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New Mathematical Approach for Economic Power Dispatch Problem with Quadratic Objective Function

S. Kavitha[1], Nirmala P. Ratchagar [2]
[1]Assistant Professor, Department of Mathematics, Annamalai University, Chidambaram-608001, India
[2]Professor, Department of Mathematics, Annamalai University, Chidambaram-608001, India
Corresponding Author: S. Kavitha

Abstract: A new mathematical approach is proposed for the solution of optimization problem with quadratic objective function subject to satisfying equality and inequality constraints. The convergence of the conventional lambda iteration method is improved by the superlinear bracketing method. Lambda value is updated in each iteration by the superlinear bracketing method. The practical application of the proposed method is illustrated with economic power dispatch problem. Economic power dispatch problem is one of the fundamental issues in an electrical power system operation. It is a nonlinear optimization problem and their objective is to minimize the total fuel cost of a thermal power generating units subject to satisfying various operating constraints. The proposed mathematical approach is implemented for the solution of economic power dispatch problem. The result shows that the proposed method provides the optimal solution and it requires lesser number of iterations for the convergence.

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I. INTRODUCTION

The numerous mathematical methods are developed in the literature to find solution for the optimization problem with quadratic objective function. There are several disadvantages of these methods for finding solution for the non-linear optimization problem. In λ-iteration method, suitable initial value must be selected and this method needs more number of iterations for the convergence.

Similarly, gradient, dynamic programming and quadratic programming have certain difficulties to solve this problem. The computational difficulty is prominent in dynamic programming approach which is referred as the curse of dimensionality[1-4]. Mullers method[5], Brent's method[6], Bisection method[7], Regula-falsi[8], Bracketing method[9] were developed in the literature to solve the nonlinear equations. Recently, Superlinear bracketing method[10] is reported in the literature to solve the nonlinear equations. The Superlinear bracketing method is able to find the roots for the nonlinear equations on the given interval.

A new method based on superlinear bracketing approach is proposed to find the solution of quadratic objective function with equality and inequality constraints. The lambda value of lagragian multiplier method is updated using the superlinear bracketing method[10]. The problem considered in this paper resembles the practical economic power scheduling problem in an electrical power system[11]. Economic power dispatch is the important optimization problem in thermal power stations. The solution of this problem gives the optimal power generation schedule of the committed electrical generators that minimize the total fuel cost subject to satisfying the equality and inequality constraints[12]. The equality constraint is the total power generation should be equal to total power demand and transmission losses. The inequality constraints are the power generation of the committed generating units should lie within the operating limits.

II. PROBLEM FORMULATION

The objective function of the optimization problem is formulated by $Min\,Z = F(X)$, the objective is expressed as the sum of n single-variable function $F_1(x_1)$, $F_2(x_2)$, …, $F_n(x_n)$. i .e, $F(x_1, x_2, \dots ,x_n) = F_1(x_1)+F_2(x_2)+ \dots + F_n(x_n)$. The minimization objective function is stated as

$$Min\,Z = \sum_{i=1}^{n} F_i(x_i) \tag{1}$$

where $F_i(x_i)$ is quadratic function. It is defined as

$$F_i(x_i) = a_i x_i^2 + b_i x_i + c_i \tag{2}$$

a_i, b_i, and c_i are coefficients of decision variable x_i, and n is the total number of variables. The above objective is subject to the following constraints,

(i) Equality constraint

$$\sum_{i=1}^{n} x_i = X_T + X_L \qquad (3)$$

where X_T is fixed value and nonlinear equation X_L is given by

$$X_L = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i A_{ij} x_j + \sum_{i=1}^{n} A_{0i} x_i + A_{00} \qquad (4)$$

The coefficients A_{ij}, A_{0i} and A_{00} are constants and A_{ij} is a symmetrical matrix.

(ii) Inequality constraint

$$x_i^{min} \le x_i \le x_i^{max}, \ for \ i = 1, 2, \dots, n \qquad (5)$$

where $x_i^{\min}$ and $x_i^{\max}$ are the lower and upper limits of the *i*th decision variable.

(iii) Non negativity constraint

$$x_i \ge 0, \text{ for i} = 1, 2, \dots, n \qquad (6)$$

III. COMPUTATIONAL PROCEDURE

The equality constraint is added with objective function using lagrangian multiplier λ,

$$\overline{Z} = \sum_{i=1}^{n} F_i(x_i) - \lambda\left(\sum_{i=1}^{n} x_i - X_T - X_L\right) \qquad (7)$$

Therefore

$$\overline{Z} = \sum_{i=1}^{n}(a_i x_i^2 + b_i x_i + c_i) - \lambda\left(\sum_{i=1}^{n} x_i - X_T - X_L\right) \qquad (8)$$

Differentiating the equation (8) with respect to control variable x_i and then equated to zero.

That is, $$\frac{\partial \overline{Z}}{\partial x_i} = 0 \qquad (9)$$

$$b_i + 2a_i x_i + \lambda\left(\sum_{j=1}^{n} 2A_{ij} x_j + A_{0i}\right) = \lambda \qquad (10)$$

$$b_i + 2a_i x_i + 2\lambda A_{ii} x_i + \lambda\left(\sum_{\substack{j=1 \\ j\neq i}}^{n} 2A_{ij} x_j + A_{0i}\right) = \lambda \qquad (11)$$

By rearranging, we get

$$2x_i(a_i + \lambda A_{ii}) = \lambda - \lambda\sum_{\substack{j=1 \\ j\neq i}}^{n} 2A_{ij} x_j - b_i - A_{0i}\lambda \qquad (12)$$

Hence $$x_i = \frac{\lambda(1 - A_{0i}) - b_i - 2\lambda\sum_{\substack{j=1 \\ j\neq i}}^{n} A_{ij} x_j}{2(a_i + \lambda A_{ii})}$$

for i = 1, 2, … , n (13)

and $$\lambda = \frac{b_i + 2a_i x_i}{1 - \left(\sum_{j=1}^{n} 2A_{ij} x_j + A_{0i}\right)} \qquad (14)$$

The computational procedure to calculate the limits of λ for $x_i^{\min}$, is given below:

1. Determine the λ value using eqn. (14). In eqn. (14), x_i is substituted by $x_i^{\min}$ and x_j is substituted by $x_j^{\min}$.
2. Fix the iteration number k=0. The initial value of of x_i^k is calculated by

$$x_i^k = \frac{(x_i^{\min} + x_i^{\max})}{2},$$

for i = 1, 2, … , n. (15)

3. Calculate x_i^{k+1} from the equation (13).
4. Verify the convergence by calculating Δx from the current and previous iterations, i.e. $\Delta x = x_i^{k+1} - x_i^k$. If Δx is less than a pre-specified tolerance go to step-6 .

5. Increase the iteration by 1, that is k=k+1. The decision variables calculated in step-3 will be taken as an initial value and new values are calculated using step-3 and check the convergence using step-4.
6. Using equation (14), calculate the λ-value, calculate X_T from the following equation,

$$X_T = \sum_{i=1}^{n} x_i - X_L \tag{16}$$

The computational steps given above is repeated for x_i^{max}. Therefore for n-variables, 2n λ-values and their corresponding X_T are computed. The lookup table is prepared by arranging these values in ascending order. Using the superlinear bracketing method [10], the optimal value of lambda is calculated.
The steps involved to solve the optimization problem described in equations(1) to (6) are detailed below:
Step1: For the given value of X_T, determine the limits of λ from the look-up Table.
Step 2: Determine the values of variable x_i for λ_1 and for λ_2 using eqn. (13).
Step 3: Calculate the function values $f_1=f(\lambda_1)$ using variables x_i corresponding to λ_1,

i.e $$f(\lambda_1) = \sum_{i=1}^{n} x_i - X_T - X_L \tag{17}$$

and $f_2=f(\lambda_2)$ using variables x_i corresponding to λ_2,

i.e $$f(\lambda_2) = \sum_{i=1}^{n} x_i - X_T - X_L \tag{18}$$

Step 4: If sign(f_1) is equal to sign(f_2) then adjust the values of λ_1 and λ_2 such that sign of $f(\lambda_1)$ is negative and sign of $f(\lambda_2)$ is positive. Check the initial condition. If sign(f_1) == sign(f_2) then adjust the values of λ_1 and λ_2 such that sign of $f(\lambda_1)$ is –ve and sign of $f(\lambda_2)$ is +ve.
Step 5: Set maximum number of iteration IT_{max}, tolerance ε and iteration counter k1=0. Calculate the initial mid-point $\lambda_0 = \frac{\lambda_1+\lambda_2}{2}$ and the function value $f_0=f(\lambda_0)$. Set $\lambda_m=\lambda_0$.
Step 6: Calculate

$$A_1 = \frac{f_1 - f_0}{(\lambda_1 - \lambda_0)(\lambda_1 - \lambda_2)} + \frac{f_0 - f_2}{(\lambda_2 - \lambda_0)(\lambda_1 - \lambda_2)} \tag{19}$$

$$B_1 = \frac{(f_0 - f_1)(\lambda_2 - \lambda_0)}{(\lambda_1 - \lambda_0)(\lambda_1 - \lambda_2)} - \frac{(f_0 - f_2)(\lambda_1 - \lambda_0)}{(\lambda_2 - \lambda_0)(\lambda_1 - \lambda_2)} \tag{20}$$

$$C_1 = f_0 \tag{21}$$

Step 7: Determine

$$\lambda_p = \lambda_0 - \frac{2C_1}{B_1 + \mathrm{sgn}(B_1)\sqrt{B_1^2 - 4A_1C_1}} \tag{22}$$

Step 8: If $\lambda_p>\lambda_2$ or $\lambda_p<\lambda_1$ then

$$\lambda_p = \lambda_0 - \frac{B_1 + \mathrm{sgn}(B_1)\sqrt{B_1^2 - 4A_1C_1}}{2A_1} \tag{23}$$

Step 9: For λ_p, calculate the values of variables x_i using eqn. (13), if x_i violates its maximum or minimum then fix the value of x_i to upper or lower limit, repectively, and find the function value $f_p = f(\lambda_p)$.
Step10: If $f_1f_0 < 0$ then $\lambda_2 = \lambda_0$, $f_2 = f_0$ else $\lambda_1 = \lambda_0$, $f_1 = f_0$.
Step 11: Set $\lambda_0 = \lambda_p$, $f_0 = f_p$, If k1 > IT_{max}. then stop.
Step 12: If k1 is greater than 1 and $|\lambda_m - \lambda_0| < \varepsilon$ then print the results λ_0, decision variables x_i, and f_0, and stop.
Step14: Increase the iteration by 1, that is k1= k1 + 1 and update $\lambda_m=\lambda_0$.
Step 15: Go to Step 6.

IV. ECONOMIC POWER DISPATCH PROBLEM

In thermal power plants, unit commitment problem identifies the scheduled combination units at each specific period of operation, the economic dispatch planning must perform the optimal generation dispatch among the operating units to satisfy the system power demand and practical operation constraints of thermal generating units.

The economic dispatch problem is formulated as

$$Min\ F_T = \sum_{i=1}^{n} F_i(P_i) = \sum_{i=1}^{n} a_i P_i^2 + b_i P_i + c_i \quad \$/h \tag{24}$$

subject to

(i) power balance constraints

At a particular dispatch interval (usually 1h), the total generation of committed units must be equal to system load demand P_D plus transmission loss P_L are met

$$\sum_{i=1}^{n} P_i = P_D + P_L \quad MW \tag{25}$$

The transmission loss P_L is a function of generator's power output and is calculated using B- matrix loss formula. The general form of the loss formula using B-coefficients is

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n}\sum_{j=1}^{n} B_{0i} P_i + B_{00}\ MW \tag{26}$$

(ii) generating capacity constraints

The physical restrictions on the real power output of generating units constitute the following constraint

$$P_i^{\min} \le P_i \le P_i^{\max}\ \mathrm{MW} \qquad i = 1, 2, 3, ..., n \tag{27}$$

where $P_i^{\min}$ and $P_i^{\max}$ are the minimum and maximum power outputs of the *i*th unit.

V. RESULT AND DISCUSSION

In order to verify the effectiveness of the proposed mathematical approach, a six-generating units sample test system including transmission loss is considered[13]. The operating ranges of all committed generating units are restricted by their minimum and maximum power generation limits. The sample system contains of six thermal generating units. The total load demand of the system is 1263 MW. The cost coefficients, generation limits of each generating unit and transmission loss coefficient are given in Appendix. The proposed approach is applied to sample test system using Matlab 6.5 programming language. The optimal generation schedule obtained through the proposed method minimizes the total fuel cost and satisfies power balance constraint i.e. total output minus loss should be equal to total load demand of the system and other operating constraints. The number of iterations taken by the method for the convergence is four. The optimal solution obtained through the proposed mathematical method is Table 1.

Table 1: Optimal solution obtained through proposed recursive method for the Load demand 1263 MW

Unit power output (MW)	**Proposed method**
P_1	450.9555
P_2	173.0184
P_3	263.6370
P_4	138.0655
P_5	164.9937
P_6	85.3094
Total output	1275.98
Loss(MW)	12.98
Total cost ($/h)	15450

VI. CONCLUSION

In this paper, a new mathematical approach based on superlinear bracketing method is proposed for the optimization problem with quadratic objective function. Mathematical approach developed in this paper provides the optimal solution within few iteration, hence the computation time is minimized in the proposed method. The proposed method can be practically implemented for the solution of various power generation scheduling problems in a thermal power station.

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Appendix

Table A.1: Generating unit capacity and coefficients

Generating unit	a_i $/(MW)^{2}h	b_i $/MWh	c_i $/h	P_i^{min} MW	P_i^{max} MW
1	0.0070	7.0	240	100	500
2	0.0095	10.0	200	50	200
3	0.0090	8.5	220	80	300
4	0.0090	11.0	200	50	150
5	0.0080	10.5	220	50	200
6	0.0075	12.0	190	50	120

B-Loss coefficients matrix

$$B_{ij} = \begin{matrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & -0.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0.0000 & -0.0010 & -0.0006 \\ -0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.0150 \end{matrix}$$

$B_{Oi} = 1.0e^{-03} * [-0.3908\ -0.1297\ 0.7047\ 0.0591\ 0.2161\ -0.6635]$

$B_{OO} = 0.0056$