**On Fsgb-Connectedness and Fsgb-Disconnectedness in Fuzzy Topological Spaces**

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**Abstract:** The theme of this article is to introduce and investigate a new type of fuzzy strongly generalized b-connectedness namely fsgb-connectedness and fsgb-disconnectedness. Some of their properties and characteristics have been determined.

**AMS subject Classification: 54A40**

**Keywords:** fsgb-connectedness , extremally fsgb-disconnectedness, fts.

**1.Introduction:**

Several real-world issues in economics, medicine, engineering and social science contain imprecise data, and their solutions rely on uncertainty. L.A.Zadeh[17] established the concepts of fuzzy sets and fuzzy operations to deal with such uncertainty. C.L.Chang[6] , who introduced fuzzy topological spaces , presented the analytical aspect of fuzzy set theory practically. The theory of fts was developed by several authors. The concept of b-open sets in general topology was first developed by Andrejevic [1].

Jenifer and Megha introduced the fsgb-closed sets concepts in [9], the concept of fsgb-continuous , fsgb-irresolute, fsgb-open and fsgb-closed mappings in [10] and some new forms of fsgb-continuous maps namely fuzzy strongly generalized b-continuous functions namely strongly fsgb-continuous, perfectly fsgb-continuous and completely fsgb-continuous mappings in fts [12].Also a new weaker form of continuous functions known as upper fsgb-continuous multifunctions and lower fsgb-continuous multifunctions in [13].In this article, the concepts of fsgb-connectedness and fsgb-disconnectedness are introduced and their properties are investigated.

**2.Preliminaries:**

Throughout this study , and (or simply *L, M* and *N*) are fuzzy topological spaces (in-short as fts). The interior, closure and compliment of a fuzzy subset *P* of are denoted by Int*(P*), Cl(*P*) and *Pc* respectively. Unless specifically specifies, no separation axiom are expected.

***2*.1 Definition[9]** A fuzzy set(in short f-set) *P* in a fts *L* is called fb-open iffP)).

**2.2 Definition[9]** Fb-interior and Fb-closure of a fuzzy set *P* is as follows

(i)bInt(*P)* =.

(ii) bCl(*P*) =.

**2.3Defintion [9]** A f-set *P* in an fts *L* is known as fuzzy generalized closed set (in short(fg-CS) if , whenever and *Q* is f-OS in *L.*

**2.4Definition** **[9]**A fuzzy open set (in short f-OS) *P* in a fts *L* is called a fsgb-CS that is fsgb-closed set if , whenever and *Q* is -open set in *L*.

**2.5Definition [9]**A f-OS *P* in a fts *L* is called a fsgb-open set(in short fsgb-OS) if , whenever and *Q* is -open set in *L*.

**2.6Defintion[10]**A mapping is said to be fsgb-continuous if is fsgb -closed set in *L*, for every fuzzy closed set *P* in *M*.

**2.7Definition[10]**A map is known as fsgb-irr that is fsgb-irresolute map, if is fsgb-CS in *L* for every fsgb-CS *P* in *M.*

**2.8Definition[14]** A fuzzy point is known as quasi-coincident with f-set Q denoted by qQ iff . A f-set Q is quasi-coincident with a f-set R denoted by iff there exists such that . If Q and R are not quasi-coincident the we denote it as R.Note that .

**3.Fuzzy Strongly Generalized b-Connectedness in fts.**

**Definition 3.1.** A fuzzy topological spaces is known as fsgb-CdS that is fuzzy strongly generalized b-connected space iff 0 and 1 are the only f-sets which are fsgb-closed and fsgb-open (in short fsgb-clopen) sets.

**Definition 3.2.** A fts is known as fsgb-Cds between f-sets *P* and *Q* if there does not exist fsgb-clopen set *R* in *L* such that and .

**Theorem 3.3.** A fts is fsgb-connected iff is fsgb-CdS between each pair of its non-zero f-sets.

**Proof.** Consider *P* and *Q* are pair of non-zero f-sets of *L*. Let ( is not fsgb-Cds between *P* and *Q*. Then there exist a fsgb-clopen set *R* of *L* such that and . As *P* and *Q* are non-zero f-sets and *R* is proper fsgb-clopen set of *L*. Hence is not fsgb-CdS, which the contradicts the hypothesis.

Conversely, consider is not fsgb-CdS. Then there is a proper f-set R of *L* that is fsgb-clopen set. Thus is not fsgb-CdS between *R* and , which contradicts the hypothesis.

**Theorem 3.4**. A fts is fsgb-connected iff is fsgb-CdS between P and Q iff there is no fsgb-clopen set R such that .

**Proof:** It is evident.

**Theorem 3.5.** If a fts is fsgb-CdS between f-sets *P* and *Q* such that and ,then is fsgb-Cds between and .

**Proof.** Consider is not fsgb-CdS between and .Then there exists a fsgb-clopen set *R* of *L* such that and . Thus . Now .If , then there exists a point such that . Hence and , which contradicts the hypothesis.

**Theorem 3.6.** If a fts is fsgb-CdS between f-sets *P* and *Q*, then *P* and *Q* are non-zero.

**Proof.** Assume that , then P is fsgb-clopen set of L such that and .Thus cannot be a fsgb-CdS, which contradicts the hypothesis.

**Theorem 3.7.** Every fsgb-CdS is f-CdS.

**Proof.** Consider be fsgb-CdS. Let is not f-CdS and so a proper f-set such that *P* is f-clopen set. As every f-CS is fsgb-CS. Thus is not fsgb-CdS, which contradicts the hypothesis. Therefore is f-CdS.

**Theorem 3.8.** A fts is fsgb-CdS iff has no non-zero fsgb-OS *P* and *Q* such that .

**Proof.** Consider is fsgb-CdS. If has 2 non-zero fsgb-OS *P* and *Q* such that , so *P* is a proper f-set that is fsgb-clopen set of *L.* Thus is not fsgb-CdS, which contradicts the hypothesis.

Conversely, consider is not fsgb-CdS , then ithas a proper f-set *P* of *L* that is fsgb-clopen set. Thus , is a fsgb-OS of *L* so that , which contradicts hypothesis.

**Remark 3.9.** A fts is fsgb-CdS iff it has no non-zero f-set *P* and *Q* such that , .

**Theorem 3.10.** Consider is fsgb-irr, surjection and *L* is fsgb-Cds, then *M* is fsgb-CdS.

**Proof.** Consider *L* be a fsgb-CdS.Let *M* is not fsgb-CdS and then there is a proper f-set *P* of *M* such that *P* is fsgb-clopen set. As is fsgb-irr, is fsgb-clopen set of *L*such that and . Therefore is not fsgb-CdS, which contradicts the hypothesis. Thus is fsgb-CdS.

**Theorem 3.11.** Consider is fsgb- map , surjection and *L* is fsgb-CdS, then *M* is fsgb-CdS.

**Proof.** Consider *L* be a fsgb-CdS. Let *M* is not fsgb-CdS and then there is a proper f-set *P* of *M*  such that *P* is fsgb-clopen set. As is fsgb-map, is fsgb-clopen set of *L*such that and .Therefore is not fsgb-CdS, which contradicts the hypothesis. Thus is fsgb-CdS.

**Theorem 3.12.** Consider be fsgbspace and f-CdS then is fsgb-CdS.

**Proof.** Consider is fsgb space and f-CdS. Let is not fsgb-CdS and then a proper f-set *P* of *L*  such that *P* is fsgb-clopen set. As is fsgbspace,*P* is f-clopen set. Thus is not f-CdS, which contradicts the hypothesis. Therefore is fsgb-CdS.

**Theorem 3.13.** Every fsgb-CdS is fb-CdS (fgb-Cd and fbg-Cd)

**Proof.** Consider be a fsgb-CdS. Suppose that is not fb-cd (fgb-Cd and fbg-Cd) and then there exists a fuzzy set so that is fb-open (fgb-open and fbg-open) and also fb-close (fgb-close and fbg-close). Since every fb-close (fgb-close and fbg-close) is fsgb-close, is not fsgb-cd, which contraducts the assumption. Thus is fb-connected (fgb-close and fbg-close).

The inverse implication is untrue, as it can be seen from the below illustrations.

**Example 3.14.** Consider .Let the fuzzy sets be

Consider , then the fuzzy sets is not a f-OS and f-CS of .

Thus is f-CdS but not fsgb-CdS.

**Example 3.15.** Consider . Let the fuzzy set be

Consider , then the FS is not a fb-OS and a fb-CS of .

Thus is fb-cd but not fsgb-CdS.

**Example 3.16.** Consider

Also consider the fuzzy sets .

Let , then the fuzzy set is fsgb-CS but not fsgb-OS of .

Thus is fsgb connected.

**Fig. 3.1. Interrelations of fsgb-connected spaces in fts.**

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| Where, A B indicates A implies B  A B indicates A and B are independent to each other.  fbg-CdS  fsgb-CdS  f-CdS  fb-CdS  fgb-CdS |

**4. Extremally fuzzy strongly generalized b-disconnectedness.**

**Definition 4.1.** A fts is called as extremally fsgb-disconnected (briefly e-fsgb-d) if is fsgb-OS, whenever is fsgb-OS.

**Theorem 4.2.** For a fts the following statements are equivalent.

1. is e-fsgb-d.
2. For every fsgb-CS , fsgb- is fsgb-CS.
3. For every fsgb-OS , we have .
4. For each pair of fsgb-OS and in with , we have .

**Proof.**

(i)(ii)

Consider be any fsgb-CS. Let us prove that is . Now =. As is fsgb-CS, is fsgb-OS and so by assumption (i) is fsgb-OS, which implies that is fsgb-OS. Thus is fsgb-CS.

(ii)(iii)

Let be any fsgb-OS. Now . Thus,

by (ii)

.

(iii)(iv)

Let and be any two fsgb-OS such that

------------(1).

Then by (iii) --------------- (2).

But from (1)

and from (1) and (2),

i.e., .

Thus .

(iv)(i)

Let be any fsgb-OS in

Put ----------------- (3)

Now by assumption (iv)

i.e., -------------(4)

From (3) and (4), .

Hence is fsgb-CS and so is fsgb-CS. Then is fsgb-OS and from (4) is fsgb-OS in . Therefore, is e-fsgb-d.

**Theorem 4.3.** A fts is an e-fsgb-d space iff for each .

**Proof.** Consider be a fsgb-OS in e-fsgb-d space . Then is a fsgb-OS in . Therefore .

Conversely, if be a fsgb-OS then . Thus is a fsgb-OS. Hence is a e-fsgb-d space.

**Theorem 4.4.** A fts is a e-fsgb-d space iff for every .

**Proof.** Consider be a fsgb-CS in e-fsgb-d space . Then is a fsgb-OS and is fsgb-OS in . Thus , . This implies that . Therefore, .

Conversely, if is a fsgb-OS then is fsgb-CS in and by hypothesis we get

and .

Thus, . Hence, is a e-fsgb-d space.

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