**On Fsgb-Connectedness and Fsgb-Disconnectedness in Fuzzy Topological Spaces**

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**Abstract:** The theme of this article is to introduce and investigate a new type of fuzzy strongly generalized b-connectedness namely fsgb-connectedness and fsgb-disconnectedness. Some of their properties and characteristics have been determined.

**AMS subject Classification: 54A40**

**Keywords:** fsgb-connectedness , extremally fsgb-disconnectedness, fts.

**1.Introduction:**

Several real-world issues in economics, medicine, engineering and social science contain imprecise data, and their solutions rely on uncertainty. L.A.Zadeh[17] established the concepts of fuzzy sets and fuzzy operations to deal with such uncertainty. C.L.Chang[6] , who introduced fuzzy topological spaces , presented the analytical aspect of fuzzy set theory practically. The theory of fts was developed by several authors. The concept of b-open sets in general topology was first developed by Andrejevic [1].

Jenifer and Megha introduced the fsgb-closed sets concepts in [9], the concept of fsgb-continuous , fsgb-irresolute, fsgb-open and fsgb-closed mappings in [10] and some new forms of fsgb-continuous maps namely fuzzy strongly generalized b-continuous functions namely strongly fsgb-continuous, perfectly fsgb-continuous and completely fsgb-continuous mappings in fts [12].Also a new weaker form of continuous functions known as upper fsgb-continuous multifunctions and lower fsgb-continuous multifunctions in [13].In this article, the concepts of fsgb-connectedness and fsgb-disconnectedness are introduced and their properties are investigated.

**2.Preliminaries:**

Throughout this study $\left(L,τ\right)$,$(M,σ)$ and $(N,γ)$(or simply *L, M* and *N*) are fuzzy topological spaces (in-short as fts). The interior, closure and compliment of a fuzzy subset *P* of $(L,τ)$ are denoted by Int*(P*), Cl(*P*) and *Pc* respectively. Unless specifically specifies, no separation axiom are expected.

***2*.1 Definition[9]** A fuzzy set(in short f-set) *P* in a fts *L* is called fb-open iff$ P\leq (IntCl(P)∨ClInt($P)).

 **2.2 Definition[9]** Fb-interior and Fb-closure of a fuzzy set *P* is as follows

(i)bInt(*P)* =$˅\left\{Q:Q is a fb-open set of L and P\geq Q\right\}$.

(ii) bCl(*P*) =$˄\left\{R:R is a fb-closed set of L and R\geq P\right\}$.

**2.3Defintion [9]** A f-set *P* in an fts *L* is known as fuzzy generalized closed set (in short(fg-CS) if $Cl(P)\leq Q$ , whenever $P\leq Q$ and *Q* is f-OS in *L.*

**2.4Definition** **[9]**A fuzzy open set (in short f-OS) *P* in a fts *L* is called a fsgb-CS that is fsgb-closed set if $bCl(P)\leq Q$, whenever $P\leq Q$ and *Q* is -open set in *L*.

**2.5Definition [9]**A f-OS *P* in a fts *L* is called a fsgb-open set(in short fsgb-OS) if $bInt(P)\geq Q$, whenever $P\geq Q$and *Q* is -open set in *L*.

**2.6Defintion[10]**A mapping $f:L\rightarrow M$ is said to be fsgb-continuous if $f^{-1}\left(P\right)$ is fsgb -closed set in *L*, for every fuzzy closed set *P* in *M*.

**2.7Definition[10]**A map$ g:L\rightarrow M$ is known as fsgb-irr that is fsgb-irresolute map, if $g^{-1}\left(P\right)$ is fsgb-CS in *L* for every fsgb-CS *P* in *M.*

**2.8Definition[14]** A fuzzy point $l\_{p}\in Q$ is known as quasi-coincident with f-set Q denoted by $l\_{p}$qQ iff $p+Q\left(l\right)>1$. A f-set Q is quasi-coincident with a f-set R denoted by $Q\_{q}R$ iff there exists $l\in L$ such that $\left(l\right)+R\left(l\right)>1$ . If Q and R are not quasi-coincident the we denote it as $Q\_{\overbar{q}}$R.Note that $Q\leq R\leftrightarrow QQ\_{\overbar{q}}(1-R)$.

**3.Fuzzy Strongly Generalized b-Connectedness in fts.**

**Definition 3.1.** A fuzzy topological spaces$\left(L,τ\right)$ is known as fsgb-CdS that is fuzzy strongly generalized b-connected space iff 0 and 1 are the only f-sets which are fsgb-closed and fsgb-open (in short fsgb-clopen) sets.

**Definition 3.2.** A fts $\left(L,τ\right)$ is known as fsgb-Cds between f-sets *P* and *Q* if there does not exist fsgb-clopen set *R* in *L* such that $P\leq R$ and $R\_{\overbar{q}}Q$.

**Theorem 3.3.** A fts$\left(L,τ\right)$ is fsgb-connected iff $\left(L,τ\right)$ is fsgb-CdS between each pair of its non-zero f-sets.

**Proof.** Consider *P* and *Q* are pair of non-zero f-sets of *L*. Let ($L,τ)$ is not fsgb-Cds between *P* and *Q*. Then there exist a fsgb-clopen set *R* of *L* such that $P\leq R$ and $R\_{\overbar{q}}Q$ . As *P* and *Q* are non-zero f-sets and *R* is proper fsgb-clopen set of *L*. Hence$\left(L,τ\right)$ is not fsgb-CdS, which the contradicts the hypothesis.

Conversely, consider$\left(L,τ\right)$ is not fsgb-CdS. Then there is a proper f-set R of *L* that is fsgb-clopen set. Thus$\left(L,τ\right)$ is not fsgb-CdS between *R* and $1-R $ , which contradicts the hypothesis.

**Theorem 3.4**. A fts$\left(L,τ\right)$ is fsgb-connected iff $\left(L,τ\right)$ is fsgb-CdS between P and Q iff there is no fsgb-clopen set R such that $P\leq R\leq 1-Q$.

**Proof:** It is evident.

**Theorem 3.5.** If a fts $\left(L,τ\right)$ is fsgb-CdS between f-sets *P* and *Q* such that $P\leq P\_{1}$ and $Q\leq Q\_{1}$ ,then $\left(L,τ\right)$ is fsgb-Cds between $P\_{1}$ and $Q\_{1}$.

**Proof.** Consider $\left(L,τ\right)$ is not fsgb-CdS between $P\_{1}$ and $Q\_{1}$ .Then there exists a fsgb-clopen set *R* of *L* such that $P\_{1}\leq R$ and $R\_{\overbar{q}}Q\_{1}$. Thus $P\leq R$. Now $R\_{\overbar{q}}Q$ .If $R\_{\overbar{q}}Q$ , then there exists a point $a\in L$ such that $R\left(a\right)+Q\left(a\right)>1$ . Hence $R\left(a\right)+Q\_{1}\left(a\right)>R\left(a\right)+Q\left(a\right)>1$ and $R\_{\overbar{q}}Q\_{1}$ , which contradicts the hypothesis.

**Theorem 3.6.** If a fts $\left(L,τ\right)$ is fsgb-CdS between f-sets *P* and *Q*, then *P* and *Q* are non-zero.

**Proof.** Assume that $=0$ , then P is fsgb-clopen set of L such that $P\leq P$ and $P\_{\overbar{q}}Q$ .Thus $\left(L,τ\right)$ cannot be a fsgb-CdS, which contradicts the hypothesis.

**Theorem 3.7.** Every fsgb-CdS is f-CdS.

**Proof.** Consider $\left(L,τ\right)$ be fsgb-CdS. Let $\left(L,τ\right)$ is not f-CdS and so $∃$ a proper f-set $P(P\ne 0,P\ne 1)$ such that *P* is f-clopen set. As every f-CS is fsgb-CS. Thus $\left(L,τ\right)$ is not fsgb-CdS, which contradicts the hypothesis. Therefore $\left(L,τ\right)$ is f-CdS.

**Theorem 3.8.** A fts $\left(L,τ\right)$ is fsgb-CdS iff $\left(L,τ\right)$ has no non-zero fsgb-OS *P* and *Q* such that $P+Q=1$.

**Proof.** Consider $\left(L,τ\right)$ is fsgb-CdS. If$\left(L,τ\right)$ has 2 non-zero fsgb-OS *P* and *Q* such that $+Q=1$ , so *P* is a proper f-set that is fsgb-clopen set of *L.* Thus$\left(L,τ\right)$ is not fsgb-CdS, which contradicts the hypothesis.

Conversely, consider $\left(L,τ\right)$ is not fsgb-CdS , then ithas a proper f-set *P* of *L* that is fsgb-clopen set. Thus $=1-P$ , is a fsgb-OS of *L* so that $P+Q=1$ , which contradicts hypothesis.

**Remark 3.9.** A fts $\left(L,τ\right)$ is fsgb-CdS iff it has no non-zero f-set *P* and *Q* such that $P+Q=1$ , $fsgb-Cl\left(P\right)+Q=P+fsgb-Cl\left(Q\right)=1$.

**Theorem 3.10.** Consider$g:\left(L,τ\right)\rightarrow (M,σ)$ is fsgb-irr, surjection and *L* is fsgb-Cds, then *M* is fsgb-CdS.

**Proof.** Consider *L* be a fsgb-CdS.Let *M* is not fsgb-CdS and then there is a proper f-set *P* of *M*$(P\ne 0,P\ne 1)$ such that *P* is fsgb-clopen set. As $g$ is fsgb-irr,$g^{-1}(P)$ is fsgb-clopen set of *L*such that $g^{-1}(P)\ne 0$ and $g^{-1}(P)\ne 1$. Therefore $\left(L,τ\right)$ is not fsgb-CdS, which contradicts the hypothesis. Thus $(M,σ)$ is fsgb-CdS.

**Theorem 3.11.** Consider $ g:\left(L,τ\right)\rightarrow (M,σ)$ is fsgb-$CN$ map , surjection and *L* is fsgb-CdS, then *M* is fsgb-CdS.

**Proof.** Consider *L* be a fsgb-CdS. Let *M* is not fsgb-CdS and then there is a proper f-set *P* of *M* $(P\ne 0,P\ne 1)$ such that *P* is fsgb-clopen set. As $g$ is fsgb-$CN$map,$g^{-1}(P)$ is fsgb-clopen set of *L*such that $g^{-1}(P)\ne 0$ and $g^{-1}(P)\ne 1$.Therefore $\left(L,τ\right)$ is not fsgb-CdS, which contradicts the hypothesis. Thus $(M,σ)$ is fsgb-CdS.

**Theorem 3.12.** Consider$\left(L,τ\right)$ be fsgb$T\_{1/2}$space and f-CdS then $\left(L,τ\right)$ is fsgb-CdS.

**Proof.** Consider$\left(L,τ\right)$ is fsgb$T\_{1/2}$ space and f-CdS. Let$\left(L,τ\right)$ is not fsgb-CdS and then $∃$ a proper f-set *P* of *L* $(P\ne 0,P\ne 1)$ such that *P* is fsgb-clopen set. As$\left(L,τ\right)$ is fsgb$T\_{1/2}$space,*P* is f-clopen set. Thus$\left(L,τ\right)$ is not f-CdS, which contradicts the hypothesis. Therefore$\left(L,τ\right)$ is fsgb-CdS.

**Theorem 3.13.** Every fsgb-CdS is fb-CdS (fgb-Cd and fbg-Cd)

**Proof.** Consider $\left(L,τ\right)$ be a fsgb-CdS. Suppose that $\left(L,τ\right)$ is not fb-cd (fgb-Cd and fbg-Cd) and then there exists a fuzzy set $P (P\ne 0,P\ne 1)$ so that $P$ is fb-open (fgb-open and fbg-open) and also fb-close (fgb-close and fbg-close). Since every fb-close (fgb-close and fbg-close) is fsgb-close, $\left(L,τ\right)$ is not fsgb-cd, which contraducts the assumption. Thus $\left(L,τ\right)$ is fb-connected (fgb-close and fbg-close).

The inverse implication is untrue, as it can be seen from the below illustrations.

**Example 3.14.** Consider $L= \{x, y, z\}$.Let the fuzzy sets be$ P=\{\left(x,0.4\right),\left(y,0.3\right),\left(z,0.5\right)\}$ $Q=\{\left(x,0.2\right),\left(y,0.6\right),\left(z,0.1\right)\}$

Consider $τ=\{0,P,1\}$, then the fuzzy sets $Q$ is not a f-OS and f-CS of $L$.

Thus $(L,τ)$ is f-CdS but not fsgb-CdS.

**Example 3.15.** Consider $L=\{x, y, z\}$. Let the fuzzy set be$ P=\{\left(x,0.3\right),\left(y,0.6\right),\left(z,0.2\right)\}$ $Q=\{\left(x,0.1\right),\left(y,0.4\right),\left(z,0.5\right)\}$ $R=\{\left(x,0.2\right),\left(y,0.5\right),\left(z,0.3\right)\}$

Consider $τ=\{0.P,Q,1\}$, then the FS $R$ is not a fb-OS and a fb-CS of $L$.

Thus $(L,τ)$ is fb-cd but not fsgb-CdS.

**Example 3.16.** Consider $L= \{x, y, z\}.$

 Also consider the fuzzy sets $P=\{\left(x,0\right),\left(y,1\right),\left(z,0\right)\}$ $Q=\{\left(x,1\right),\left(y,1\right),\left(z,0\right)\}$ $R=\{\left(x,0\right),\left(y,1\right),\left(z,1\right)\}$.

Let $τ=\{0,P,Q,1\}$, then the fuzzy set $R$ is fsgb-CS but not fsgb-OS of $L$.

Thus $\left(L,τ\right)$ is fsgb connected.

**Fig. 3.1. Interrelations of fsgb-connected spaces in fts.**

|  |
| --- |
| Where, A B indicates A implies B A B indicates A and B are independent to each other.fbg-CdSfsgb-CdSf-CdSfb-CdSfgb-CdS |

**4. Extremally fuzzy strongly generalized b-disconnectedness.**

**Definition 4.1.** A fts $(L,τ)$ is called as extremally fsgb-disconnected (briefly e-fsgb-d) if $fsgb-cl(P)$ is fsgb-OS, whenever $P$ is fsgb-OS.

**Theorem 4.2.** For a fts $(L,τ)$ the following statements are equivalent.

1. $(L,τ)$ is e-fsgb-d.
2. For every fsgb-CS $P$, fsgb-$int(P)$ is fsgb-CS.
3. For every fsgb-OS $P$, we have $fsgb-cl\left(P\right)+fsgb-Cl\left[1-fsgb-Cl\left(P\right)\right]=1$.
4. For each pair of fsgb-OS $P$ and $Q$ in $(L,τ)$ with $fsgb-Cl\left(P\right)+Q=1$, we have $fsgb-Cl\left(P\right)+fsgb-Cl\left(Q\right)=1$.

**Proof.**

(i)$\rightarrow $(ii)

Consider $P$ be any fsgb-CS. Let us prove that $fsgb-int(P)$ is $fsgb-CS$. Now $1-fsgb-int(P)$=$fsgb-Cl(1-P)$. As $P$ is fsgb-CS, $1-P$ is fsgb-OS and so by assumption (i) $fsgb-Cl(1-P)$ is fsgb-OS, which implies that $1-fsgb-int(P)$ is fsgb-OS. Thus $fsgb-int(P)$ is fsgb-CS.

(ii)$\rightarrow $(iii)

Let $P$ be any fsgb-OS. Now $1-fsgb-Cl\left(P\right)=fsgb-int\left(1-P\right)$. Thus, $ fsgb-Cl\left(P\right)+fsgb-Cl\left[1-fsgb-Cl\left(P\right)\right]=fsgb-Cl\left(P\right)+fsgb-Cl\left[fsgb-int\left(1-P\right)\right]$

$=fsgb-Cl\left(P\right)+fsgb-int(1-P)$ by (ii)

$=fsgb-Cl\left(P\right)+1-fsgb-Cl(P)=1$.

(iii)$ \rightarrow $(iv)

Let $P$ and $Q$ be any two fsgb-OS such that

$fsgb-Cl\left(P\right)=1$ ------------(1).

Then by (iii) $fsgb-Cl\left(P\right)+fsgb-Cl[1-fsgb-Cl\left(P\right)]$ --------------- (2).

 But from (1) $Q=1-fsgb-Cl(P)$

and from (1) and (2),

 $1-fsgb-Cl\left(P\right)=fsgb-Cl[1-fsgb-Cl\left(P\right)]$

i.e., $1-fsgb-Cl\left(P\right)=fsgb-Cl(Q)$.

Thus $fsgb-Cl\left(P\right)+fsgb-Cl\left(Q\right)=1$.

(iv)$ \rightarrow $(i)

Let $P$ be any fsgb-OS in $(L,τ)$

Put $Q=1-fsgb-Cl(P)$ ----------------- (3)

Now by assumption (iv) $fsgb-Cl\left(P\right)+fsgb-Cl\left(Q\right)=1$

i.e., $fsgb-Cl\left(Q\right)=1-fsgb-Cl(P)$ -------------(4)

 From (3) and (4), $Q=fsgb-Cl(Q)$.

Hence $Q$ is fsgb-CS and so $fsgb-cl(Q)$ is fsgb-CS. Then $1-fsgb-Cl(Q)$ is fsgb-OS and from (4) $fsgb-Cl(P)$ is fsgb-OS in $(L,τ)$. Therefore, $(L,τ)$ is e-fsgb-d.

**Theorem 4.3.** A fts $(L,τ)$ is an e-fsgb-d space iff $fsgb-Cl\left(P\right)=fsgb-int[fsgb-Cl\left(P\right)]$ for each $P\in fsgb-O(L,τ)$.

**Proof.** Consider $P$ be a fsgb-OS in e-fsgb-d space $(L,τ)$. Then $fsgb-cl(P)$ is a fsgb-OS in $(L,τ)$. Therefore $fsgb-Cl\left(P\right)=fsgb-int[fsgb-Cl\left(P\right)]$.

Conversely, if $P$ be a fsgb-OS then $fsgb-Cl\left(P\right)=fsgb-int[fsgb-cl\left(P\right)]$. Thus $fsgb-Cl(P)$ is a fsgb-OS. Hence $(L,τ)$ is a e-fsgb-d space.

**Theorem 4.4.** A fts $(L,τ)$ is a e-fsgb-d space iff $fsgb-int\left(Q\right)=fsgb-Cl[fsgb-int\left(Q\right)]$ for every $Q\in fsgb-C(L,τ)$.

**Proof.** Consider$Q$ be a fsgb-CS in e-fsgb-d space $(L,τ)$. Then$(1-Q)$ is a fsgb-OS and $fsgb-Cl(1-Q)$ is fsgb-OS in $(L,τ)$. Thus , $fsgb-Cl\left(1-Q\right)=fsgb-int[fsgb-Cl\left(1-Q\right)]$. This implies that $1-fsgb-Cl\left(1-Q\right)=1-fsgb-int[fsgb-Cl\left(1-Q\right)]$. Therefore, $fsgb-int\left(Q\right)=fsgb-Cl[fsgb-int\left(Q\right)]$.

Conversely, if $Q$ is a fsgb-OS then $1-Q$ is fsgb-CS in $L$ and by hypothesis we get $fsgb-int\left(1-Q\right)=fsgb-Cl[fsgb-int\left(1-Q\right)]$

and $1-fsgb-int\left(1-Q\right)=1-fsgb-Cl[fsgb-int\left(1-Q\right)]$.

Thus, $fsgb-Cl\left(Q\right)=fsgb-int[fsgb-Cl\left(Q\right)]$. Hence, $(L,τ)$ is a e-fsgb-d space.

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