cos(\xi x)d\xi, j=1,2\end{aligned} \tag{53}$$

$$\begin{aligned}\frac{1}{2\mu}\tau_{xy}(x,y)=\frac{2}{\pi}\int_0^{\infty}\sum_{i=1}^{4}\left[\left\{-\frac{\xi}{2}M_i+\frac{m_i}{2}\right\}A_ie^{m_iy}\right.\\ \left.+\left\{-\frac{\xi}{2}N_if_i^{(j)}+\frac{1}{2}D_1\left(f_i^{(j)}\right)\right\}\right]sin(\xi x)d\xi, j=1,2\end{aligned} \tag{54}$$

From the boundary conditions (15)-(18),the unknown constants A_i $(i=1,\ldots,4)$ can be found out from the following linear algebraic system of equations expressed in matrix form:

$$LA=B, \tag{55}$$

where the matrices L, A, B are

$$L=\begin{bmatrix}S_1e^{m_1h} & S_2e^{m_2h} & S_3e^{m_3h} & S_4e^{m_4h}\\ G_1e^{m_1h} & G_2e^{m_2h} & G_3e^{m_3h} & G_4e^{m_4h}\\ m_1-G_1 & m_2-G_2 & m_3-G_3 & m_4-G_4\\ G_1 & G_2 & G_3 & G_4\end{bmatrix}, \quad A=\begin{bmatrix}A_1\\A_2\\A_3\\A_4\end{bmatrix}, \qquad B=\begin{bmatrix}-R_1-P_1\\P_2\\-R_2-P_3\\P_4\end{bmatrix}$$

and

$$R_1(\xi) = \pm\frac{(\kappa-1)}{2\mu}P[cos(a\xi)+cos(-a\xi)]\,, P_1 = \sum_{i=1}^{4} D'_{4i} f_{ih}^{(j)}\,,$$

$$R_2(\xi) = \int_0^\infty f(x) sin(\xi x)dx\,, \qquad P_2 = \sum_{i=1}^{4} D_{4i} f_{ih}^{(j)}\,,$$

$$P_3 = \sum_{i=1}^{4} N_i f_{i0}^{(j)}\,, \qquad P_4 = \sum_{i=1}^{4} D_{4i} f_{i0}^{(j)}\,, j=1,2, \tag{56}$$

$$S_i = (1+\kappa)m_i M_i + (3-\kappa)\xi\,,$$

$$G_i = -\xi M_i + m_i\,, \quad (i = 1,\ldots\ldots,4).$$

$$D_{4i} = \xi\left[N_i - \frac{1}{\xi}D_1\right], i = 1,\ldots\ldots,4\,, \tag{57}$$

$$D'_{4i} = (1+\kappa)D_1 N_i + (3-\kappa)\xi\,, i = 1,\ldots\ldots,4\,. \tag{58}$$

Eq. (55) yields

$$A_i(\xi) = D_{1i}(\xi)R_1(\xi) + D_{2i}(\xi)R_2(\xi) + D_{3i}(\xi)\,, i = 1,\ldots\ldots\ldots,4\,, \tag{59}$$

where $D_{ki}(k = 1,2,3 \;\; and \;\; i = 1,\ldots\ldots\ldots,4)$ are shown in **Appendix.** Substitution of these values into the Eq. (19) will lead to the following singular integral equation:

$$\frac{1}{\pi}\int_{-b}^{b} f(t)\left[\frac{1}{t-x} + k_2(x,t)\right]dt = \frac{(\kappa-1)}{\mu\chi}\left[-p_0 \pm \frac{P}{2\pi}k_3(x) + p_0 k_4(x)\right], \qquad (-b < x < b) \tag{60}$$

where

$$k_2(x,t) = \frac{1}{\chi}\int_0^\infty \sum_{i=1}^{4}\{D_{2i}S_i - \chi\} \sin\xi(t-x)d\,\xi\,, \chi = \lim_{\xi\to\infty}\sum_{i=1}^{4} D_{2i}S_i = 4\nu\,, \tag{61}$$

$$k_3(x) = -2\int_0^\infty \sum_{i=1}^{4} D_{1i}S_i \cos\xi(a+x)d\xi\,, \tag{62}$$

$$k_4(x) = -\frac{2\mu}{\pi p_0(\kappa-1)}\int_0^\infty \sum_{i=1}^{4}\left[D_{3i}S_i - D'_{4i}f_{i0}^{(j)}\right]cos(\xi x)d\xi\,, j = 1,2 \tag{63}$$

The kernels $k_2(x\,,t)$ and $k_3(x)$, $k_4(x)$ are bounded and continuous in the closed interval $-b \le x \le b$. The integral equation must be solved under the following single-valuedness condition

$$\int_{-b}^{b} f(t)dt = 0\,, \tag{64}$$

Before further proceeding it will be convenient to introduce non-dimensional variables r and s by rescaling all lengths in the problem by length scale b:

$$x = br\,, t = bs\,, \tag{65}$$

$$f(t) = f(bs) = \frac{p_0(\kappa-1)}{\mu\chi}\phi(s), \omega = \xi b\,, \tag{66}$$

In terms of non-dimensional variables the integral equation (60) and single valuedness condition (64) become

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{s-r}+k_2^*(r,s)\right]\phi(s)ds=-1\pm\frac{Q}{\pi}k_3^*(r)+k_4^*(r)\,,(-1<r<1)\,, \tag{67}$$

where

$$k_2^*(r,s)=\frac{1}{\chi}\int_0^\infty\left\{\sum_{i=1}^{4}D_{2i}S_i-\chi\right\}\sin\omega(s-r)d\omega\,, \tag{68}$$

$$k_3^*(r)=-2\int_0^\infty\sum_{i=1}^{4}D_{1i}S_i\cos\omega(r+a^*)d\omega\,, \tag{69}$$

$$k_4^*(r)=-\frac{2\mu_0}{\pi p_0 b(\kappa-1)}\int_0^\infty\sum_{i=1}^{4}\left[D_{3i}S_i-D_{4i}^{'}f_{i0}^{(j)}\right]\cos(\omega r)d\omega\,, \tag{70}$$

$a^*=\frac{a}{b}$ and Q is the load ratio defined as:

$$Q=\frac{P}{2bp_0}\,, \tag{71}$$

$$f_{i0}^{(j)}\equiv f_i^{(j)}\Big|_{y=0\,,}\quad f_{ih}^{(j)}\equiv f_i^{(j)}\Big|_{y=h,}\quad(i=1,2,3\,;j=1,2),D_1\equiv\frac{\partial}{\partial y}\,. \tag{72}$$

V. SOLUTION OF THE INTEGRAL EQUATIONS

(a) **Thermal part:**

The singular integral equation (32) is a Cauchy-type singular integral equation for an unknown function $\Theta(s)$. For the evaluation of thermal stress it is necessary to solve the integral equation (32). For this purpose we write

$$\Theta(s)=\frac{\Upsilon(s)}{\sqrt{1-s^2}}\,,\qquad(-1<s<1)\,, \tag{73}$$

where $\Upsilon(s)$ is a regular and bounded unknown function. Substituting Eq. (73) into Eq. (32) and using Gauss-Chebyshev formula[28] , we obtain

$$\frac{1}{N}\left[\sum_{k=1}^{N}\left\{\frac{1}{s_k-x_i}+k_1(x_i,s_k)\right\}\Upsilon(s_k)\right]=\frac{T_1-T_2}{h}\,,i=1,2,\dots\dots\dots,N-1\,, \tag{74}$$

and

$$\frac{\pi}{N}\sum_{k=1}^{N}\Upsilon(s_k)=0\,, \tag{75}$$

where s_k and x_i are given by

$$s_k=\cos\left(\frac{2k-1}{2N}\pi\right),(k=1,2,3,\dots\dots,N) \tag{76}$$

$$x_i=\cos\left(\frac{\pi i}{N}\right),(i=1,2,3,\dots\dots,N-1) \tag{77}$$

We observe that corresponding to $(N-1)$ collocation points $x_i=\cos\left(\frac{i\pi}{2(N+1)}\right)$ $,i=1,2,\dots\dots\dots,(N-1)$ we have a set of N linear equations in N unknowns $\Upsilon(s_1),\Upsilon(s_2),\dots\dots\dots\dots\dots\dots,\Upsilon(s_N)$. This linear algebraic system of equations is solved numerically by utilizing Gaussian elimination method.

(b) **Elastic part:**

The singular integral equation (67) is a Cauchy-type singular integral equation for an unknown function $\phi(s)$. Expressing now the solution of Eq. (67) in the form

$$\phi(s) = \frac{\psi(s)}{\sqrt{1-s^2}}\,,(-1 < s < 1)\,, \tag{78}$$

where $\psi(s)$ is a regular and bounded unknown function and using Gauss-Chebyshev formula[28] to evaluate the integral equation (67) , we obtain ,

$$\frac{1}{N}\left[\sum_{k=1}^{N}\left[\frac{1}{s_k - r_i} + k_2^*(r_i, s_k)\right]\psi(s_k)\right] = -1 \pm \frac{Q}{\pi}k_3^*(r_i) + k_4^*(r_i), i = 1,2, \dots \dots .., N-1 \tag{79}$$

$$\frac{\pi}{N}\sum_{k=1}^{N}\psi(s_k) = 0\,, \tag{80}$$

where r_i is given by

$$r_i = cos\left(\frac{i\pi}{N}\right), (i = 1,2,3, \dots \dots ., N-1) \tag{81}$$

We observe that corresponding to $(N-1)$ collocation points $x_r = cos\left(\frac{r\pi}{2(N+1)}\right)$, $r = 1,2, \dots \dots \dots, (N-1)$ the Eqs. (79) and (80) represent a set of N linear equations in N unknowns $\psi(s_1)\,, \psi(s_2)\,, \dots \dots \dots \dots \dots \dots .., \psi(s_N)$. This linear algebraic system of equations are solved numerically by utilizing Gaussian elimination method.

VI. DETERMINATION OF STRESS-INTENSITY FACTOR

Presence of a crack in a solid significantly affects the stress distribution compared to the state when there is no crack. While the stress distribution in a solid with a crack in the region far away from the crack is not much disturbed , the stresses in the neighbourhood of the crack tip assumes a very high magnitude. In order to predict whether the crack has a tendency to expand further , the stress intensity factor (**SIF**) , a quantity of physical interest , has been defined in fracture mechanics. In our present problem the solid under consideration is acted upon by two types of loading: (a) Thermal loading (b) Mechanical loading (concentrated forces of magnitude $\frac{P}{2}$ applied symmetrically on the crack faces). The stress components $\sigma_x\,, \sigma_y\,, \tau_{xy}$ given by Eqs. (52)-(54) do not have closed form expressions. As they are expressed in taking of infinite integrals , they are to be evaluated numerically. In our present discussion we shall be interested to determine the SIF when both the thermal and mechanical loadings are present (thermomechanical stress intensity factor) and also the stress intensity factor when only thermal loading is present (thermal stress intensity factor). The stress intensity factor is defined as

$$k(b) = \lim_{r\to 1}\sqrt{2b(r-1)}\sigma_y^*(r,0).$$

The non-dimensional stress intensity factor when both mechanical and thermal load are present (TMSIF) , can be obtained interms of the solution of the integral equation (67) as

$$TMSIF = k^{'}(b) = \frac{1}{p_0\sqrt{b}}\;\lim_{r\to 1}\sqrt{2b(r-1)}\sigma_y^*(r,0) = -\psi(1), \tag{82}$$

When there is only thermal loading , the TSIF can be extracted following the same procedure as discussed above. In this case the Eq.(67) will be modified to

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{s-r} + k_2^*(r,s)\right]\phi(s)ds = -1 + k_4^*(r)\,, (-1 < r < 1)\,, \tag{83}$$

The TSIF at the crack tip can be expressed in terms of the solution of the integral equation (83) as

$$TSIF = \frac{1}{p_0\sqrt{b}}\;\lim_{r\to 1}\sqrt{2b(r-1)}\sigma_y^*(r,0) = -\Omega(1), \tag{84}$$

in this case we assume

$$\phi(s) = \frac{\Omega(s)}{\sqrt{1-s^2}}\,, -1 < s < 1, \tag{85}$$

where $\Omega(s)$ is regular and bounded unknown function and $\psi(1), \Omega(1)$ can be found out from $\psi(s_k)$ and $\Omega(s_k)$ $(k = 1,2,3, \dots \dots \dots .., N)$ using the interpolation formulas given by Krenk[29] .

Following the method as in Gupta and Erdogan[30] we obtain the crack surface displacement in the form

$$v^{'}(r,0) = \int_{-1}^{r}\frac{\psi(s)}{\sqrt{1-s^2}}dt\,, (-1 < s < 1), \tag{86}$$

where

$$v'(r,0) = \frac{v(r,0)}{b} \frac{4\mu\nu}{p_0(\kappa-1)}, \tag{87}$$

which can be obtained numerically , using say , Simpson's $\frac{1}{3}$ integration formula and appropriate interpolation formula.

VII. NUMERICAL RESULTS AND DISCUSSIONS

The present study is related to the study of an internal crack problem in an infinite functionally graded layer under thermal effect. The main objective of the present discussion is to study the effects of temperature , graded parameter as well as of applied loads on stress intensity factor and crack opening displacement. Following the standard numerical method described , the normal displacement component and the stress intensity factor are computed and shown graphically.

Before analyzing our numerical results we denote the regions $-h \leq y \leq 0$ and $0 \leq y \leq h$ below and above the line of crack in the layer by R_1 and R_2 respectively. Fig. 2(a) shows the temperature distribution on the crack faces for various values of $\frac{b}{h}$. As expected , the result shows that the temperature increases with the decrease of layer thickness. Fig. 2(b) shows the temperature distribution along $x = 0$,taking $T_1 > T_2$. Temperature decreases linearly from the region R_1 to the region R_2 . There is one point to note here that the variations of temperature at a particular point on $x = 0$ below and above the line of crack are opposite in nature in respect of the values of $\frac{b}{h}$.

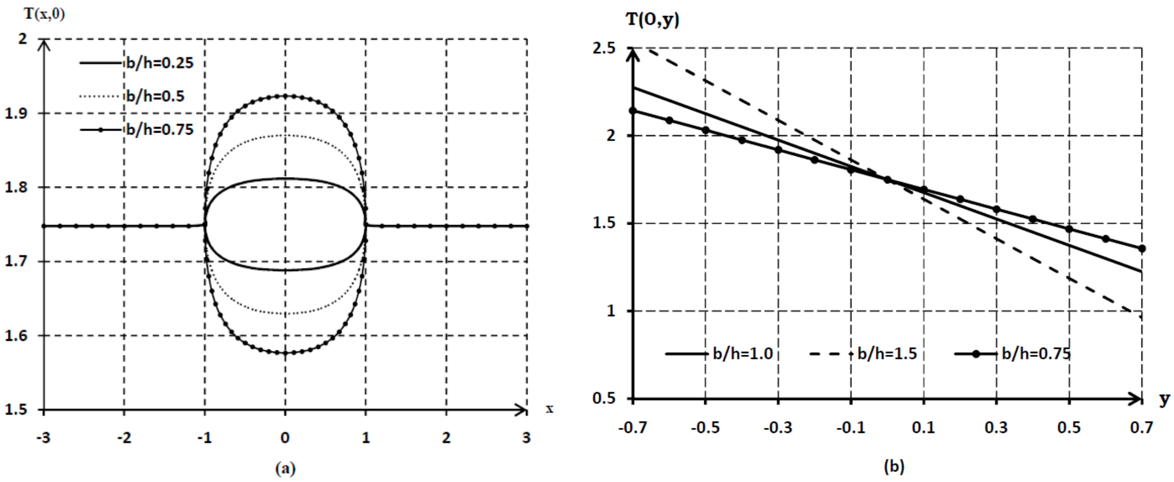
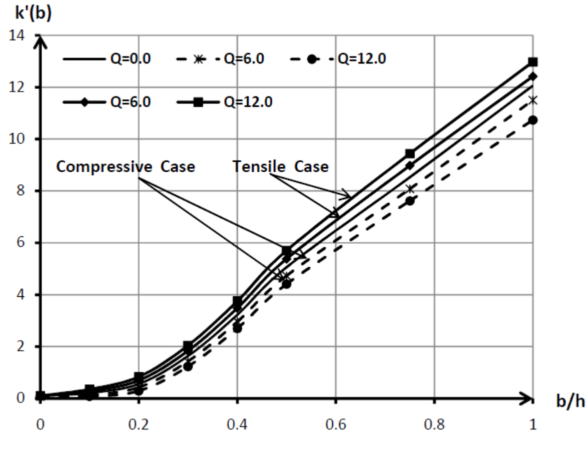


Fig.2(a) Temperature distribution on crack face and extension line for different $\frac{b}{h}$ when $\kappa = 1.8$, $H = 1.0, \alpha_t = 1.5, T_1 = 2.5, T_2 = 1.0$. (b) Temperature distribution on the line $x = 0$ for different $\frac{b}{h}$ when $\kappa = 1.8, H = 1.0$, $\alpha_t = 1.5, T_1 = 2.5, T_2 = 1.0$.

The variation of normalized thermo-mechanical stress intensity factor (TMSIF) $k'(b)$ with crack length $\frac{b}{h}$ are shown in Fig. 3 for both the cases of two symmetric transverse pair of compressive and tensile concentrated forces. It is observed from Figs. 3(a , b) that for compressive concentrated forces the TMSIF decreases with the increase of the load ratio Q, and the increase of $k'(b)$ is quite significant for smaller values of Q. It is also observed from Figs. 3(a,b) that the load ratio Q does not have much effect on $k'(b)$ when the crack length is sufficiently small. Contrary to this , where the force is of tensile nature , $k'(b)$ increases with Q. For small crack length , the behavior of $k'(b)$ is similar to the case of compressive concentrated load.

In Fig. 4 , TMSIF experiences the effect of graded parameter β for fixed load ratio Q for both the regions R_1 and R_2. It is observed that in both compressive and tensile load conditions $k'(b)$ increases slightly with graded parameter β upto a certain distance from the center of the crack , while the effect is reversed and significant afterward. Fig. 5 displays the variation of $k'(b)$ for different position of loading. It is noted that in the case of compressive concentrated forces , $k'(b)$ increases with increasing $\frac{a}{b}$, but it decreases in the case of tensile concentrated forces.



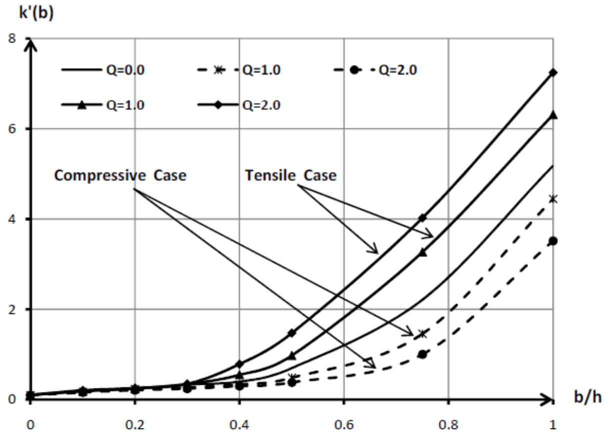


Fig.3(a) Variation of TMSIF, $k'(b)$ with $\frac{b}{h}$ for different loads Q in both cases in the region R_2 when ($\frac{a}{b} = 0.0, \kappa = 1.8, \beta = 0.1, H = 1.0, \alpha_t = 1.5, T_1 = 2.5, T_2 = 1.0$). (b) Variation of TMSIF, $k'(b)$ with $\frac{b}{h}$ for different loads Q in both cases in the region R_1 when ($\frac{a}{b} = 0.0, \kappa = 1.8, \beta = 0.1, H = 1.0, \alpha_t = 1.5, T_1 = 2.5, T_2 = 1.0$).

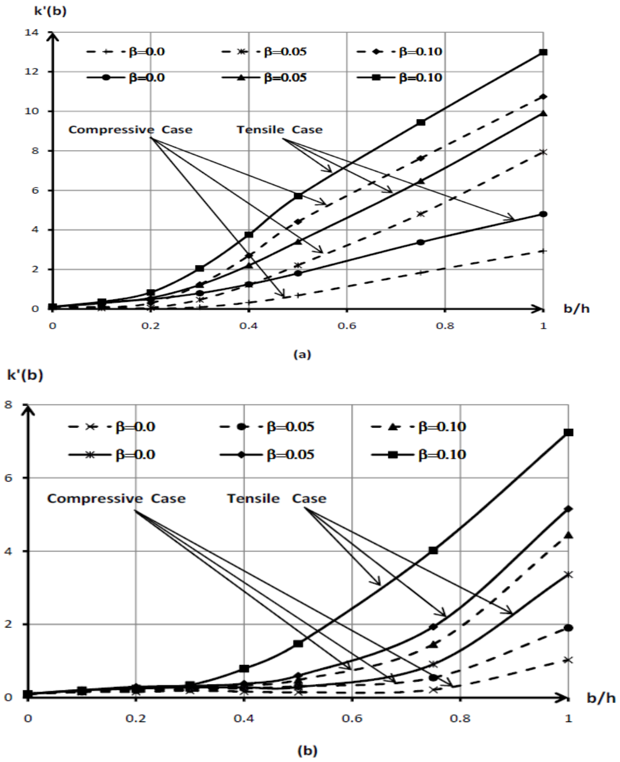


Fig.4(a) Effect of graded parameter β on TMSIF, $k'(b)$ for both cases in the region R_2 ($\frac{a}{b} = 0.0, Q = 12.0, \kappa = 1.8, H = 1.0, \alpha_t = 1.5, T_1 = 2.5, T_2 = 1.0$). (b) Effect of graded parameter β on TMSIF,$k'(b)$ for both cases in the region R_1($\frac{a}{b} = 0.0, Q = 1.0$ (Compressive),$Q = 2.0$ (Tensile),$\kappa = 1.8, H = 1.0, \alpha_t = 1.5, T_1 = 2.5, T_2 = 1.0$).

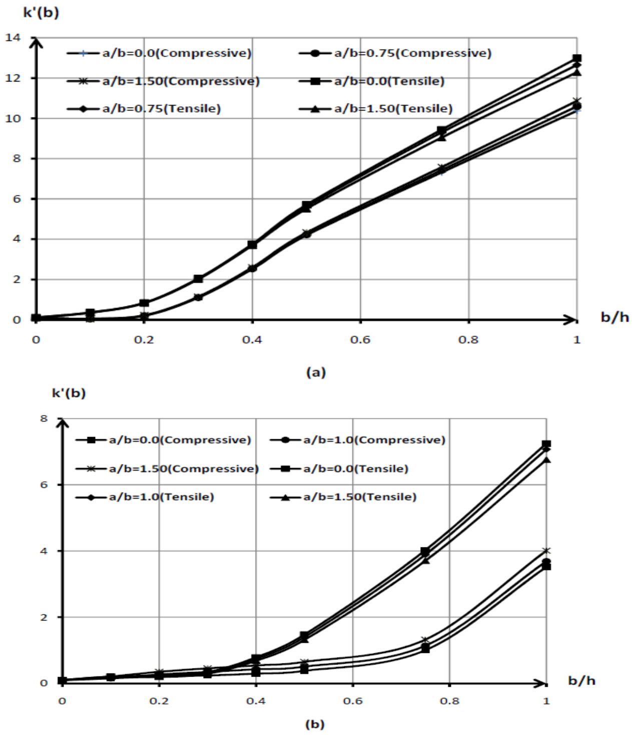
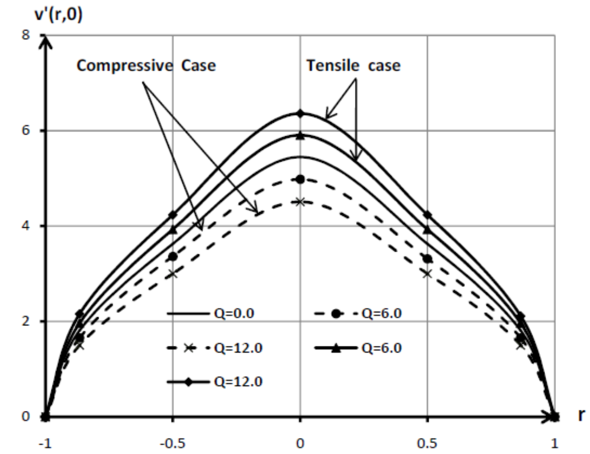
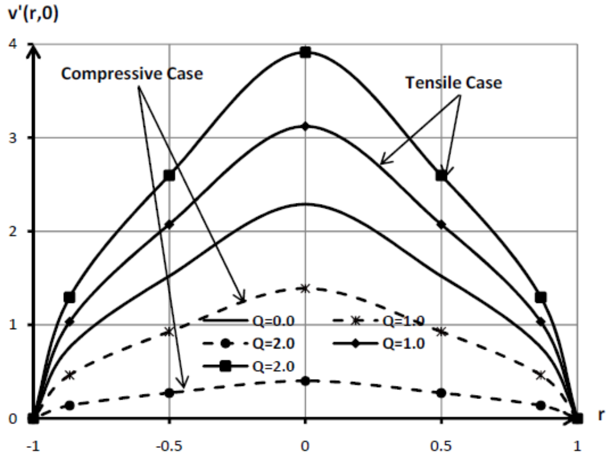


Fig.5(a)Variation of TMSIF,$k'(b)$ for different values of $\frac{a}{b}$ for both cases in the region R_2 ($Q = 16.0$ (Compressive), $Q = 12.0$(Tensile),$\alpha = 1.8, \beta = 0.1, H = 1.0, \alpha_t = 1.5, T_1 = 2.5, T_2 = 1.0$). (b) Variation of TMSIF, $k'(b)$ for different values of $\frac{a}{b}$ for both cases in the region R_1($Q = 2.0, \kappa = 1.8, \beta = 0.1, H = 1.0, \alpha_t = 1.5, T_1 = 2.5, T_2 = 1.0$).

Fig. 6 depicts the variation of normalized crack surface displacement $v'(r,0)$ with r for diffeerent values of load ratio Q. It is clear from Figs. 6(a , b) that for compressive nature of forces $v'(r,0)$ decreases as load ratio Q increases, but for tensile nature of loading $v'(r,0)$ decreases as load ratio Q decreases. For both the cases of compressive and tensile concentrated forces the graphs show that the normalized crack surface displacement is symmetrical with respect to origin. The effect of graded parameter β on $v'(r,0)$ is observed in Fig. 7 for both the cases of compressive and tensile concentrated forces for the both the regions R_1 and R_2.



(a)



(b)

Fig.6(a) Normalized crack surface displacement $v'(r,0)$ for various values of Q for both cases in the region R_2 $(\frac{a}{b}=0.0, \frac{b}{h}=1.0$ $, \kappa=1.8,$ $\beta=0.1,$ $H=1.0, \alpha_t=1.5, T_1=2.5, T_2=1.0)$. (b) Normalized crack surface displacement $v'(r,0)$for various values of Q for both cases in the region $R_1(\frac{a}{b}=0.0, \frac{b}{h}=1.0, \kappa=1.8, \beta=0.1, H=1.0, \alpha_t=1.5, T_1=2.5, T_2=1.0)$.

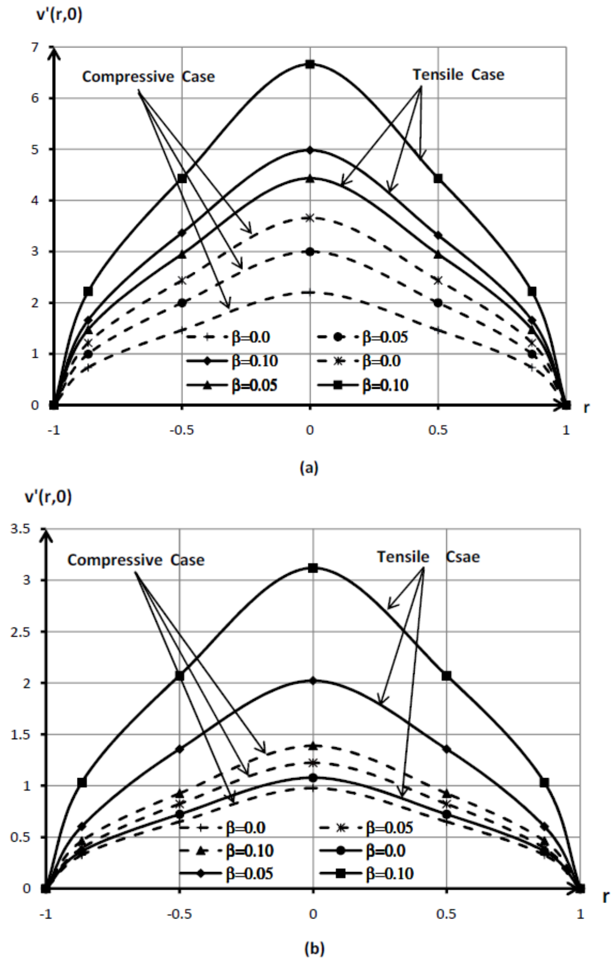


Fig.7(a) Effect of graded parameter β on normalized crack surface displacement $v'(r,0)$ for both cases in the region R_2 ($\frac{a}{b} = 0.0,\ \frac{b}{h} = 1.0,\ Q = 6$ (Compressive), $Q = 16$ (Tensile), $\kappa = 1.8,\ H = 1.0,\ \alpha_t = 1.5,\ T_1 = 2.5,\ T_2 = 1.0$). (b) Effect of graded parameter β on normalized crack surface displacement $v'(r,0)$ for both cases in the region R_1 ($\frac{a}{b} = 0.0,\ \frac{b}{h} = 1.0,\ Q = 1, \kappa = 1.8,\ H = 1.0,\ \alpha_t = 1.5,\ T_1 = 2.5, T_2 = 1.0$).

Fig. 8 illustrates the role of the point of application of loading on the normalized crack surface displacement for a particular load ratio $Q = 6.0$ for the compressive forces while $Q = 12.0$ for the tensile forces for the region R_2 and the load ratio $Q = 1.0$ for both types of compressive and tensile forces for the region R_1 with $\frac{b}{h} = 1.0$. It is observed in Figs. 6.8 (a , b) , that for compressive concentrated loading the normalized crack surface displacement increases with the increased values of $\frac{a}{b}$ but behavior is just opposite for tensile concentrated loading.

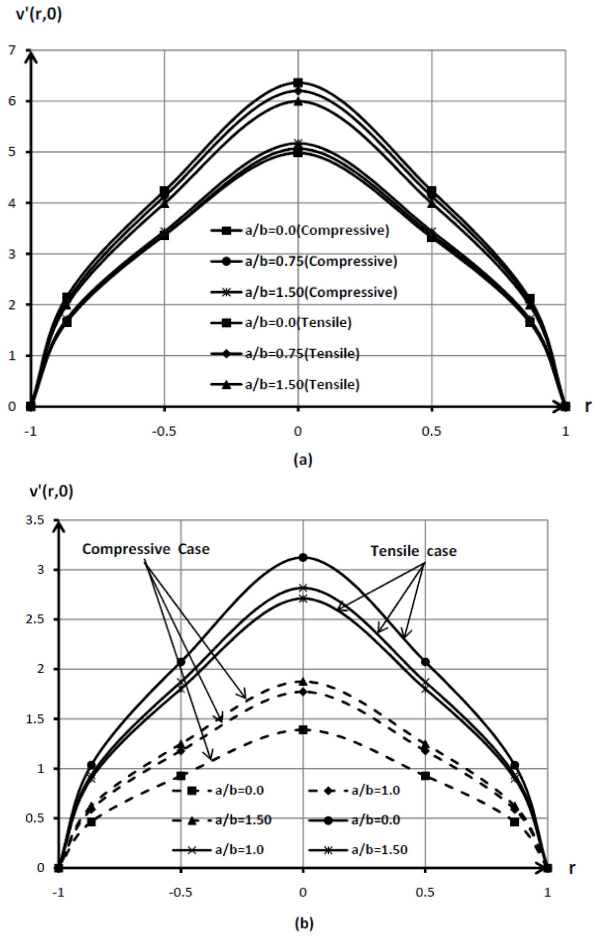


Fig.8(a)Normalized crack surface displacement $v'(r,0)$ for various values of $\frac{a}{b}$ for both cases in the region R_2 ($Q = 6.0$ (Compressive),$Q = 12$(Tensile), $\frac{b}{h} = 1.0, \kappa = 1.8,\ \beta = 0.1,\ H = 1.0,\ \alpha_t = 1.5,\ T_1 = 2.5,\ T_2 = 1.0$). (b) Normalized crack surface displacement $v'(r,0)$ for various values of $\frac{a}{b}$ for both cases in the region R_1 ($Q = 1.0,\ \frac{b}{h} = 1.0,\ \kappa = 1.8,\ \beta = 0.1,\ H = 1.0, \alpha_t = 1.5, T_1 = 2.5, T_2 = 1.0$).

Fig. 9 displays the comparison of the variation of thermo-mechanical stress intensity factor (TMSIF) and thermal stress intensity factor (TSIF) with $\frac{b}{h}$ for both the regions R_1 and R_2 .

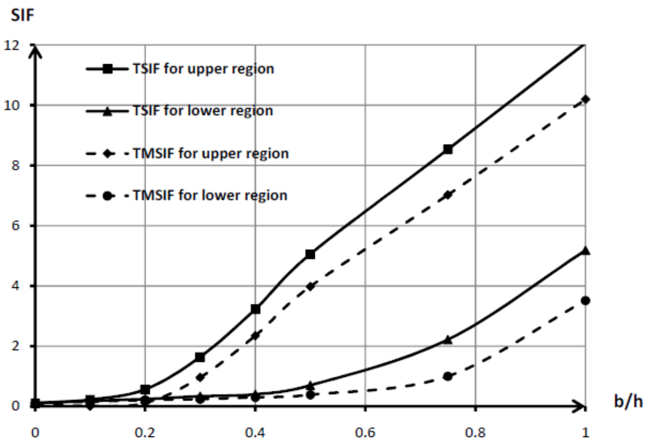


Fig.9 Comparison of TMSIF and TSIF for both the regions R_1 and R_2 ($\frac{a}{b} = 0.0$, $\kappa = 1.8,\ \beta = 0.1,\ H = 1.0,\ \alpha_t = 1.5,\ T_1 = 2.5,\ T_2 = 1.0$).

VIII. CONCLUSION

The present discussion relating thermo-mechanical loading on a functionally graded layer with a crack in it yielded the following observations:

(a) The SIF and the crack surface displacement $v(x,0)$ are very much affected by thermal loading.

(b) The position and the magnitude of mechanical loading affects the SIF and $v(x,0)$.

(c) Grading of medium has significant effects on SIF and $v(x,0)$. Both $v(x,0)$ and SIF increase with increase of rigidity.

(d) The crack length to layer thickness ratio also plays important roles on the behavior of SIF and $v(x,0)$.

IX. APPENDIX

We set $\Delta = |L|^{-1}$, then for D_{ki} ($k = 1,2,3$ and $i = 1,\ldots\ldots,4$) we have the following expressions:

$$D_{11}(\xi) = \Delta[(m_4G_2G_3 - m_3G_2G_4)e^{m_2h} + (m_2G_3G_4 - m_4G_2G_3)e^{m_3h} + (m_3G_2G_4 - m_2G_3G_4)e^{m_4h}],$$

$$D_{12}(\xi) = \Delta[(m_3G_1G_4 - m_4G_1G_3)e^{m_1h} + (m_4G_1G_3 - m_1G_3G_4)e^{m_3h} + (m_1G_3G_4 - m_3G_1G_4)e^{m_4h}],$$

$$D_{13}(\xi) = \Delta[(m_4G_1G_2 - m_2G_1G_4)e^{m_1h} + (m_1G_2G_4 - m_4G_1G_2)e^{m_2h} + (m_2G_1G_4 - m_1G_2G_4)e^{m_4h}],$$

$$D_{14}(\xi) = \Delta[(m_2G_1G_3 - m_3G_1G_2)e^{m_1h} + (m_3G_1G_2 - m_1G_2G_3)e^{m_2h} + (m_1G_2G_3 - m_2G_1G_3)e^{m_3h}],$$

$$D_{21}(\xi) = \Delta\big[(G_3G_4S_2 - G_2G_3S_4)e^{(m_2+m_4)h} + (G_2G_3S_4 - G_2G_4S_3)e^{(m_3+m_4)h} + (G_2G_4S_3 - G_3G_4S_2)e^{(m_2+m_3)h}\big],$$

$$D_{22}(\xi) = \Delta\big[(G_1G_3S_4 - G_3G_4S_1)e^{(m_1+m_4)h} + (G_1G_4S_3 - G_1G_3S_4)e^{(m_3+m_4)h} + (G_3G_4S_1 - G_1G_4S_3)e^{(m_1+m_3)h}\big],$$

$$D_{23}(\xi) = \Delta\big[(G_2G_4S_1 - G_1G_2S_4)e^{(m_1+m_4)h} + (G_1G_2S_4 - G_1G_4S_2)e^{(m_2+m_4)h} + (G_1G_4S_2 - G_2G_4S_1)e^{(m_1+m_2)h}\big],$$

$$D_{24}(\xi) = \Delta\big[(G_1G_2S_3 - G_2G_3S_1)e^{(m_1+m_3)h} + (G_1G_3S_2 - G_1G_2S_3)e^{(m_2+m_3)h} + (G_2G_3S_1 - G_1G_3S_2)e^{(m_1+m_2)h}\big],$$

$$\begin{aligned}D_{31}(\xi) = \Delta[&(m_4P_2S_2G_3 + m_4P_1G_2G_3 - m_3P_2S_2G_4 - m_3P_1G_2G_4)e^{m_2h}\\ &+(m_2P_2S_3G_4 + m_2P_1G_3G_4 - m_4P_2S_3G_2 - m_4P_1G_2G_3)e^{m_3h}\\ &+(m_3P_2S_4G_2 + m_3P_1G_2G_4 - m_2P_2S_4G_3 - m_2P_1G_3G_4)e^{m_4h}\\ &+(m_4P_4S_3G_2 - m_4P_4S_2G_3 + P_3S_3G_2G_4 - P_4S_3G_2G_4 - P_3S_2G_3G_4 + P_4S_2G_3G_4)e^{(m_2+m_3)h}\\ &+(-m_3P_4S_4G_2 - P_3S_4G_2G_3 + P_4S_4G_2G_3 + m_3P_4S_2G_4 + P_3S_2G_3G_4 - P_4S_2G_3G_4)e^{(m_2+m_3)h}\\ &+(m_2P_4S_4G_3 + P_3S_4G_2G_3 - P_4S_4G_2G_3 - m_2P_4S_3G_4 - P_3S_3G_2G_4 + P_4S_3G_2G_4)e^{(m_3+m_4)h}\big],\end{aligned}$$

$$\begin{aligned}D_{32}(\xi) = \Delta[&(-m_4P_2S_1G_3 - m_4P_1G_1G_3 + m_3P_2S_1G_4 + m_3P_1G_1G_4)e^{m_1h}\\ &+(m_4P_2S_3G_1 + m_4P_1G_1G_3 - m_1P_2S_3G_4 - m_1P_1G_3G_4)e^{m_3h}\\ &+(-m_3P_2S_4G_1 + m_1P_2S_4G_3 - m_3P_1G_1G_4 + m_1P_1G_3G_4)e^{m_4h}\\ &+(-m_4P_4S_3G_1 + m_4P_4S_1G_3 - P_3S_3G_1G_4 + P_4S_3G_1G_4 + P_3S_1G_3G_4 - P_4S_1G_3G_4)e^{(m_1+m_3)h}\\ &+(m_3P_4S_4G_1 + P_3S_4G_1G_3 - P_4S_4G_1G_3 - m_3P_4S_1G_4 - P_3S_1G_3G_4 + P_4S_1G_3G_4)e^{(m_1+m_4)h}\\ &+(-m_4P_4S_4G_3 - P_3S_4G_1G_3 + P_4S_4G_1G_3 + m_1P_4S_3G_4 + P_3S_3G_1G_4 - P_4S_3G_1G_4)e^{(m_3+m_4)h}\big],\end{aligned}$$

$$\begin{aligned}D_{33}(\xi) = \Delta[&(m_4P_2S_1G_2 + m_4P_1G_1G_2 - m_2P_2S_1G_4 - m_2P_1G_1G_4)e^{m_1h}\\ &+(-m_4P_2S_2G_1 - m_4P_1G_1G_2 + m_1P_2S_2G_4 + m_1P_1G_2G_4)e^{m_2h}\\ &+(m_2P_2S_4G_1 - m_1P_2S_4G_2 + m_2P_1G_1G_4 - m_1P_1G_2G_4)e^{m_4h}\\ &+(m_4P_4S_2G_1 - m_4P_4S_1G_2 + P_3S_2G_1G_4 - P_4S_2G_1G_4 - P_3S_1G_2G_4 + P_4S_1G_2G_4)e^{(m_1+m_2)h}\\ &+(-m_2P_4S_4G_1 - P_3S_4G_1G_2 + P_4S_4G_1G_2 + m_2P_4S_1G_4 + P_3S_1G_2G_4 - P_4S_1G_2G_4)e^{(m_1+m_4)h}\\ &+(m_1P_4S_4G_2 + P_3S_4G_1G_2 - P_4S_4G_1G_2 - m_1P_4S_2G_4 - P_3S_2G_1G_4 + P_4S_2G_1G_4)e^{(m_2+m_4)h}\big],\end{aligned}$$

$$\begin{aligned}D_{34}(\xi) = \Delta[&(-m_3P_2S_1G_2 - m_3P_1G_1G_2 + m_2P_2S_1G_3 + m_2P_1G_1G_3)e^{m_1h}\\ &+(m_3P_2S_2G_1 + m_3P_1G_1G_2 - m_1P_2S_2G_3 - m_1P_1G_2G_3)e^{m_2h}\end{aligned}$$

$$+(-m_2P_2S_3G_1 + m_1P_2S_3G_2 - m_2P_1G_1G_3 + m_1P_1G_2G_3)e^{m_3h}$$
$$+(-m_3P_4S_2G_1 + m_3P_4S_1G_2 - P_3S_2G_1G_3 + P_4S_2G_1G_3 + P_3S_1G_2G_3 - P_4S_1G_2G_3)e^{(m_1+m_2)h}$$
$$+(m_2P_4S_3G_1 + P_3S_3G_1G_2 - P_4S_3G_1G_2 - m_2P_4S_1G_3 - P_3S_1G_2G_3 + P_4S_1G_2G_3)e^{(m_1+m_3)h}$$
$$+(-m_1P_4S_3G_2 - P_3S_3G_1G_2 + P_4S_3G_1G_2 + m_1P_4S_2G_3 + P_3S_2G_1G_3 - P_4S_2G_1G_3)e^{(m_2+m_3)h}].$$

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