Leap Zagreb Eccentricity Connectivity Indices of Pseudo-Regular Graphs

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Abstract

For a graph $G = (V, E)$, the second degree of a vertex v in a graph G is the number of its second neighbors, that is the number of vertices in G having distance 2 to v. In this manuscript we have computed the leap zagreb eccentricity of a connected graph G is defined as, $LM_1\xi^c(G) = \sum_{v\in V(G)} d_2^2(v).e(v)$, $LM_2\xi^c(G) = \sum_{u,v\in V(G)} \Big(d_2(u).e(v)\Big)\Big(d_2(v).e(v)\Big)$, $LM_3\xi^c(G) = \sum_{v\in V(G)} \Big(d(v).d_2(v)\Big).e(v)$, where, $d(u)$, $d(v)$, $d_2(u)$, $d_2(v)$ and $d_2^2(v)$ are the degree of the vertices u and v and $e(v)$ is a eccentricity of the vertices u and v. In this paper the leap zagreb eccentricity connectivity indices of pseudo-regular graphs are determined.

Key words: Leap Zagreb Indices , Connectivity, Eccentricity index.

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1 Introduction

Throughout this paper, by $G = (V, E)$ we mean an undirected simple graph with vertex set V and edge set E. As usual, we denote the number of vertices and edges in a graph G by n and m respectively. The distance $d_G(u, v)$ between any two vertices $u, v \in V$ of G is equal to the length of a shortest path between u and v. For a vertex v of G, the eccentricity of v is $e(v) = \max\{d_G(v, u) : u \in V(G)\}$. The diameter of G is $diam(G) = \max\{e(v) : v \in V(G)\}$ and the radius of G is $rad(G) = \min\{e(v) : v \in V(G)\}$.

A topological index of a G is a graph invariant number calculated from G. Various topological indices represent molecule structures and have got greater applications in chemistry. The Zagreb indices have been introduced, more than fifty years ago by Gutman and Trinajestic [9], in 1972, and studied by various authors in [4, 2, 5, 10, 11, 19, 12, 3, 6]. Recently several graph invariants are defined based on vertex eccentricities and studied by so many authors. Analogously to Zagreb indices, Ghorbani et al.[8] and Vukičević et al.[18], defined the Zagreb eccentricity indices by replacing degrees by the eccentricity of vertices. In 2017, Naji et. al., [14], had introduced three new distance-degree-based topological indices depending on the second degrees of vertices (number of their second neighbours), and are so-called leap Zagreb indices of graph G and are respectively defined by

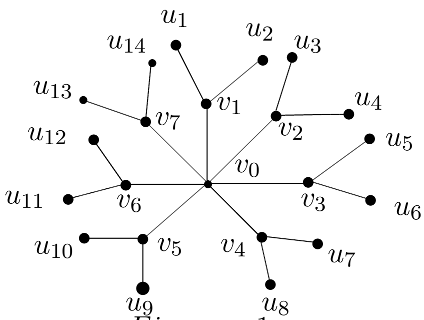
$$LM_1(G) = \sum_{v \in V(G)} d_2^2(v),$$

$$LM_2(G) = \sum_{uv \in E(G)} d_2(u) d_2(v),$$

$$LM_3(G) = \sum_{v \in V(G)} d(v) d_2(v).$$

2 Pseudo-Regular Graphs

Let $G = (V, E)$ be a simple, connected, undirected graph with n vertices and m edges. For any vertex $v_i \in V$, the degree of v_i is the number of edges incident on v_i. It is denoted by d_i or $d(v_i)$. A graph G is called regular if every vertex of G has equal degree. A bipartite graph is called semi regular if each vertex in the same part of a bipartition has the same degree. The 2 degree v_i [1] is the sum of the degree of the vertices adjacent to v_i and denoted by t_i [7].

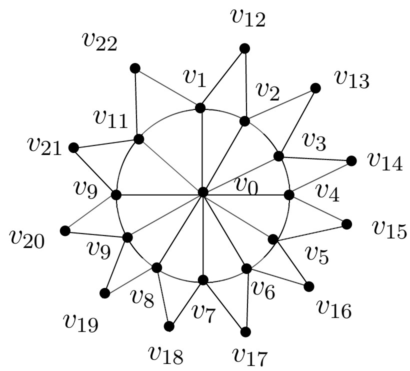


$Figure - 1$

The average degree of v_i is defined as $\frac{t_i}{d_i}$. For any vertex $v_i \in V$, the average degree of v_i is also denoted by $m(v_i) = \frac{t_i}{d_i}$.

A graph G is called pseudo-regular graph [1] if every vertex of G has equal average degree and $m(G) = (\frac{1}{n}) \sum_{v_i \in V(G)} m(u)$ is the average neighbor degree number of the graph G. A graph is said to be $r - regular$ if all its vertices are of equal degree r. Every regular graph is a pseudo-regular graph, see [15, 13]. But the pseudo-regular graph need not be a regular graph. Pseudo-regular graph is shown in Figures 1 and 2.

In Figure 1, there are 14 vertices of degree 1, 7 vertices of degree 3, and 1 vertex of degree 7. So totally there are 22 vertices in the graph. Average degree of vertices of degree 1 is equal to. Average degree of vertices of degree 2 is equal to $9/3 = 3$. Average degree of vertices of degree 7 is equal to $21/7 = 3$.



$Figure - 2$

Therefore, average degree of each vertex is 3. Hence, it is a pseudo-regular graph. In Figure 2, average degree of each vertex is 5. Hence, the graph in Figure 2 is also a pseudo regular graph.

The relevance of pseudo-regular graph for the theory of nanomolecules and nanostructure should become evident from the following. There exist polyhedral (planar, 3-connected) graphs and infinite periodic planar graphs belonging to the family of the pseudo-regular graphs. Among polyhedral, the deltoidal hexecontahedron possesses pseudo regular property. The deltoidal hexecontahedron is a Catalan polyhedron with 60 deltoid faces, 120 edges, and 62 vertices with degrees 3, 4, and 5, and average degree of its vertices is 4.

In this paper, motivated by connectivity index, we introduce the leap zagreb eccentricity connectivity indices are $LM_1\xi^c(G)$, $LM_2\xi^c(G)$ and $LM_3\xi^c(G)$ of Pseudo-Regular Graphs.

Definition 2.1. For a graph $G = (V, E)$, the leap zagreb eccentricity connectivity indice of G is defined by

$$LM_1\xi^c(G) = \sum_{v\in V(G)} d_2^2(v).e(v),$$

$$LM_2\xi^c(G) = \sum_{u,v\in V(G)} \Big(d_2(u).e(v)\Big)\Big(d_2(v).e(v)\Big),$$

$$LM_3\xi^c(G) = \sum_{v \in V(G)} \Big(d(v).d_2(v)\Big).e(v),$$

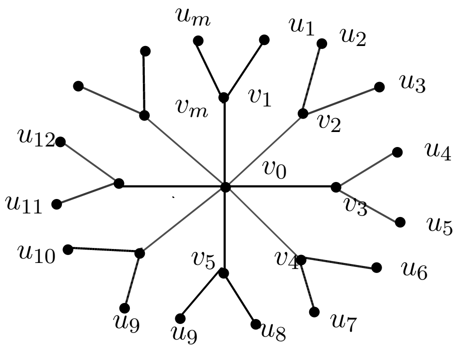
where, $d(u)$, $d(v)$, $d_2(u)$, $d_2(v)$ and $d_2^2(v)$ are the degree of the vertices u and v and $e(v)$ is a eccentricity of the vertices u and v, $(d_2(v) = |\{u \in V(G) : d(u,v) = 2\}|$ and $e(v) = max\{d(u,v)$: for every $u \in V(G)\})$.

3 Computation of the leap zagreb eccentricity connectivity indices of Pseudo -Regular graphs

Theorem 3.1. For $p \geq 7$ the leap zagreb eccentricity connectivity index of Pseudo-Regular graph is

$$LM_1\xi^c(G) = 2\Big(2(p+1)\Big)^2 + 3p^2(p+1) + 16\Big(2(p+1)\Big).$$

Proof: Let $V(G) = \{v_0, v_1, v_2, v_3...v_m,\ u_1, u_2, u_3, ..., u_{m(p-1)}\}$, be the vertex set of G and v_0 be the central vertex of star graph $K_{1,m}$ $\{v_1, v_2, v_3...v_m\}$ are pendent vertices of $K_{1,m}$, where $m = p^2 - p + 1$ and the $(p-1)$ pendent vertices of $\{u_1, u_2, u_3, ..., u_{m(p-1)}\}$ are attached with m pendent vertices $v_1, v_2, v_3...v_m$. Hence, $d_2(v_0) = 2(p+1)$, $d_2(v_i) = 2(p+1)$, $d_2(u_j) = 2$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$, is shown in Figure 3.



$Figure - 3$

The leap zagreb eccentricity connectivity index of G is given by $LM_1\xi^c(G) = \sum_{v \in V(G)} d_2^2(v)e(v)$. Now

$$\begin{aligned} LM_1\xi^c(G) &= d_2^2(v_0)e(v_0) + \sum_{i=1}^{p+1} d_2^2(v_i)e(v_i) + \sum_{j=1}^{2(p+1)} d_2^2(u_i)e(u_i) \\ &= \Big(2(p+1)\Big)^2.2.1 + \sum_{i=1}^{p+1} p^2.3 + \sum_{j=1}^{2(p+1)} 2^2.4 \\ &= 2\Big(2(p+1)\Big)^2 + 3p^2(p+1) + 16\Big(2(p+1)\Big), \quad for\ p \geq 1. \end{aligned}$$

∎

Theorem 3.2. For $p \geq 7$, the leap zagreb eccentricity connectivity index of Pseudo-Regular graph is

$$LM_2\xi^c(G) = \Big(4(p+1)\Big)\Big(3p(p+1)\Big) + \Big(3p(p+1)\Big)8\Big(2(p+1)\Big).$$

Proof: By Theorem 2.1, and from definition of $LM_2\xi^c(G)$. Hence, $d_2(v_0) = 2(p+1)$, $d_2(v_i) = p$, $d_2(u_j) = 2$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$, is shown in Figure 3. The leap zagreb eccentricity connectivity index of G is given by $LM_2\xi^c(G) = \sum_{u,v\in V(G)} \Big(d_2(u)e(u)\Big)\Big(d_2(v)e(v)\Big)$. Now

$$\begin{aligned}
LM_2\xi^c(G) &= \Big(d_2(v_0)e(v_0)\Big)\left(\sum_{i=1}^{p+1} d_2(v_i)e(v_i)\right) + \left(\sum_{i=1}^{p+1} d_2(v_i)e(v_i)\right)\left(\sum_{j=1}^{2(p+1)} d_2(u_i)e(u_i)\right)\\
&= \Big((2(p+1)).2.1\Big)\left(\sum_{i=1}^{p+1} p.3\right) + \left(\sum_{i=1}^{p+1} p.3\right)\left(\sum_{j=1}^{2(p+1)} 2.4\right)\\
&= \Big(4(p+1)\Big)\Big(3p(p+1)\Big) + \Big(3p(p+1)\Big)8\Big(2(p+1)\Big), \quad for\ p \geq 1.
\end{aligned}$$

∎

Theorem 3.3. For $p \geq 7$, the leap zagreb eccentricity connectivity index of Pseudo-Regular graph is

$$LM_3\xi^c(G) = \Big(2(p+1)^2\Big)\Big(2(p+1)\Big) + 9p(p+1) + 16(p+1).$$

Proof: By Theorem 2.1, and from definition of $LM_3\xi^c(G)$. Hence, $d(v_0) = p+1$, $d_2(v_0) = 2(p+1)$, $d(v_i) = 3$, $d_2(v_i) = p$, $d(u_j) = 1$, $d_2(u_j) = 2$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$, is shown in Figure 3. The leap zagreb eccentricity connectivity index of G is given by $LM_3\xi^c(G) = \sum_{v\in V(G)} \Big(d(v)d_2(v)\Big)e(v)$. Now

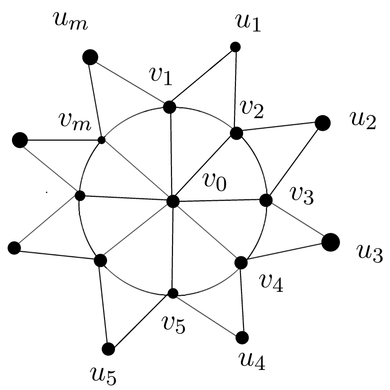
$$\begin{aligned}
LM_3\xi^c(G) &= \Big(d(v_0)d_2(v_0)\Big)e(v_0) + \sum_{i=1}^{p+1} \Big(d(v_i)d_2(v_i)\Big)e(v_i) + \sum_{j=1}^{2(p+1)} \Big(d(u_j)d_2(u_j)\Big)e(u_j)\\
&= (p+1)(2(p+1))(p+1)2 + \sum_{i=1}^{p+1} 3.p.3 + \sum_{j=1}^{2(p+1)} 1.2.4\\
&= \Big(2(p+1)^2\Big)\Big(2(p+1)\Big) + 9p(p+1) + 16(p+1) \quad for\ p \geq 1.
\end{aligned}$$

∎

Theorem 3.4. For $p \geq 7$, the leap zagreb eccentricity connectivity index of Pseudo-Regular graph is

$$LM_1\xi^c(G) = \begin{cases} 120, & \text{if } |p| = 7, \\ 404, & \text{if } |p| = 9, \\ 665, & \text{if } |p| = 11, \\ 2\Big((p-1)+3\Big)^2 + 3\Big((p-2)+3\Big)^2((p-1)+3) + 100((p-1)+3), & \text{if } p \geq 4. \end{cases}$$

Proof: Let $V(G) = \{v_0, v_1, v_2, v_3 ... v_m,\ u_1, u_2, u_3, ..., u_{m(p-3)}\}$, be the vertex set of G and v_0 be the central vertex of wheel graph W_m and $\{v_1, v_2, v_3 ... v_m\}$ are vertex of cycle C_m in the clockwise direction and $\{u_1, u_2, u_3 ... u_{m(p-3)}\}$ are the pendant vertices joined to every vertex in the cycle except the central vertex, where $m = p^2 - 3p + 3$. Hence, $d_2(v_0) = ((p-1)+3)$, $d_2(v_i) = ((p-2)+3)$, $d_2(u_j) = 5$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$, is shown in Figure 4.



$Figure - 4$

The leap zagreb eccentricity connectivity index of G is given by $LM_1\xi^c(G) = \sum_{v \in V(G)} d_2^2(v)e(v)$. Now

$$\begin{aligned} LM_1\xi^c(G) &= (d_2^2(v_0))e(v_0).1 + \sum_{i=1}^{((p-1)+3)} (d_2^2(v_i))e(v_i) + \sum_{j=1}^{((p-1)+3)} (d_2^2(u_i))e(u_i) \\ &= \Big((p-1)+3\Big)^2.2.1 + \sum_{i=1}^{((p-1)+3)} ((p-2)+3)^2.3 + \sum_{j=1}^{((p-1)+3)} 5^2.4 \\ &= 2\Big((p-1)+3\Big)^2 + 3\Big((p-2)+3\Big)^2\Big((p-1)+3\Big) + 100\Big((p-1)+3\Big). \end{aligned}$$

■

Theorem 3.5. For $p \geq 7$, the leap zagreb eccentricity connectivity index of Pseudo-Regular graph is

$$LM_2\xi^c(G) = \begin{cases} 180, & \text{if } |p| = 7, \\ 1632, & \text{if } |p| = 9, \\ 5100, & \text{if } |p| = 11, \\ 6\Big((p-1)+3\Big)^2\Big((p-2)+3\Big) + 60\Big((p-1)+3\Big)^2\Big((p-2)+3\Big), & \text{if } p \geq 4. \end{cases}$$

Proof: By Theorem 2.4. and from definition of $LM_2\xi^c(G)$. Hence, $d_2(v_0) = ((p-1)+3)$, $d_2(v_i) = ((p-2)+3)$, $d_2(u_j) = 5$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$, is shown in Figure 4. The leap zagreb eccentricity connectivity index of G is given by

$$LM_2\xi^c(G) = \sum_{u,v \in V(G)} (d_2(u)e(u))(d_2(v)e(v)).$$

Now

$$\begin{aligned} LM_2\xi^c(G) &= \Big(d_2(v_0)e(v_0)\Big)\Big(d_2(v_i)e(v_i)\Big) + \sum_{i=1}^{((p-1)+3)} \Big(d_2(v_i)e(v_i)\Big) \sum_{j=1}^{((p-1)+3)} \Big(d_2(u_i)e(u_i)\Big) \\ &= \Big(((p-1)+3).2.1\Big)\Big((p-2)+3\Big).3.\Big((p-1)+3\Big) + \\ &\sum_{i=1}^{((p-1)+3)} \Big((p-2)+3\Big)3\Big((p-1)+3)\Big) \sum_{j=1}^{((p-1)+3)} 5.4 \\ &= 6\Big((p-1)+3\Big)^2\Big((p-2)+3\Big) + 60\Big((p-1)+3\Big)^2\Big((p-2)+3\Big) \end{aligned}$$

■

Theorem 3.6. For $p \geq 7$, the leap zagreb eccentricity connectivity index of Pseudo-Regular graph is

$$LM_3\xi^c(G) = \begin{cases} 96, & \text{if } |p| = 7, \\ 272, & \text{if } |p| = 9, \\ 500, & \text{if } |p| = 11, \\ 2\Big((p-1)+3\Big)^2 + 15\Big((p-2)+3\Big)\Big((p-1)+3\Big) + 40\Big((p-1)+3\Big), & \text{if } p \geq 4. \end{cases}$$

Proof: By Theorem 2.4. and from definition of $LM_3\xi^c(G)$. Hence, $d(v_0) = ((p-1)+3)$, $d_2(v_0) = ((p-1)+3)$, $d_2(v_i) = 5$, $d_2(v_i) = ((p-2)+3)$, $d(u_j) = 2$, $d_2(u_j) = 5$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$, is shown in Figure 4. The leap zagreb eccentricity connectivity index of G is given by $LM_3\xi^c(G) = \sum_{v \in V(G)} \Big(d(v))(d_2(v)\Big).e(v))$. Now

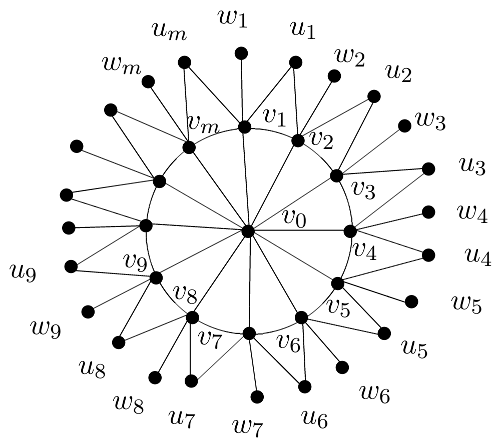
$$LM_3\xi^c(G) = (d(v_0))(d_2(_0)\Big).e(v_0)) + \sum_{i=1}^{((p-1)+3)} (d(v_i))(d_2(v_i)\Big).e(v_i))$$

$$+\sum_{j=1}^{((p-1)+3)}(d(u_i))(d_2(u_i)\Big).e(v_i))$$
$$=((p-1)+3)((p-1)+3)e(v_0).1+\sum_{i=1}^{((p-1)+3)}5((p-2)+3).3$$
$$+\sum_{j=1}^{((p-1)+3)}2.5.4$$
$$=2\Big((p-1)+3\Big)^2+15\Big((p-2)+3\Big).\Big((p-1)+3\Big)+40\Big((p-1)+3\Big).$$

■

Theorem 3.7. For $p \geq 10$, the leap zagreb eccentricity connectivity of pseudo-regular graph is $LM_1\xi^c(G) = \begin{cases} 675, & \text{if } |p| = 10, \\ 1316, & \text{if } |p| = 13, \\ 2(2p+8)^2+3(p+5)^2(p+4)+296(n+4), & \text{if } p \geq 1. \end{cases}$

Proof: Let $V(G) = \{v_0, v_1, v_2, ..., v_m, u_1, u_2, ..., u_{m(p-5)}, w_1, w_2, ..., w_m\}$ be the vertex set of G and v_0 as the central vertex of wheel graph w_m , where $m = p^2-3p+1$ and $\{u_1, u_2, ..., u_{m(p-5)}\}$ are the pendant vertices and $\{w_1, w_2, ..., w_m\}$ are the vertices joined to the end vertices of each edge of a wheel graph except the central vertex is shown in Figure 5 [13]. Hence $d_2(v_0) = 2p+8$, $d_2(v_i) = p+5$, $d_2(u_i) = 7$, $d_2(w_i) = 5$, $e(v_0) = 2$, $e(v_i) = 3$, $e(u_i) = 4$, $e(w_i) = 4$.



$Figure - 5$

The leap zagreb eccentricity connectivity index of G is given by $LM_1\xi^c(G) = \sum_{v \in V(G)} d_2^2(v)e(v)$.

Now

$$
\begin{aligned}
LM_3\xi^c(G) =& d_2(v_0).e(v).1 + \sum_{i=1}^{p+4} d_2(v_i).e(v_i) + \sum_{i=1}^{p+4} d_2(u_i).e(u_i) + \sum_{i=1}^{p+4} d_2(w_i).e(w_i) \\
=& (2p+8)^2.2.1 + \sum_{i=1}^{p+4} (p+5)^2.3 + \sum_{i=1}^{p+4} 7^2.4 + \sum_{i=1}^{p+4} 5^2.4 \\
Thus, =& 2(2p+8)^2 + 3(p+5)^2.(p+4) + 296(n+4)
\end{aligned}
$$

■

Theorem 3.8. For $p \geq 10$, the leap zagreb eccentricity connectivity of pseudo-regular graph is

$$
LM_2\xi^c(G) = \begin{cases} 3618, & \text{if } |p| = 10, \\ 11040, & \text{if } |p| = 13, \\ 6(2p+8)(p+5)(p+4) + (3(p+5)(p+4))(28(p+4)) + (28(p+4))(20(p+4)), \\ \quad \text{if } p \geq 1. \end{cases}
$$

Proof: By Theorem 2.7. and from definition of $LM_2\xi^c(G)$. Hence, $d_2(v_0) = 2p+8$, $d_2(v_i) = p+5$, $d_2(u_i) = 7$, $d_2(w_i) = 5$, $e(v_0) = 2$, $e(v_i) = 3$, $e(u_i) = 4$, and $e(w_i) = 4$, is shown in Figure-5. The leap zagreb eccentricity connectivity index of G is given by $LM_2\xi^c(G) = \sum_{u,v \in V(G)} (d_2(u_i)e(u_i))((d_2(v_i)e(v_i)))$. Now

$$
\begin{aligned}
LM_2\xi^c(G) =& \left(d_2(v_0)e(v_0).1 \right) \left(d_2(v_i)e(v_i)(p+4) \right) + \left(\sum_{i=1}^{p+4} d_2(v_i)e(v_i \right) \left(\sum_{i=1}^{p+4} d_2(u_i)e(u_i) \right) + \\
& \left(\sum_{i=1}^{p+4} d_2(u_i)e(u_i) \right) \left(\sum_{i=1}^{p+4} d_2(w_i)e(w_i) \right) \\
=& \left((2p+8).2.1 \right) \left((p+5).3.(p+4) \right) + \left(\sum_{i=1}^{p+4} (p+5).3 \right) \left(\sum_{i=1}^{p+4} 7.4 \right) + \\
& \left(\sum_{i=1}^{p+4} 7.4 \right) \left(\sum_{i=1}^{p+4} 5.4 \right) \\
=& 6(2p+8)(p+5)(p+4) + \Big(3(p+5)(p+4) \Big) \Big(28(p+4) \Big) + \\
& \Big(28(p+4) \Big) \Big(20(p+4) \Big).
\end{aligned}
$$

■

Theorem 3.9. For $p \geq 10$, the leap zagreb eccentricity connectivity of pseudo-regular graph is

$$
LM_3\xi^c(G) = \begin{cases} 819, & \text{if } |p| = 10, \\ 2416, & \text{if } |p| = 13, \\ 2(p+4)(2p+8) + 18(p+5)(p+4)^2 + 76(p+4)^2, & \text{if } p \geq 1. \end{cases}
$$

Proof: By Theorem 2.7. and from definition of $LM_2\xi^c(G)$. Hence, $d(v_0) = p+4$, $d_2(v_0) = 2p+8$, $d(v_i) = 6$, $d_2(v_i) = p+5$, $d(u_i) = 2$ $d_2(u_i) = 7$, $(w_i) = 1$ $d_2(w_i) = 5$, $e(v_0) = 2$, $e(v_i) = 3$, $e(u_i) = 4$, and $e(w_i) = 4$, is shown in Figure-5. The leap zagreb eccentricity connectivity index of G is given by $LM_3\xi^c(G) = \sum_{v\in V(G)}(d(v_i)d_2(v_i))e(v_i)$. Now

$$\begin{aligned}
LM_3\xi^c(G) =& \Big(d(v_0)d_2(v_0)\Big)e(v_0) + \sum_{i=1}^{p+4}\sum_{i=1}^{p+4}\Big(d(v_i)d_2(v_i)\Big)e(v_i) + \sum_{i=1}^{p+4}\sum_{i=1}^{p+4}\Big(d(u_i)d_2(u_i)\Big)e(v_i)\\
&+ \sum_{i=1}^{p+4}\sum_{i=1}^{p+4}\Big(d(w_i)d_2(w_i)\Big)e(w_i)\\
=&((p+4)(2p+8))2.1 + \sum_{i=1}^{p+4}\sum_{i=1}^{p+4}6.(p+5).3 + \sum_{i=1}^{p+4}\sum_{i=1}^{p+4}(2.7).4 + \sum_{i=1}^{p+4}\sum_{i=1}^{p+4}(1.5).4\\
=&2(p+4)(2p+8) + 18(p+5)(p+4)^2 + 76(p+4)^2.
\end{aligned}$$

■

4 Conclusion

In this manuscript, we clearly determined the Leap zagreb eccentricity connectivity indices of pseudo-regular graphs.

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