

## Review of Fracture Toughness (G, K, J, CTOD, CTOA)

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**Abstract:** The present paper gives a technical review of fracture toughness testing, evaluation and standardization for metallic materials in terms of the linear elastic fracture mechanics as well as the elastic-plastic fracture mechanics. This includes the early investigations and recent advances of fracture toughness test methods and practices developed by American Society for Testing and Materials (ASTM). The review describes the most important fracture mechanics parameters: the elastic energy release rate  $G$ , the stress intensity factor  $K$ , the  $J$ -integral, the crack-tip opening displacement (CTOD) and the crack-tip opening angle (CTOA) from the basic concept, definition, to experimental estimation, test methods and ASTM standardizing practices. Attention is paid to guidelines on how to choose an appropriate fracture parameter to characterize fracture toughness for the material of interest, and how to measure the fracture toughness value defined either at a critical point or in a resistance curve format using laboratory specimens. The relevant ASTM fracture toughness test standards considered in this paper are E399 for  $K_{Ic}$  testing, E561 for  $K-R$  curve testing, E813 for  $J_{Ic}$  testing, E1152 for  $J-R$  curve testing, E1737 for  $J_{Ic}$  and  $J-R$  curve testing, E1290 for CTOD ( $d$ ) testing, a combined common test standard E1820 for measuring the three parameters of  $K$ ,  $J$  and  $d$ , E1921 for the transition reference temperature  $T_0$  testing and the master curve of cleavage toughness  $K_{Ic}$  testing, and E2472 for CTOA testing. The effects of loading rate, temperature and crack-tip constraint on fracture toughness as well as fracture instability analysis are also reviewed.

**Keywords:** CTOA, CTOD,  $J$ -Integer,  $K_{Ic}$

### I. Introduction

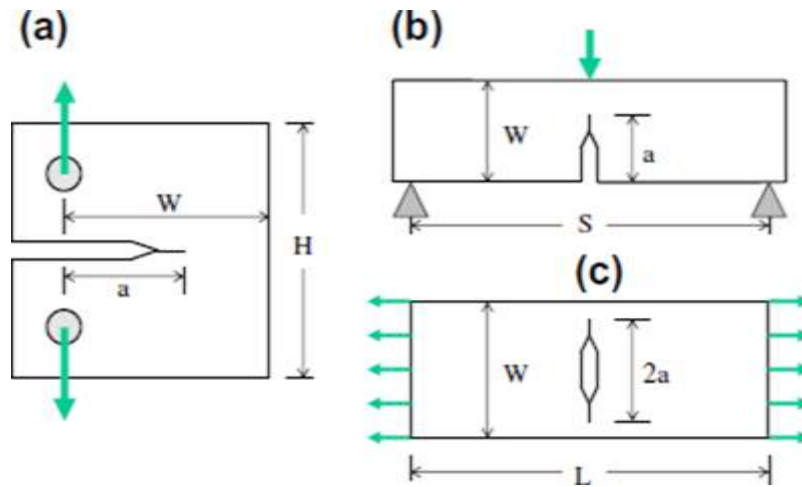
Fracture toughness is usually used as a generic term for measures of material resistance to extension of a crack. It is restricted to results of fracture mechanics tests in this work, which are directly applicable to fracture control and to fracture test in describing the material property for a crack to resist fracture. The experimental measurement and standardization of fracture toughness play an imperative role in application of fracture mechanics methods to structural integrity assessment, damage tolerance design, fitness-for-service evaluation, and residual strength analysis for different engineering components and structures. The fracture toughness values may also serve as a basis in material characterization, performance evaluation, and quality assurance for typical engineering structures, including nuclear pressure vessels and piping, petrochemical vessels and tanks, oil and gas pipelines, and automotive, ship and aircraft structures. Therefore, fracture toughness testing and evaluation has been a very important subject in development of fracture mechanics method and its engineering applications.

The stress intensity factor  $K$  (or its equivalent partner – the elastic energy release rate  $G$ ), the  $J$ -integral, the crack-tip opening displacement (CTOD), and the crack-tip opening angle (CTOA) are the most important parameters used in fracture mechanics. The  $K$  factor was proposed in 1957 by Irwin [1] to describe the intensity of elastic crack-tip fields, and symbolizes the linear elastic fracture mechanics. The  $J$ -integral was proposed in 1968 by Rice [2] to characterize the intensity plastic crack-tip fields, and symbolizes the elastic-plastic fracture mechanics.

The CTOD concept was proposed in 1963 by Wells [3] to serve as an engineering fracture parameter, and can be equivalently used as  $K$  or  $J$  in practical applications. The CTOA parameter was used in the recent decade to describe fracture behavior of stable crack extension for thin-walled materials. Different experimental methods have been developed for measuring these parameters to describe fracture toughness of materials. The detailed descriptions of these fracture mechanics parameters and their applications can be found in the textbooks of fracture mechanics, such as those by Broek [4], Kanninen and Popelar [5], Hertzberg [6], Anderson [7] and others.

The basic fracture mechanics concepts were summarized by Irwin and Dewit [8]. Recently, Erdogan [9] and Cotterell [10] reviewed the history and development of fracture mechanics. Extensive applications of fracture mechanics methods via fracture toughness in structural integrity and assessment were documented in a set of 11-volume comprehensive books compiled by Milne et al. [11]. Standard terminology relating to fracture toughness testing and evaluation has been defined in E1823 [12] by the American Society for Testing and Materials (ASTM) in the United States. All terms and concepts pertaining to fracture tests used in this work are

consistent with those defined by ASTM E1823. This paper reviews the historical investigation and state-of-the-art development of fracture toughness testing, evaluation and standardization for structural metallic materials in terms of the linear elastic fracture mechanics and the elastic-plastic fracture mechanics.



**Fig 1.** The conventional fracture test specimens. (a) Compact tension (C(T)) specimen, (b) single edge-notched bend specimen (SE(B)) in three-point bending, and (c) middle-cracked tension (M(T)) specimen.

Particular attention is paid to the practices of fracture toughness test methods developed by ASTM for measuring the fracture mechanics parameters of the stress intensity factor  $K$  (or the elastic energy release rate  $G$ ), the  $J$ -integral, the crack-tip opening displacement and the crack-tip opening angle. The effects of loading rate, temperature and crack-tip constraint on fracture toughness measurements as well as fracture instability analysis are also reviewed. Six types of conventional fracture test specimens are permitted in ASTM fracture test standards, but no single standard allows all six configurations. These include compact tension (C(T)) specimen, single edge-notched bend (SE(B)) specimen in three-point bending, middle-cracked tension (M(T)) panel, disk-shaped compact tension (DC(T)) specimen, arc-shaped tension (A(T)) specimen and arc-shaped bend (A(B)) specimen. This paper focuses on the mostly often used C(T), SE(B) and M(T) specimens containing a through-thickness tensile crack, i.e., mode-I crack, as illustrated in Fig. 1. In this figure,  $W$  is the specimen width,  $B$  is the specimen thickness,  $H$  is the height of C(T) specimen,  $S$  is the span of SE(B) specimen,  $L$  is the length of M(T) specimen  $a$  is the crack length of the two bending specimens and  $2a$  is the crack length of the tensile specimen.

In most cases,  $W = 2B$ ,  $H = 1.2W$ ,  $S = 4W$ ,  $LP3W$  and  $a/W = 0.5$ . However, different specimen size requirements are prescribed in different fracture test standards in order to obtain valid fracture toughness and to limit the effects of cracktip constraint on that fracture toughness parameter.

## II. K-Based Fracture Testing

### 2.1 Early fracture testing

Early fracture testing and analysis was based on the energy approach that described the occurrence of fracture when the energy available for crack growth is sufficient to overcome the resistance of the material. Griffith [31] at the British Royal Aircraft Establishment was the first to propose the fundamental energy theory for brittle materials like glass, where the resistance was assumed to come exclusively from the surface formation energy of the material. For metallic materials, plastic deformation generated at the crack tip absorbs much more applied energy than the surface energy, and thus Griffith's energy theory underestimates severely the fracture strength of metals. Irwin [36] and Orowan [37] modified Griffith's theory by including the local plastic dissipation energy for its application to metals. Irwin [38] further modified this theory using the energy release rate  $G$  as a measure of the energy available for an increment of crack extension. On these bases, Irwin [1] obtained the simple relationship of Eq. (3) between  $G$  and  $K$  for the infinite plate in tension. This relationship is significant to connect the global energy concept of Griffith to a more readily calculable crack-tip parameter. Because of the leading roles they played in its development, the linear elastic fracture mechanics is often known as Griffith–Irwin theory. The concept of fracture resistance was first introduced by Irwin and Kies [39] using the energy approach. As illustrated in Fig. 2a, the 1954 concept regards the fracture resistance  $R$  of the material as a constant or slightly decreasing value, when plotted against the crack length, whereas the driving force  $G$  was an increasing function of crack length. Fracture instability was defined by a critical value  $G_c$  at the intersection of these two curves

## 2.2. $K_{Ic}$ testing and ASTM E399 development

As defined in ASTM E1823, the plane strain fracture toughness,  $K_{Ic}$ , is the crack-extension resistance under conditions of crack-tip plane strain in mode I for slow rates of loading under predominantly linear-elastic conditions and negligible plastic-zone adjustment.  $K_{Ic}$  provides for the measurement of crack-extension resistance at the onset (2% or less) of crack extension. This section discusses the  $K_{Ic}$  test standard ASTM E399 and its development history. The test procedures for measuring fracture toughness of materials have been developed and standardized by the American Society for Testing and Materials (ASTM) in the United States. In 1958, a special ASTM Technical Committee E24 on Fracture Testing of Metals was established for the purpose to develop and write test methods for determination of fracture properties. Kaufman [44] reviewed the progress of the committee effort over the early 10 years. Note that ASTM Technical Committees E09 on fatigue and E24 on fracture merged 25 years later in 1993 as the present ASTM Technical Committee E08 on Fatigue and Fracture Mechanics.

Much of the early fracture toughness testing was directed at relatively thin sections and the procedures prescribed in the first and second ASTM E24 committee reports [45, 46] dealt almost entirely with thin-section problems. It was indicated that thickness-dependent apparent toughness  $K_c$  might not be a single-value property, and thickness-independent plane strain toughness  $K_{Ic}$  might be a more fundamental property of materials. Thus the committee effort shifted from the thin-section problem to concentration on the thick-section problem to develop test methods for determining  $K_{Ic}$ . It was anticipated that once the plane strain problem was solved, attention would return to the thin-section (or plane stress) problem.

## III. J-Based Fracture Testing

J-integral concept and HRR field For structural steels in the presence of large-scale plasticity, linear elastic fracture mechanics cannot accurately characterize the fracture behavior, and thus an alternative nonlinear fracture mechanics model is needed. Based on the deformation theory of plasticity, Rice [2] at Brown University (now at Harvard University) proposed a new fracture parameter that was called J integral.  $C$  is an arbitrary curve around the tip of a crack,  $w$  is the strain energy density,  $T_i$  is the components of the traction vector,  $u_i$  is the displacement vector components,  $ds$  is the length increment along the contour,  $x$  and  $y$  are the rectangular coordinates with the  $y$  direction taken normal to the crack line and the origin at the crack tip. Rice [2] showed that for deformation plasticity (i.e., nonlinear elasticity) the J-integral is independent of the path of integration around the crack tip. This path independence was first verified by Kobayashi et al. [89] using the finite element analysis (FEA). Thus, J is called a path-independent integral.

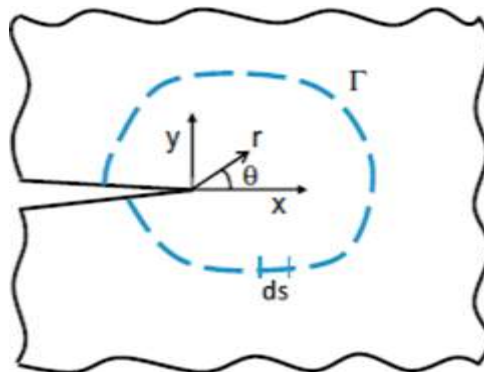


Fig 2. An arbitrary contour around the crack tip used in the definition of J-integral.

## IV. $CTOD$ & $CTOA$ – Based Fracture Testing

In addition to the  $K$  and  $J$  concepts described above, another important fracture mechanics parameter is the crack tip opening displacement ( $CTOD$ ) first proposed by Wells [3] at the British Welding Institute. Initially, Wells referred to this quantity as the crack opening displacement ( $COD$ ), but more recently the name has been changed to  $CTOD$  to distinguish the quantity from the crack mouth opening displacement ( $CMOD$ ), a physical crack opening displacement measured across the crack mouth at the specimen surface. Wells developed the  $CTOD$  approach in order to extend the elastic stress intensity factor approach into elastic-plastic yielding conditions. The use of  $CTOD$  criterion requires a laboratory measurement of a critical  $CTOD$  value, usually associated with the onset of cleavage fracture under plane strain conditions. Such a measurement near the vicinity of the blunting crack tip is difficult and subjective. Initial measurements were made using injection and removal of dental implant compound. The early approaches for  $CTOD$  measurements were reviewed by Burdekin. The subsequent measurements were estimated using geometrical models inputting displacement

measurements made remotely from the crack tip. In particular, a plastic hinge model was developed by Hollstein and Blauel to determine CTOD by assuming that two arms of the specimen rotate rigidly about a plastic hinge point in the uncracked ligament. In order to apply the plastic hinge model to both elastic and elastic-plastic conditions, the total  $d$  is separated into elastic and plastic components, just like the  $J$  separation. The plastic component  $d_{pl}$  is determined from the plastic CMOD in terms of the plastic hinge model, and the elastic component  $d_{el}$  is calculated from the applied stress intensity factor  $K$ .

## V. Conclusion

This paper gave a systematic technical review of fracture toughness testing, experimental evaluation, test methods and standardization for metallic materials in reference to both the linear elastic fracture mechanics and the elastic-plastic fracture mechanics. This review described the most important fracture parameters of the elastic energy release rate  $G$ , the stress intensity factor  $K$ , the  $J$ -integral, the crack-tip opening displacement  $d$ , and the crack-tip opening angle (CTOA) and presented, basically in the chronological order, the historic and state-of-the-art developments of these fracture parameter test and evaluation methods. The basic concept, definition, experimental estimation, early fracture test practice, test method, recent development, critical point-value toughness evaluation, and resistance curve testing as well as ASTM standardization effort of fracture test methods were described in detail for each fracture parameter of  $K$  (or  $G$ ),  $J$ ,  $CTOD$ , and  $CTOA$ . The effects of loading rate, temperature, crack-tip constraint and fracture instability on fracture toughness measurements were also reviewed. Three typical fracture mechanics constraint theories, i.e. the  $J$ - $T$  approach, the  $J$ - $Q$  theory and the  $J$  A2 three-term solution and their applications to quantifying the constraint effect on fracture toughness were briefly reviewed.

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