IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 09, Issue 8 (August. 2019), ||S (I) || PP 17-25

An Evaluation of the System Reliability Using Fuzzy Lifetime Distribution Emphasising Hexagonal Fuzzy Number

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Abstract: – The fuzzy set theory has been applied in many fields such as Management, Engineering etc. In this paper we evaluate system reliability in which the lifetimes of components are described using a fuzzy exponential distribution where formula of a fuzzy reliability function and its α – cut set are presented. The fuzzy reliability of structures is defined on the basis of hexagonal fuzzy number. Using the concept of α-cut of hexagonal fuzzy number, the fuzzy reliability functions of *k-out-of-m* system, series system, parallel systems and their fuzzy mean time to failure are discussed respectively. Finally, some numerical examples are presented to illustrate how to calculate the fuzzy reliability and its α-cut of FMTTF.

Keywords: Fuzzy Reliability, Fuzzy Exponential Distribution, Survival Function, Fuzzy Mean Time To Failure.

Date of Submission: 26-07-2019 Date of Acceptance: 12-08-2019

I. INTRODUCTION

The reliability of an item is the probability that the item will perform its stated mission satisfactorily for the specified condition when used under specific time period. The most frequently used function in lifetime data analysis is the reliability function. This function gives the probability of an item operating for a given time without failure. Exponential distribution is mostly used in determining the lifetime reliability of components. However, in real situation, collections of data or system parameters are often fuzzy/imprecise because of incomplete or vague information. Therefore conventional reliability analysis is inadequate to account for such uncertainties in information. For this reason, the concept of fuzzy reliability has been proposed and developed by several authors [2, 3, 4 & 15].

In other hand a fuzzy number [9] is quantity whose values are imprecise, rather than exact as in the case with single-valued function. Dubois D. and Prade H. in 1983 has defined any of the fuzzy numbers as a fuzzy subset of the real line [7, 8, 9 & 10]. So far fuzzy numbers like triangular fuzzy numbers [5], trapezoidal fuzzy numbers [12 &16], pentagonal fuzzy numbers [13] and hexagonal fuzzy numbers [14] have been introduced with its membership functions. These numbers have got many applications [11] like non-linear equations, risk analysis and reliability. In few cases triangular or trapezoidal is not applicable to solve the problem if it has six different points; hence we make use operation of hexagonal fuzzy number to evaluate fuzzy reliability. In a particular case of the growth rate in bacteria which consists of six points is difficult to solve using trapezoidal or triangular fuzzy numbers, therefore hexagonal fuzzy numbers plays a vital role in solving the problem.

Therefore, in this paper we construct the fuzzy reliability function of systems and its α-cut set using exponential lifetime distribution. The lifetime rate of system is represented by hexagonal fuzzy number. The rest of the paper is organised as follows: Section 2 introduces the fuzzy sets and fuzzy numbers, section 3 describes the fuzzy probability theory. In section 4, we discuss formation of fuzzy survival function and Fuzzy mean time to failure (FMTTF) for series and parallel system and section 5 discuss the FMTTF and fuzzy reliability function of k -out-of-m system. Finally, Section 6 concludes the paper.

II. PRELIMINARIES

To move forward in this section we have a brief introduction about fuzzy sets and fuzzy numbers.

Definition 2.1 (Zadeh (1965)): If a set X be fixed, then a fuzzy set A is given by

$$A=\left\{\left\langle x,\mu_A(x)\right\rangle \middle| x\in X\right\}$$

where $\mu_A(x)\in[0,1]$ is the membership degree of the element $x \in X$.

Definition 2.2 [24]: A Fuzzy number "A" is a convex normalized fuzzy set on the real line R such that:

• There exist at least one $x_0 \in X$ with $\mu_A(x_0) = 1$

• $\mu_A(x)$ is piecewise continuous

Definition 2.3: A fuzzy number $\bar{A}_H$ is a hexagonal fuzzy number denoted by $\bar{A}_H\left(a_1, a_2, a_3, a_4, a_5, a_6\right)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function $\mu_{\bar{A}_H}\left(x\right)$ is given below

$$\mu_{\bar{A}_H}\left(x\right) = \begin{cases} 0, & for\ x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & for\ a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & for\ a_2 \le x \le a_3 \\ 1, & for\ a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & for\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & for\ x > a_6 \end{cases}$$

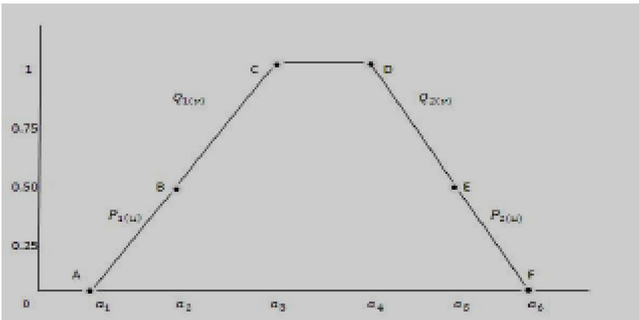

Fig 2. Graphical representation of a normal hexagonal fuzzy number for $x \in [0,1]$

Definition 2.4: An Hexagonal fuzzy number denoted by $\bar{A}_H$ is defined as

$\bar{A}_W = \left(P_1\left(u\right), Q_1\left(v\right), Q_2\left(v\right), P_2\left(u\right)\right)$ for $u \in \left[0, 0.5\right]$ and $v \in \left[0.5, w\right]$ where,

(i) $P_1\left(u\right)$ is a bounded left continuous non decreasing function over $\left[0, 0.5\right]$

(ii) $Q_1\left(v\right)$ is a bounded left continuous non decreasing function over $\left[0.5, w\right]$

(iii) $Q_2\left(v\right)$ is a bounded a continuous non increasing function over $\left[w, 0.5\right]$

(iv) $P_2\left(u\right)$ is a bounded left continuous non increasing function over $\left[0.5, 0\right]$

Remark:

If $w = 1$, then the hexagonal fuzzy number is called a normal hexagonal fuzzy number. Here $\bar{A}_w$ represents a fuzzy number in which "w" is the maximum membership value that a fuzzy number takes on whenever a normal fuzzy number is meant, the fuzzy number is shown by $\bar{A}_H$ for convenience.

Remark:

Hexagonal fuzzy number $\bar{A}_H$ is the ordered quadruple $P_1\left(u\right), Q_1\left(v\right), Q_2\left(v\right), P_2\left(u\right)$ for $u \in \left[0, 0.5\right]$ and $v \in \left[0.5, w\right]$ where,

$$P_1(u) = \frac{1}{2}\left(\frac{u-a_1}{a_2-a_1}\right)$$

$$Q_1(v) = \frac{1}{2}+\frac{1}{2}\left(\frac{v-a_2}{a_3-a_2}\right)$$

$$Q_2(v) = 1-\frac{1}{2}\left(\frac{v-a_4}{a_5-a_4}\right)$$

$$P_2(u) = \frac{1}{2}\left(\frac{a_6-u}{a_6-a_5}\right)$$

ALPHA CUT:

The classical set $\bar{A}_\alpha$ called alpha cut set is the set of elements whose degree of membership is the set of elements whose degree of membership in $\bar{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is no less than, α is defined as

$$A_\alpha = \{x \in X / \mu_{\bar{A}_H}(x) \geq \alpha\}$$

$$= \begin{cases} [P_1(\alpha), P_2(\alpha)] & for\ \alpha \in [0, 0.5) \\ [Q_1(\alpha), Q_2(\alpha)] & for\ \alpha \in [0.5, 1] \end{cases}$$

α- cut operations:

If we get crisp interval by α cut operations interval A_α shall be obtained as follows for all $\alpha \in [0,1]$

Consider $Q_1(x) = \alpha,$

$$(i.e)\frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) = \alpha$$

$$x = 2\alpha(a_3-a_2)-a_3+2a_2$$

$$(i.e)Q_1(\alpha) = 2\alpha(a_3-a_2)-a_3+2a_2$$

Similarly from $Q_2(x) = \alpha,$

$$1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) = \alpha$$

$$x = -2\alpha(a_5-a_4)+2a_5-a_4$$

$$(i.e)Q_1(\alpha) = 2\alpha(a_3-a_2)-a_3+2a$$

This implies

$$[Q_1(\alpha), Q_2(\alpha)] = [2\alpha(a_3-a_2)-a_3+2a, -2\alpha(a_5-a_4)+2a_5-a_4]$$

Consider $P_1(\alpha) = x = 2\alpha(a_2-a_1)+a_1$

Similarly from $P_2(x) = \alpha$

We get, $P_2(\alpha) = -2\alpha(a_6-a_5)+a_6$

This implies

$$[P_1(\alpha), P_2(\alpha)] = [2\alpha(a_2-a_1)+a_1, -2\alpha(a_6-a_5)+a_6]$$

Hence

$$\bar{A}_{H_\alpha} = \begin{cases} \left[2\alpha\left(a_2-a_1\right)+a_1, -2\alpha\left(a_6-a_5\right)+a_6\right] & for\ \alpha\in[0.0.5) \\ \left[2\alpha\left(a_3-a_2\right)-a_3+2a_2, -2\alpha\left(a_5-a_4\right)+2a_5-a_4\right] & for\ \alpha\in[0.0.5] \end{cases}$$

III. FUZZY PROBABILITY THEORY

In this section, we shall present the fuzzy distribution of both Binomial and exponential attributed to Backly [1].

3.1 Fuzzy Binomial distribution

In m independent Bernoulli experiment let us assume that p, probability of a "success" in each experiment is not known precisely and needs to be estimated, or obtained from expert opinion. So that p value is uncertain and we substitute $\bar{p}$ for p and $\bar{q}$ for q so that there is a $p \in p[1]$ and a $p \in p[1]$ with $p+q=1$. Now let $\tilde{p}(r)$ be the fuzzy probability of r success in m independent trials of the experiment. Under our restricted fuzzy algebra we obtain

$$\tilde{P}(r)[\alpha]=\left\{C_m^r p^r q^{m-r} \middle| S_\alpha\right\},$$

For $0\le\alpha\le1$, where now S_α is the statement, "$S_\alpha=\left\{(p,q)\middle| p\in\bar{p}(\alpha), q\in\bar{q}(\alpha), p+q=1\right\}$".

If $\bar{P}(r)[\alpha]=\left[P^L[\alpha], P^U[\alpha]\right]$ then

$P^L[\alpha]=\min\left\{C_m^r p^r q^{m-r}\middle|S\right\}$ and $P^U[\alpha]=\max\left\{C_m^r p^r q^{m-r}\middle|S\right\}$

And if $\tilde{P}[a,b]$ be the fuzzy probability of x success so that $a\le x\le b$, then

$$\tilde{P}([a,b])[\alpha]=\left\{\sum_{x=a}^{b} C_m^x p^x q^{m-x}\middle| S_\alpha\right\}$$

If $\tilde{P}([a,b])[\alpha]=\left[P^L([a,b])[\alpha], P^U([a,b])[\alpha]\right]$ then

$P^L([a,b])[\alpha]=\min\left\{\sum_{x=a}^{b} C_m^x p^x q^{m-x}\middle| S_\alpha\right\}$ and $P^U([a,b])[\alpha]=\max\left\{\sum_{x=a}^{b} C_m^x p^x q^{m-x}\middle| S_\alpha\right\}$

Where S_α is the same with past case.

3.2 Fuzzy exponential distribution

In general, if the lifetime of a component (X) is modeled by an Exponential distribution, then $f(x,y)=\lambda e^{-\lambda x},\ x>0,$ where λ in Exponential distribution. In this case we show the fuzzy probability of obtaining a value in the interval $[c,d], c\ge0$ is as $\bar{P}(c\le X\le d)$ and compute its α - cut as follows

$$\bar{P}(c\le X\le d)[\alpha]=\left\{\int_c^d \lambda e^{-\lambda x}dx\middle|\lambda\in\bar{\lambda}[\alpha]\right\}=\left[P^L[\alpha], P^U[\alpha]\right] \text{ for all } \alpha,$$

Where, $P^L[\alpha]=\min\left\{\int_c^d \lambda e^{-\lambda x}dx\middle|\lambda\in\bar{\lambda}[\alpha]\right\}$ and $P^U[\alpha]=\max\left\{\int_c^d \lambda e^{-\lambda x}dx\middle|\lambda\in\bar{\lambda}[\alpha]\right\}$.

IV. FUZZY RELIABILITY FUNCTIONS

Reliability or survival function (S(t)) is the probability a unit survives beyond time t.

Let the random variable X denote lifetime of a system components, also let X has density function $f(x,\theta)$ (it is known as the lifetime density function), and cumulative distribution function $F_x(t)=P(X\le t)$, then the reliability function at time t is defined as

$S(t)=P(X>t)=1-F_x(t),\quad t>0$, and the unreliability function $Q(t)$ is the probability of failure or the probability of an item failing in the interval $[0,t]$

$$Q(t)=P(X\le t)=F_x(t),\quad t>0.$$

Suppose that we want to calculate reliability of component, such that the lifetime has fuzzy exponential distribution. So we represent parameter $\bar{\lambda}$ with a hexagonal fuzzy number as $\bar{\lambda}=(a_1,a_2,a_3,a_4,a_5,a_6)$. The α - cut $\bar{\lambda}$ denote as follows

$$\bar{\lambda}[\alpha]=\begin{cases}\left[2\alpha(a_2-a_1)+a_1,-2\alpha(a_6-a_5)+a_6\right] & for\ \alpha\in[0,0.5)\\ \left[2\alpha(a_3-a_2)-a_3+2a_2,-2\alpha(a_5-a_4)+2a_5-a_4\right] & for\ \alpha\in[0.5,1]\end{cases}$$

So fuzzy function of component reliability is as follows

$$\tilde{S}(t)[\alpha]=\left\{\int_t^{\infty}\lambda e^{-\lambda x}dx \,\middle|\, \lambda\in\bar{\lambda}[\alpha]\right\}=\left\{e^{-\lambda t}\,\middle|\,\lambda\in\bar{\lambda}[\alpha]\right\}$$

According to that the $e^{-\lambda t}$ decreasing, then

$$\tilde{S}(t)[\alpha]=\begin{cases}\left[e^{-[-2\alpha(a_6-a_5)+a_6]t},e^{-[2\alpha(a_2-a_1)+a_1]t}\right] & for\ \alpha\in[0,0.5)\\ \left[e^{-[-2\alpha(a_5-a_4)+2a_5-a_4]t},e^{-[2\alpha(a_3-a_2)-a_3+2a_2]t}\right] & for\ \alpha\in[0.5,1]\end{cases}$$

Here $\tilde{S}(t)[\alpha]$ is a two dimensional function in terms of α and t.

FMTTF is the expected the mean time to failure. According definition of Buckley [1] FMTTF of any fuzzy system is a fuzzy number and can be calculated as follows:

$$M\tilde{T}TF[\alpha]=\left\{xf(x)dx\,\middle|\,S_\alpha\right\}=\left\{\int_0^{\infty}S(t)dt\,\middle|\,S_\alpha\right\}$$

When the lifetimes have fuzzy exponential distributed then:

$$M\tilde{T}TF[\alpha]==\left\{\int_0^{\infty}\lambda xe^{-\lambda x}dx\,\middle|\,\lambda\in\tilde{\lambda}[\alpha]\right\}=\left\{\frac{1}{\lambda}\,\middle|\,\lambda\in\tilde{\lambda}[\alpha]\right\}$$

Example 1

Let lifetime of electronic component is modeled by an Exponential distribution with fuzzy parameter $\tilde{\lambda}$ that $\tilde{\lambda}$ is "about 0.70 to 0.85" $\left(\tilde{\lambda}=(0.70,0.72,0.75,0.80,0.83,0.85)\right)$. Then α -cut of fuzzy system reliability and FMTTF is given by

$$\tilde{S}(t)[\alpha]=\begin{cases}\left[e^{0.04\alpha t-0.85t},e^{-0.04\alpha t-0.70t}\right] & for\ \alpha\in[0,0.5)\\ \left[e^{0.06\alpha t-0.86t},e^{-0.06\alpha t-0.69t}\right] & for\ \alpha\in[0.5,1]\end{cases}$$

$$M\tilde{T}TF[\alpha]=\begin{cases}\left[\frac{1}{-0.04\alpha+0.85},\frac{1}{0.04\alpha+0.70}\right] & for\ \alpha\in[0,0.5)\\ \left[\frac{1}{-0.06\alpha+0.86},\frac{1}{0.06\alpha+0.69}\right] & for\ \alpha\in[0.5,1]\end{cases}$$

(i) If t =0.4, then fuzzy reliability is as:

$$\tilde{S}(0.4)[\alpha]=\begin{cases}\left[e^{0.016\alpha-0.340},e^{-0.016\alpha-0.280}\right] & for\ \alpha\in[0.0.5)\\ \left[e^{0.024\alpha-0.344},e^{-0.024\alpha-0.276}\right] & for\ \alpha\in[0.0.5]\end{cases}$$

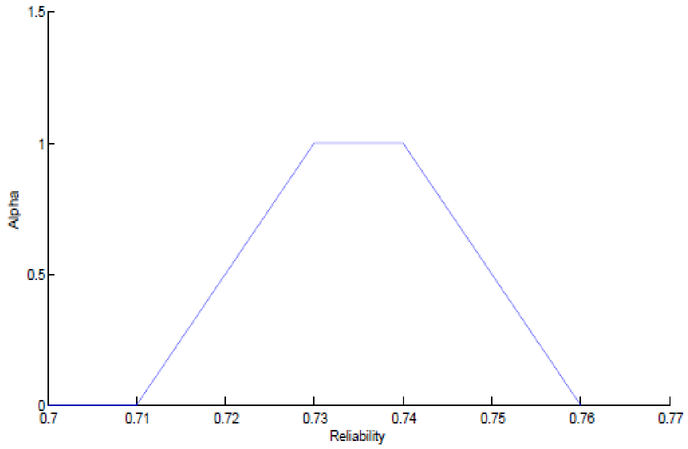

Fig 2. Fuzzy reliability with t=0.4

(ii) If α =0 then reliability band is as

$$\tilde{S}(t)[0]=\left[e^{-0.85t},e^{-0.70t}\right]$$

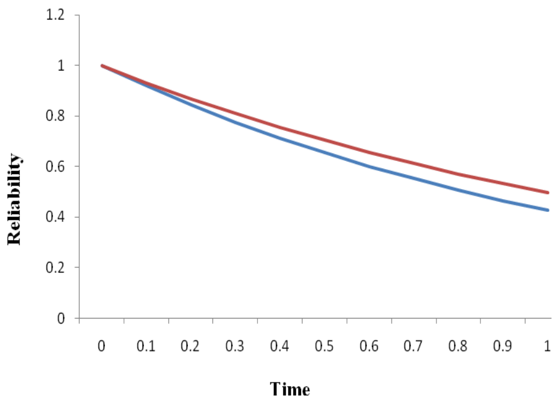

Fig 3. α -cut of fuzzy survival function ($\alpha=0$)

V. k-out-of-m system

A *k-out-of-m* system have m component such that system works if at least k out of m components is working. Each component can be represented by a Bernoulli random variable Y_i with fuzzy reliability (fuzzy survival probability) $\tilde{S}(t)$ and unreliability (fuzzy failure probability) $\tilde{Q}(t)$. That is

$$Y_i=\begin{cases}1, \text{ with fuzzy probability } \tilde{S}(t)\\ 0, \text{ with fuzzy probability } \tilde{Q}(t)\end{cases}$$

Then, $Z=\sum_{i=1}^{m}Y_i$ the number of survivors at time t has the fuzzy Binomial distribution. So, for a k-out-of-m system consisting of m independent and identical components, the fuzzy system reliability function is given by

$$\tilde{R}(t)[\alpha]=\tilde{P}(Z\geq k)=\left\{\sum_{j=k}^{m}C_m^j S(t)^j Q(t)^{m-j}\middle| S_\alpha\right\},$$

For $0\leq\alpha\leq1$ and $t>0$, where S is the statement,

$$S_\alpha=\left\{\left(S(t),Q(t)\right)\middle| S(t)\in\tilde{S}(t)[\alpha],Q(t)\in\tilde{Q}(t)[\alpha],S(t)+Q(t)=1\right\}$$

Both parallel and series systems are special cases of the *k-out-of-m* system. A series system is equivalent to a *1-out-of-m* system. In the first specially case, *k-out-of-m* system is reducing at series systems and its fuzzy reliability function with modeled fuzzy exponential is as:

$$\tilde{R}(t)[\alpha]=\tilde{P}(Z\geq m)=\left\{S(t)^{m}\middle|S_{\alpha}\right\}=\tilde{S}(t)[\alpha]$$

$$=\begin{cases}\left[e^{-\left[-2\alpha(a_6-a_5)+a_6\right]mt},e^{-\left[2\alpha(a_2-a_1)+a_1\right]mt}\right] & for\ \alpha\in[0,0.5)\\ \left[e^{-\left[-2\alpha(a_5-a_4)+2a_5-a_4\right]mt},e^{-\left[2\alpha(a_3-a_2)-a_3+2a_2\right]mt}\right] & for\ \alpha\in[0.5,1]\end{cases}$$

In the second especially case, *k-out-of-m* system is reducing at parallel systems and its α -cut fuzzy reliability function with modeled fuzzy expotential is as:

$$\tilde{R}(t)[\alpha]=\tilde{P}(Z\geq 1)=\left\{1-Q(t)^{m}\middle|S_{\alpha}\right\}$$

$$=\begin{cases}\left[1-\left(1-e^{-\left(-2\alpha(a_6-a_5)+a_6\right)t}\right)^{m},1-\left(1-e^{-\left(2\alpha(a_2-a_1)+a_1\right)t}\right)^{m}\right] & for\ \alpha\in[0,0.5)\\ \left[1-\left(1-e^{-\left(-2\alpha(a_5-a_4)+2a_5-a_4\right)t}\right)^{m},1-\left(1-e^{-\left(2\alpha(a_3-a_2)-a_3+2a_2\right)t}\right)^{m}\right] & for\ \alpha\in[0.5,1]\end{cases}$$

The FMTTF of *k-out-of-m* system calculated as follows:

$$M\tilde{T}TF[\alpha]=\left\{\int_0^{\infty}\sum_{j=k}^{m}C_m^jS(t)^jQ(t)^{m-j}\middle|S_{\alpha}\right\}=\left\{\sum_{j=k}^{m}\frac{1}{\lambda j}\middle|\lambda\in\tilde{\lambda}[\alpha]\right\}$$

$$=\begin{cases}\left[\sum_{j=k}^{m}\frac{1}{\left(-\left(-2\alpha(a_6-a_5)+a_6\right)jt\right)},\sum_{j=k}^{m}\frac{1}{\left(-\left(2\alpha(a_2-a_1)+a_1\right)jt\right)}\right] & for\ \alpha\in[0,0.5)\\ \left[\sum_{j=k}^{m}\frac{1}{\left(-\left(-2\alpha(a_5-a_4)+2a_5-a_4\right)jt\right)},\sum_{j=k}^{m}\frac{1}{\left(-\left(2\alpha(a_3-a_2)-a_3+2a_2\right)jt\right)}\right] & for\ \alpha\in[0.5,1]\end{cases}$$

Example 2

Consider a *3-out-of-4* system consisting of three independent and identical components. Lifetime is modeled as Example 1. Then the reliability function is given by

$$\tilde{R}(t)[\alpha]=\tilde{P}(Z\geq 3)=\left\{\sum_{j=3}^{4}C_4^jS(t)^j\left(1-S(t)\right)^{4-j}\middle|S(t)\in\tilde{S}(t)[\alpha]\right\},$$

$$=\left\{4S(t)^3\left(1-S(t)\right)+S(t)^3\middle|S(t)\in\tilde{S}(t)[\alpha]\right\}$$

According to that the $4S(t)^3\left(1-S(t)\right)+S(t)^3$ increasing, then the α -cut of fuzzy reliability function is given by

$$\tilde{R}(t)[\alpha]=\begin{cases}\left[e^{0.12\alpha t-2.55t}\left(4-3e^{0.04\alpha t-0.85t}\right),e^{-0.12\alpha t-2.10t}\left(4-3e^{-0.04\alpha t-0.70t}\right)\right] & for\ \alpha\in[0,0.5)\\ \left[e^{0.18\alpha t-2.58t}\left(4-3e^{0.06\alpha t-0.86t}\right),e^{-0.18\alpha t-2.07t}\left(4-3e^{-0.06\alpha t-0.69t}\right)\right] & for\ \alpha\in[0.5,1]\end{cases}$$

(i) If t=0.5, then fuzzy reliability is as:

$$\tilde{R}(0.5)[\alpha]=\begin{cases}\left[e^{0.06\alpha-1.275}\left(4-3e^{0.02\alpha-0.425}\right),e^{-0.06\alpha-1.05}\left(4-3e^{-0.02\alpha-0.35}\right)\right] & for\ \alpha\in[0,0.5)\\ \left[e^{0.09\alpha-1.29}\left(4-3e^{0.03\alpha-0.43}\right),e^{-0.09\alpha-1.035}\left(4-3e^{-0.03\alpha-0.345}\right)\right] & for\ \alpha\in[0.5,1]\end{cases}$$

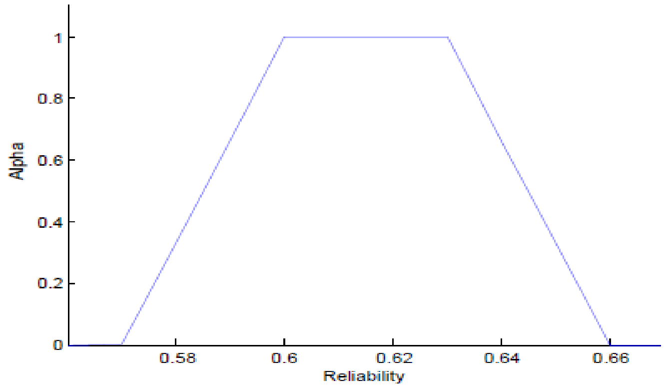

Fig 4. Fuzzy reliability with t=0.5

(ii) If $\alpha = 0$ then reliability band is as:

$$\tilde{R}(t)[0] = \left[e^{-2.55t}\left(4 - 3e^{-0.85t}\right), e^{-2.10t}\left(4 - 3e^{-0.70t}\right) \right] \quad for\ \alpha \in [0, 0.5)$$

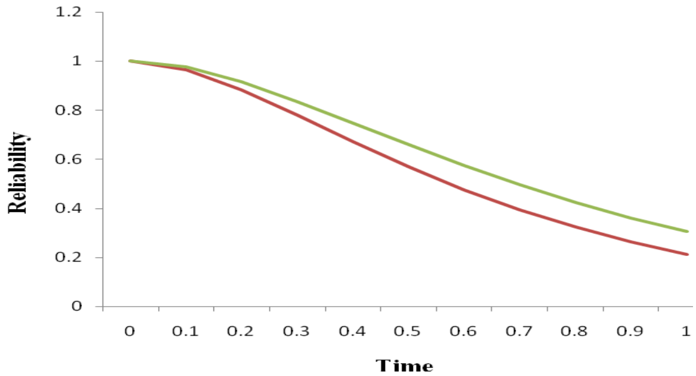

Fig 5. α -cut of fuzzy survival function ($\alpha = 0$)

The of α -cut FMTTF of *3-out-of-4* is as follows:

$$M\tilde{T}TF[\alpha] = \begin{cases} \left[\dfrac{7}{12(-0.04\alpha + 0.85)}, \dfrac{7}{12(0.04\alpha + 0.70)} \right] & for\ \alpha \in [0, 0.5) \\ \left[\dfrac{7}{12(-0.06\alpha + 0.86)}, \dfrac{7}{12(0.06\alpha + 0.69)} \right] & for\ \alpha \in [0.5, 1] \end{cases}$$

VI. CONCLUSION

In this paper hexagonal fuzzy number has been newly introduced for evaluating fuzzy reliability and FMTTF of reliability system. The fuzzy probability theory has been successfully applied to the reliability system in this paper. Whenever, the lifetime rate and lifetimes of components contain fuzziness and randomness respectively, conventional reliability system is not feasible. Thus we invoked successfully the fuzzy distribution to overcome this difficulty using α -cut of hexagonal fuzzy number. It also helps us to solve many optimization problems in future which have six parameters. Our fuzzy theory is a generalization of conventional theory since if lifetime rate is crisp; it turns in to conventional reliability system.

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IOSR Journal of Engineering (IOSRJEN) is UGC approved Journal with Sl. No. 3240, Journal no. 48995.

Deepak Kumar. “An Evaluation of the System Reliability Using Fuzzy Lifetime Distribution Emphasising Hexagonal Fuzzy Number.” IOSR Journal of Engineering (IOSRJEN), vol. 09, no. 08, 2019, pp. 17-25.