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Design and Analysis of Rocket Nozzle

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Abstract: A nozzle is a tube of varying cross-sectional area aiming at increasing the speed of an outflow, and controlling its direction and shape to produce thrust which is the result of pressure which is exerted on the wall of the combustion chamber. For this design selected lunching condition,Ethiopia, around Ambo city. approximately 6,893 feet above sea level and atmospheric pressure is around 1.012 bar. this paper used mathematical method like interpolation, extrapolation and iteration in the precision of 10-5 todefine design parametersand also select the material from different perspective like nozzle erosion and thermal-stress cracking. finally reach on the conclusion that an expansion ratio ($\frac{A_e}{A}$) of 8 and throat area, the contact point of X_{n2} = (0.01207, 0.04254 ^ 0.01207, - 0.04254) with small circle also the two circles contact point on X_{n1} = (0, 0.035 ^ 0, - 0.035).
Keyword: Nozzle, Thrust, Solid propellant, Expansion ratio, Mach number

I. INTRODUCTION

A nozzle is a tube of varying cross-sectional area (usually axisymmetric) aiming at increasing the speed of an outflow, and controlling its direction and shape. In the simplest case of a rocket nozzle, relative motion is created by ejecting mass from a chamber backwards through the nozzle, with the reaction forces acting mainly on the opposite chamber wall, with a small contribution from nozzle walls. Two types of Nozzle, Converging nozzles and Converging-Diverging nozzle.Converging nozzles are used to accelerate the fluid in subsonic gas streams (and in liquid jets), since at low speeds density do not vary too much, and $\dot{m}$ = ρ v A =const can be approximated by v A = const. Liquid jets and low speed gas flows can be studied with classical Bernoulli equation (until cavitation effects appear in liquid flows), but high-speed gas dynamics is dominated by compressibility effects in the liquid.(1)Thrust is the force that propels a rocket or spacecraft and is measured in pounds, kilograms or Newtons. Physically speaking, it is the result of pressure which is exerted on the wall of the combustion chamber. Figure 1 shows a combustion chamber with an opening, the nozzle, through which gas can escape. The pressure distribution within the chamber is asymmetric; that is, inside the chamber the pressure varies little, but near the nozzle it decreases somewhat.The force due to gas pressure on the bottom of the chamber is not compensated for from the outside. The resultant force *F* due to the internal and external pressure difference, the thrust, is opposite to the direction of the gas jet.
Applying the principle of the conservation of momentum gives

$$F=\dot{m}\, V_e + (Pe - P_a)\, Ae \qquad \text{……………………………………………………………………………………}1$$

Where
$\dot{m}$ is the rate of the ejected mass flow,
P_a = 101,325 pa,the pressure of the ambient atmosphere,
P_e the pressure of the exhaust gases and
V_e their ejection speed

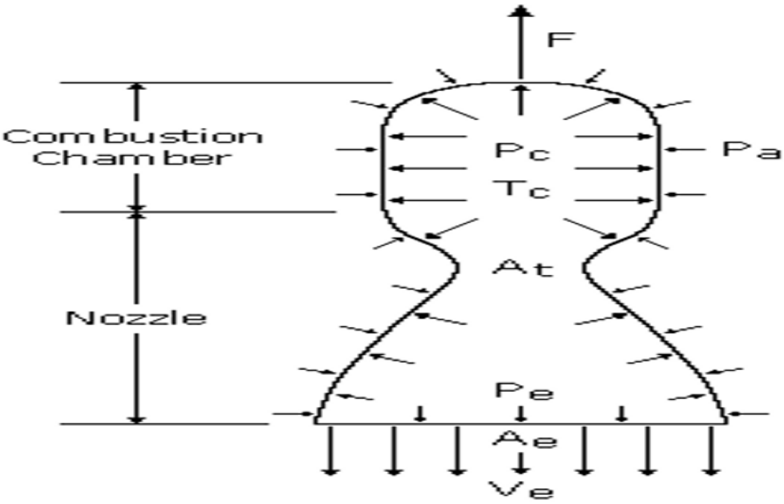


Figure 1: combustion chamber with the nozzle through which gas can escape

From equation (1) one can easily deduce that for a given mass flow rate and pressure difference (exit pressure and ambient pressure) increasing exhaust gas and exit area will increase the trust generated.

II. MATERIAL ANDMETHODS

Solid propellant:A solid propellant rocket is a simple propulsion system that consists of a high-pressure vessel that contains all the solid components needed. The fuel and oxidizer are intimately mixed together and cast into a solid mass, called grain. (3)Modern solid propellants consist of ingredients that are in a solid and liquid state during the mixing process. All solids are typically based on an oxygen rich salt like ammonium perchlorate (NH_4ClO_4) as the oxidizer and aluminum powder (Al) as fuel. Ammonium perchlorate contains 54.5% per weight oxygen. The liquid part is often an industrial long chain polymer. The purpose of the polymer is to harden and thereby fixate the powered based part of the mix. A commonly used polymer is Hydroxyl-Terminated Polybutadiene or HTPB ($C_xH_yO_z$). The hardening process is triggered by adding a hardener agent like Isoforondiisocyanat ($C_{12}H_{18}N_2O_2$) in order to start the cross-polymerization process. (2)

Table 1: Designed proportion of modern solid propellant mixture (2)

Ingredient	**Function**	**Formula**	**% per mass**
Ammonium Perchlorate	Oxidizer	NH_4ClO_4	68
Aluminum	Fuel	Al	20
HTPB	Fuel-Binder	$C_xH_yO_z$	11.5
Isoforondiisocyanat	Curative	$C_{12}H_{18}N_2O_2$	0.5

A modern composite propellant can give 2500 m/s (255 s) specific impulse. (2)

Rocket propellant laboratory: because of the high cost of a laboratory equipment andhence lack of the accessibility of the equipment for the laboratory to study the exact properties of the propellant, it is difficult to get the exact match of available propellant with their properties. There are variety of modern solid rocket propellant available in the market with different properties as seen some of them in table 2.

Table 2:Three commonproportion modern propellant with corresponding specification. (4)

Composition	**Propellant 1**	**Propellant 2**	**Propellant 3**
NH_4CLO_4 (%)	70	68	72
Aluminum	16	18	16
Binder and additives	14	14	12
Density (lbm/in^3)	0.0636	0.0635	0.0641
Burning rate at 1000 psi (in/sec)	0.349	0.276	0.280
Burning rate exponent	0.21	0.3-0.45	0.28
Temperature coefficient of	0.102	0.09	0.10

pressure (% ^{0}F)			
Adiabatic flame temperature(^{0}F)	5790	6150	5909
Characteristic velocity (ft/sec)	5180	5200	5180

Table3: Aluminized Ammonium perchlorate as a function of chamber pressure for expansion to sea level. (4)

Chamber pressure(psia)	1500	1000	750	500	200
Chamber pressure (atm) or pressure ratio p1/p2	102.07	68.046	51.034	34.023	13.609
Chamber temperature(K)	3346.9	3322.7	3304.2	3276.6	3207.7
Nozzle exit temperature (K)	2007.7	2135.6	2226.8	2327.0	2433.6
Chamber enthalpy(cal/g)	-572.17	-572.17	-572.17	-572.17	-572.17
Exit enthalpy(cal/g)	-1382.19	-1325.15	-1282.42	-1219.8	-1071.2
Entropy((cal/g-K)	2.1826	2.2101	2.2597	2.2574	2.320
Chamber molecular mass(kg/mol)	29.303	29.215	29.149	29.050	28.908
Exit molecular mass(kg/mol)	29.879	29.853	29.820	29.763	29.668
Exit Mach number	3.20	3.00	2.86	2.89	2.32
Specific heat ratio-chamber, K	1.1369	1.1351	1.1337	1.1318	1.1272
Specific impulse, vacuum (sec)	287.4	280.1	274.6	265.7	242.4
Specific impulse, sea level expansion (sec)	265.5	256.0	248.6	237.3	208.4
Characteristic velocity c* (m/sec)	1532	1529	1527	1525	1517
Nozzle area ration, A_2/A_t^a	14.297	10.541	8.507	8.531	6.300
Thrust coefficient, c_f^a	1.700	1.641	1.596	1.597	1.529

Procedure methodology: To determine the intermediate properties for the various proportion of solid modern rocket propellant available, interpolation and extrapolation methods have been employed.
From Table2 the interpolation between 256.0 and 248.6 specific impulse, the combustion pressure.

1000 X 750
256 255 248.6
Pt = 966.21622 psia = 6,661,826.33 pascal
Interpolation between 256.0 and 248.6 specific impulse, the Specific heat ratio-chamber, K
1.1351 Y 1.1337
256 255 248.6
$\gamma = 1.134925$
Interpolation between 256.0 and 248.6 specific impulse, the Chamber temperature(K)
3322.7 Z 3304.2
256 255 248.6
Tt =3320.4 K
Interpolation between 256.0 and 248.6 specific impulse, the exit molecular mass (kg/mol)
29.853 W 29.820
256 255 248.6
W =29.848 kg/mol

The Mach speed of the exhaust gases from the flow of propellant can be evaluated from knowing the chamber pressure and the ambient pressure, or desirable nozzle exit pressure (Pa=Pe). (5)

Exit Pressure $=\frac{Pe}{Pt} = (1+\frac{\gamma-1}{2}M^2)^{\frac{-\gamma}{\gamma-1}}$…………………………….……………………………………… 2

Calculated from the pressure ratio between ambient pressure and combustion pressure equals to 0.0152 at M= 3.15 figure 2 shows the relation between pressure ration verses Mach number in specified heat ration-chamber.

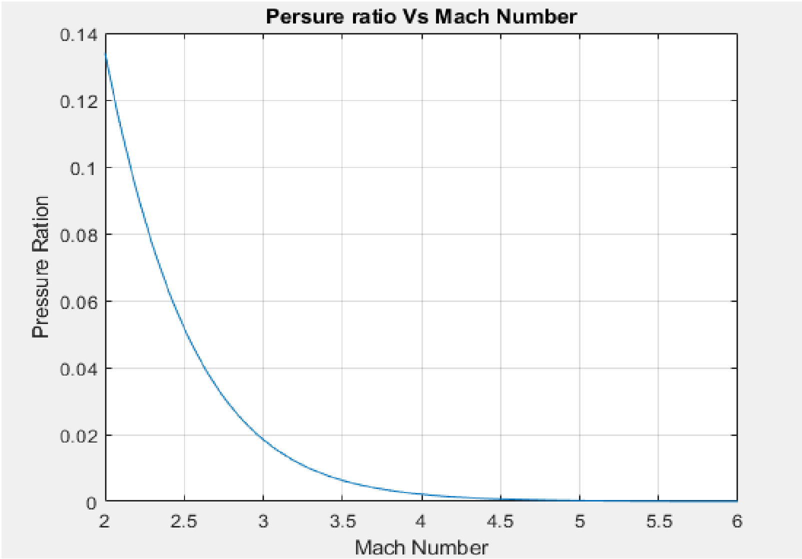


Figure 2 pressure ratio verses Mach number

Exit Temperature = $\frac{Te}{Tt} = (1 + \frac{\gamma-1}{2} M^2)^{-1}$ ……………………………..………………………………………………...…. 3
Te =1,988.98 K

The expansion ratio(ratio of the exit area to throat area) of the nozzle was optimized for the launch conditionsas well as the operating pressure of the chamber. The launch site will be approximately 5000 feet above sea level, where the atmospheric pressure is around .8 bar. A line is drawn on the graph (figure 3) at this pressure, as well as at .6 bar, representing the lower limit of pressures that will not experience flow separation testing engine operation.This expansion ratio allows the nozzle to operate in a safe range, and provide the maximum efficiency for the maximum allowable diameter of our engine. (6)

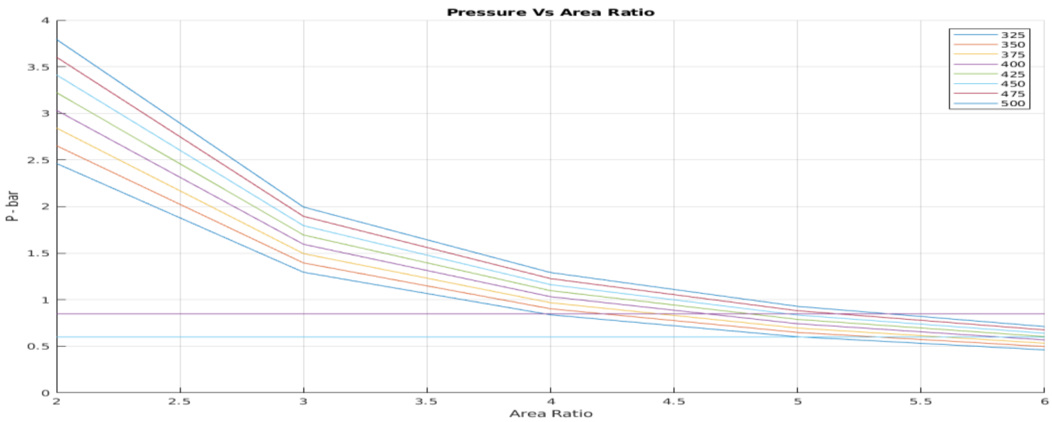


Figure 3 combustion chamber Vs Area ratio[6]

Extrapolation at area ratio: 2,2.5, 3,3.5, 4,4.5,5,5.5and 6 in 500 and 475 psi and the result shown in figure 4
$y(x) = y(1) + \frac{x-x(1)}{x(2)-x(1)} * $ [y (2) – y (1)] ……………………………………………………………….. 4

Table 4: Extrapolation parameters at area ratio 2

Y(x)	X (966.21622)
Y2(3.785)	X2(500)
Y1(3.6)	X1(475)

Y(x)= $y(1) + 19.648 *$ (y (2) – y (1)) ……………………..………….………………………………..…. 5

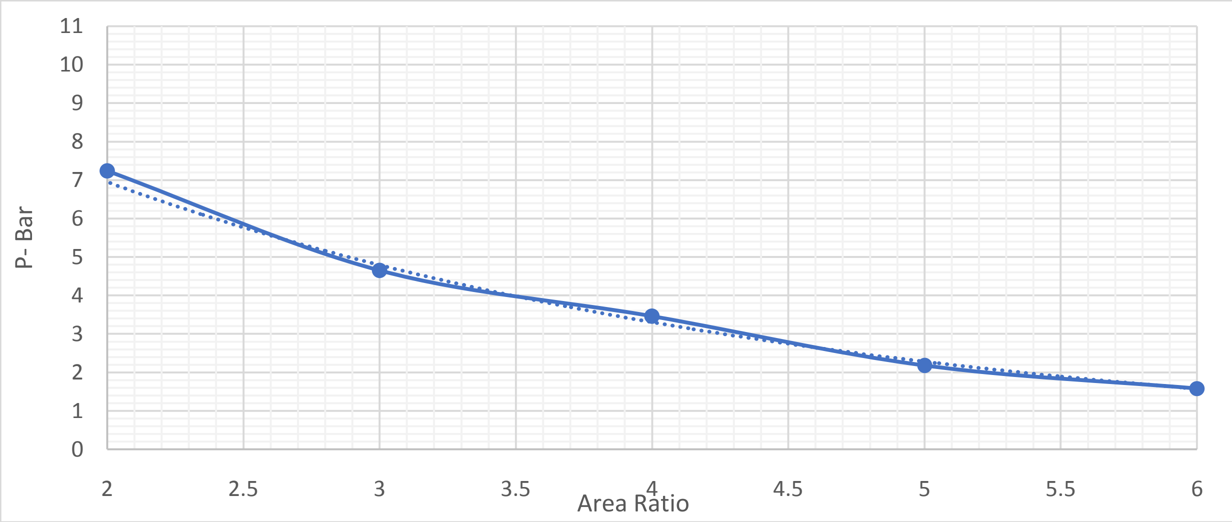


Figure 4 atmospheric pressure Vs area ratio in 966.21655 psi chamber pressure

Suppose the following data comes from an exponentially decreasing phenomena, Using the exponential transformation, we get that the best fitting exponential function is

$y(x) = 14.614\, e^{-0.372x}$ ……………………………………………………………………………………... 6

Assume lunching site is selected in Ambo, Ethiopia. Which is approximately 6,893 feet above sea level and atmospheric pressure is around 1.012 bar.

Table 5: Extrapolation value at area ratio 7 and 8

Area ratio	p- bar
7	1.081
8	0.7453

Moreover, an expansion ratio $(\frac{A_e}{A})$ of 8 was chosen for a chamber pressure of 966.21622 psi. This expansion ratio allows the nozzle to operate in a safe range, and provide the maximum efficiency for the maximum allowable diameter of our engine.The area of the nozzle throat determines the total mass flow rate and is directly correlated to total thrust. Holding the expansion ratio constant, this gives the smallest diameter and therefore lightest nozzle option. The smaller diameter is not only significant because the nozzle will be lighter, but also because the combustion chamber and other component diameters can be minimized. This decreases the weight of the entire engine and will ultimately increase efficiency and total altitude. (6)

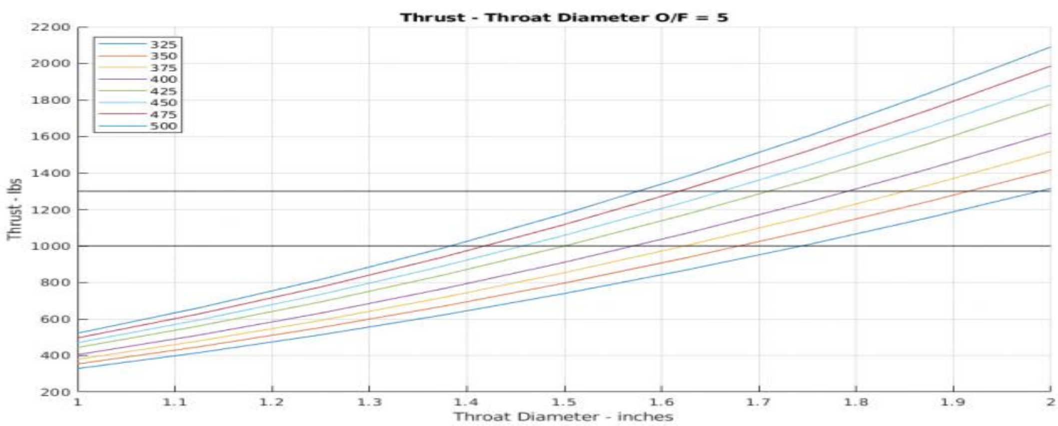


Figure 5 Rocket total trust Vs Throat diameter [6]

Extrapolation at nozzle throat diameter: 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9 and 2 in 500 and 475 psi and the result shown in figure 6:

Table 6: Extrapolation parameters of nozzle throat diameter at 1

Y(x)	X (966.21622)
Y2(525)	X2(500)
Y1(500)	X1(475)

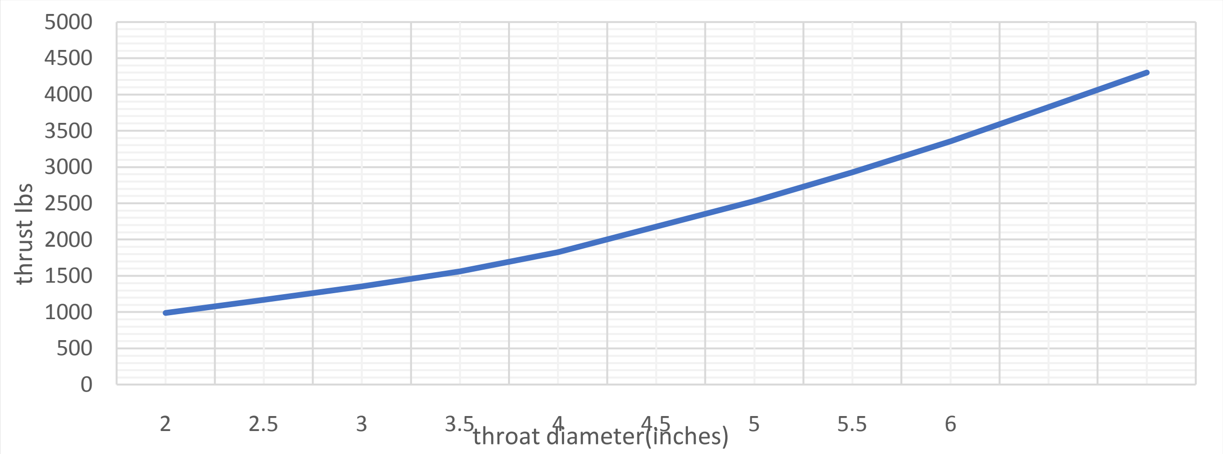


Figure 6 thrust Vs throat diameter in 966.21655 psi chamber pressure

Once the expansion ratio was chosen, the throat area was determined by optimizing the total size of the nozzle due to the area ratio and the total mass flow rate necessary to achieve a thrust of 13,483.15lbs or 60 kN.Suppose the following data comes from an exponentially increasing phenomena, Using the exponential transformation, we get that the best fitting exponential function is

$Y(x) = 228.46\ e^{1.4676x}$…………………………………………………………………………….. 7

Table 7: Extrapolation value at different nozzle throat diameter to reach 13,483.15 lbs

Nozzle throat diameter(inches)	**Thrust lbs**
2.1	4,980.67
2.2	5,768.1
2.3	6,679.8
2.4	7,735.7
2.5	8.958.53
2.6	10,374.66
2.7	12014.64
2.8	**13,913.9**

let take the nozzle throat diameters 2.8 inch =0.07112 m

Area of throat, A = $0.00397503904 m^2$

Exit Velocity= $V_e = M\sqrt{\gamma * R * Te}$……………………………….…………………………..……..... 8

$V_e = 2{,}535.4\ \frac{m}{s}$

Pressure of the exhaust gases in propellant mass flow rate of 24 kg/sec, P_e =1,726,460.841 pascal

But,

M is the Mach exit number (should always be a supersonic value),

Pt=total pressure in the combustion chamber

Tt=total temperature in the combustion chamber

γ= specific heat ratio of the exhaust,

R= Gas constant

A = throttle Area

Ae =nozzle exit area

Te = exit temperature in Kelvin;

Configuration of bell nozzle:A nozzle is constructed using three curves: an initial, large circle coming from the inlet to the throat, a smaller circle exiting the throat, and a parabola to extend the approximated bell contour to the exit plane.(7)In order to define the nozzle further, a coordinate system is defined with the axial (x) axis passing through the line of symmetry and the radial (y) axis going through the Centre of the throat. The first and second curves define the entrance and exit of the throat of the nozzle, and are based on circular curves.

$x^2+\big(y-(R_t+$
$1.5Rt2=(1.5Rt)2$…….……………………….………………………………………….……..….9

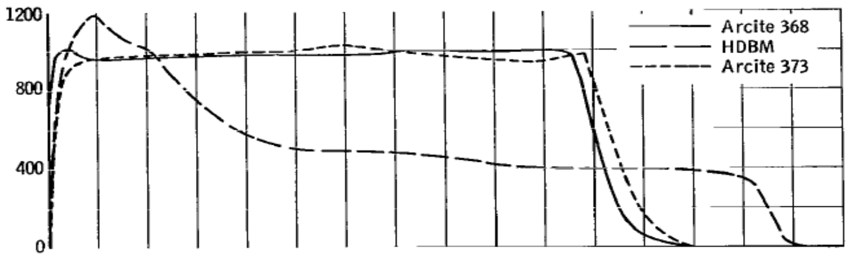
$x^2+\big(y-(R_t+$
$0.382Rt2=(0.382Rt)2$……………………………………………………..……………… 10

The equation of the parabola, curve 3, takes the form
$x=ay^2+by+c$…..……………………………………………..…………………….…... 11

Nozzle Material selection:the thermal, chemical, and mechanical environments produced by high performance solid propellants introduce many materials problems in the development of rocket nozzles. Some propellants are highly corrosive, many contain metal additives, and typical flame temperatures range from 5000' to 6400' F. The interaction of environmental conditions together with the usual requirement that dimensional stability in the nozzle throat be maintained makes the selection of suitable rocket nozzle materials extremely difficult. Usually, materials for typical large solid-propellant rocket nozzles are incorporated into suitable design configurations only after many full- scale prototype test firings. (8) Only full-scale engine tests can completely evaluate rocket nozzle materials. However, most of the important conditions encountered in full-scale engines can be simulated with small-scale engine tests. Parameters such as flame temperature, combustion products, and gas velocity are readily duplicated. However, two major conditions, the nozzle surface temperature history and the nozzle thermal stress, may be greatly influenced by size effects. Full-scale nozzle surface temperature history can be approximated in a small-scale nozzle by appropriate selection of wall thickness**.** The thermal stresses that may be encountered in full-scale nozzles, however, are markedly influenced by many interrelated factors such as size, shape, and specific installation configuration. In general, the thermal stresses encountered in small-scale engines are less severe than those in full-scale engines. The selection of the aluminized propellant dimensions was complicated by the deposit of aluminum oxide on the nozzle insert throat during firing. An example of this deposit on a molybdenum nozzle is shown in figure 10

Figure 7- Aluminum oxide deposit on throat surface of molybdenum nozzle insert. X500.
(Reduced 35 percent in printing.) (8)

Nozzle Erosion:Refractory metals,Overall, the fully densified refractory metals were the most erosion resistant group of materials. Molybdenum did not erode Figure 10 in the two lower temperature propellant environments, but it eroded catastrophically with the highest temperature propellant.Refractory compounds,the silver infiltrated columbium carbide - tungs ten material eroded only slightly with the HDBM propellant but eroded catastrophically with the more oxidizing Arcite**368.** It is possible that the greater surface area exposed as the silver was melted from the porous columbium carbide - tungsten skeleton contributed to making the nozzle more subject to oxidation than the fully densified columbium carbide - tungsten



(a) Arc-cast molybdenum.

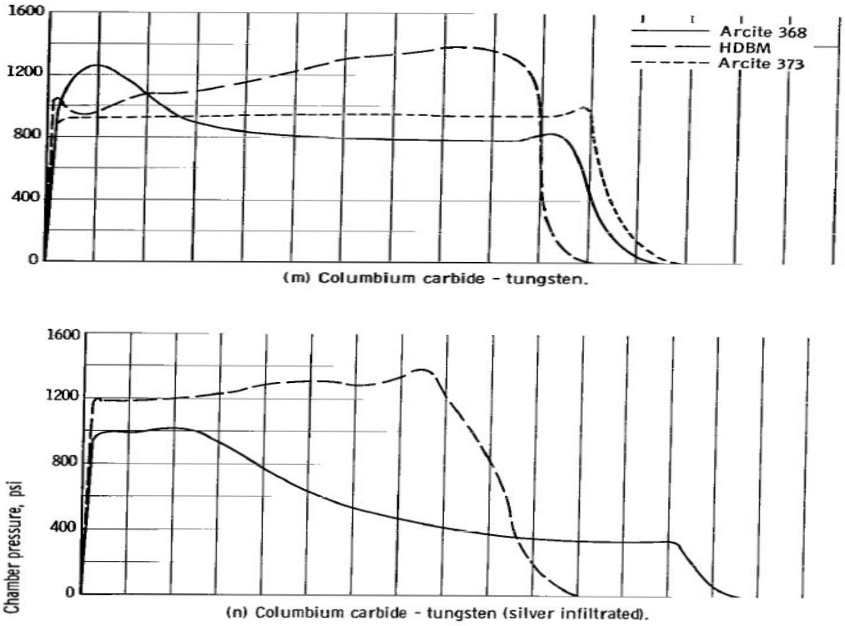


Figure 8 Chamber pressure -time traces obtained during material-evaluation firings [8]

The final chamber pressure and the following equation were used to calculate total erosion of each nozzle:

$A_t = \frac{Sr\rho}{PC_d}$.. 12

$c_d = 3.62 * 10^{-4}$

But

At = nozzle throat area

S = burning surface area

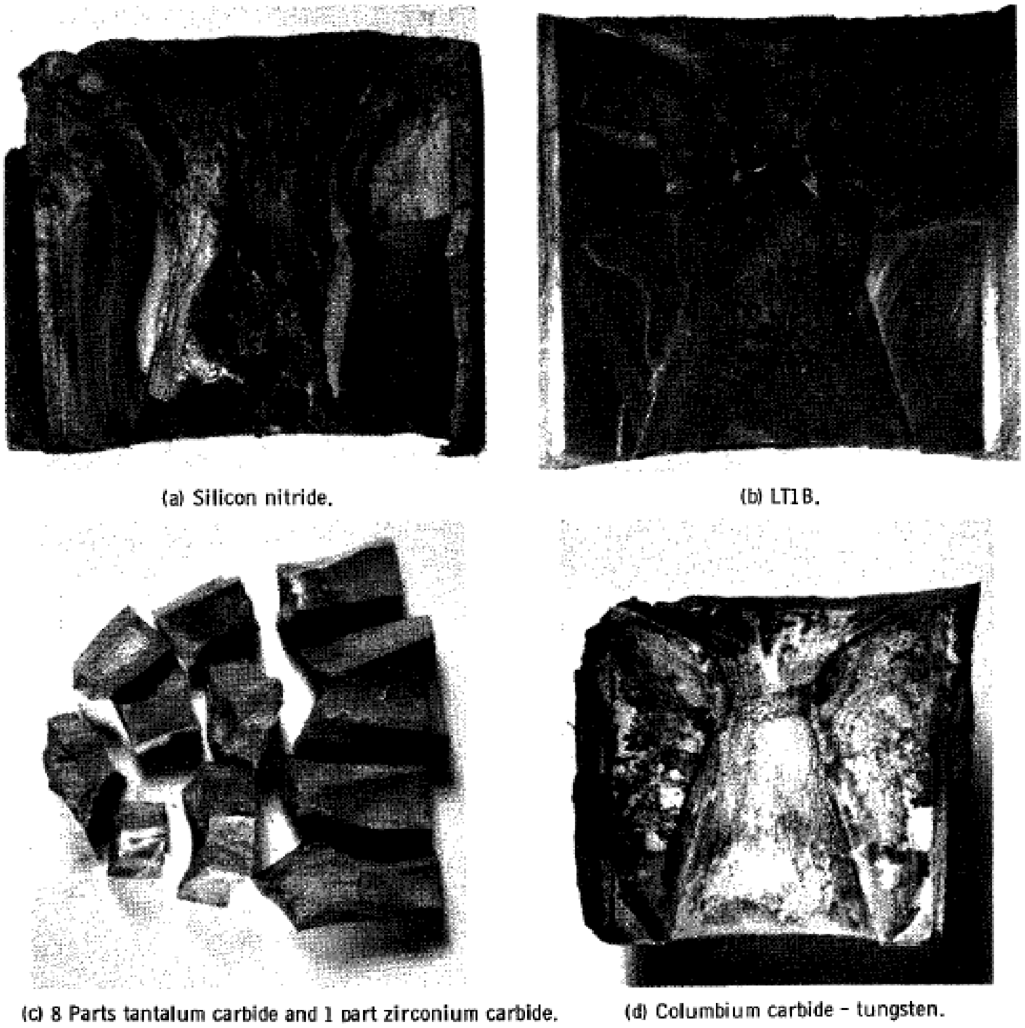
r = burning rate

p = propellant density

P = chamber pressure,

cd = nozzle discharge coefficient.

Thermal -Stress Cracking: Some nozzles were cracked extensively both radially and circumferentially so that nozzles separated into several pieces on removal from the retainer. The refractory-metal-carbide - tungsten nozzles cracked less severely than the carbide-graphite type; in some cases, only a single fracture occurred, as indicated in figure 12



(a) Silicon nitride.

(b) LT1B.

(c) 8 Parts tantalum carbide and 1 part zirconium carbide.

(d) Columbium carbide - tungsten.

Figure 9 Thermal-stress failure in nozzle inserts. (Reduced 40 percent in printing.)

III. RESULT

Iteration on pressure ration, Mach number, exit temperature, exit velocity and exit pressure with the pressure ration of 0.04632the following result are summarized in table 8

Table 8: Iteration value in the pressure ratio, Mach number, exit temperature, exit velocity and Exit pressure.

Iteration	Pressure ratio	Mach number	Exit temperature (K)	Exit velocity(m/sec)	Exit pressure(pascal)
-	0.0152	3.15	1,988.98	2,535.4	74,608.3
1	0.011199373	3.25	1,938.6	2,582.6	38,985.9
2	0.005852147	3.6	1,771.5	2,691.4	- 43,126.4
3	0.00647366	3.55	1,794.62	2,714.2	-60,333.8
4	0.009056644	3.28	1,923.99	2,596	28,872.88
5	0.0043340788	3.7	1,726.17	2,774.39	-105,764
6	0.015876137	3.1	2,014.42	2,511.08	92,959.23
7	0.013954017	3.2	1,963	2,559	56,569
8	0.008491547	3.35	1,889	2,628	4,722
9	0.000708840	4	1,596	2,884.76	- 189,062
10	0.028379905	2.78	2,182.5	2,343.94	21,9106
11	0.03288977	2.7	2,225.76	2,298.94	25,3065
12	0.037987424	2.65	2,253.01	2,270.14	27,4801
13	0.041250179	2.6	2,280.42	2,240.8	29,6936
14	0.044572762	2.58	2,291.42	2,228.9	30,5903

15	0.045918832	2.575	2,298.17	2,225.9	30,8155
16	**0.046256849**	**2.574**	**2,294.72**	**2,225.3**	**30,8605**
17	0.04632				

Configuration of bell nozzle:By substituting the value of R_t into equation 9 and 10 we reach on figure 7 it shows the curve from combustion end to throat (large circle) and from throat to the parabolic curves.figure 8 shows the complete image of nozzlewith the connect points, the point with connect first curve and second curve; X_{n1} = (0, 0.035 ^ 0, - 0.035) and the point with connect second curve and parabola; X_{n2} = (0.01207, 0.04254 ^ 0.01207, - 0.04254).

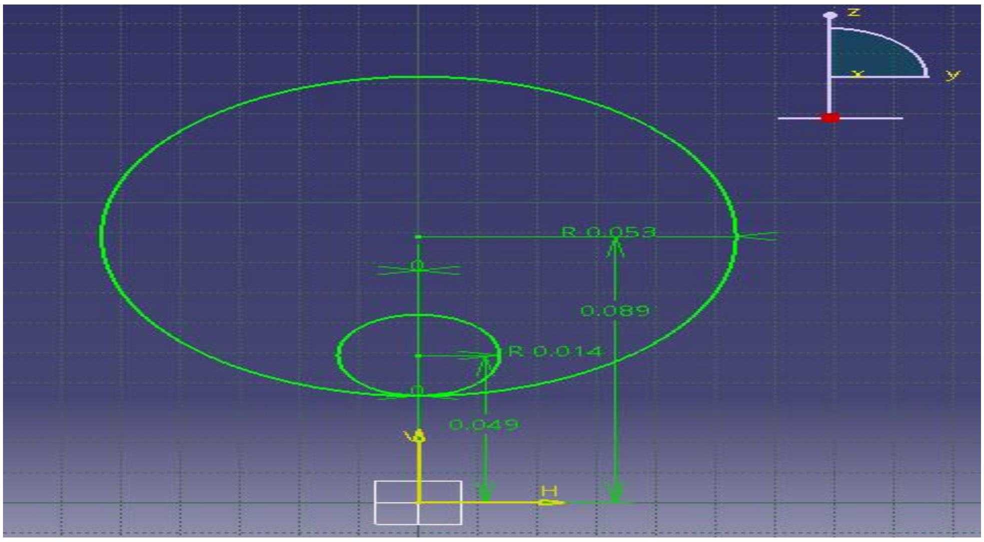


Figure 10 the first two curves are based on circular curves.

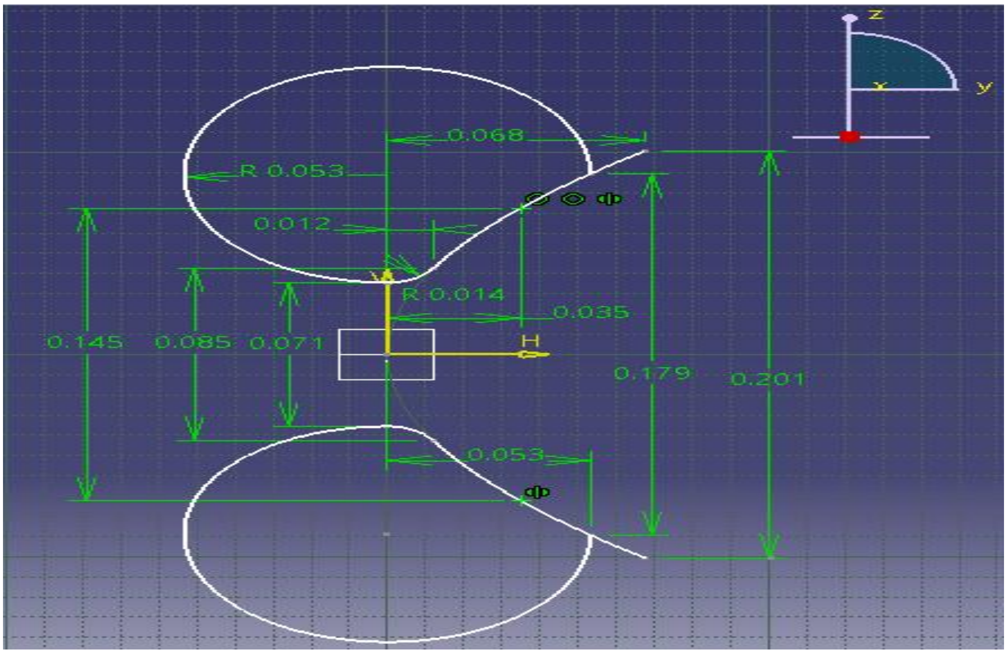


Figure 11 complete curves with for nozzle physical model

Nozzle Material selection: By considering our specific design condition with different material properties like Thermal -Stress Cracking and Nozzle Erosion Columbium Carbide- tungsten material selected.

IV. CONCLUSION

The following conclusion is drawn on Convergent-Divergent Nozzle, an expansion ratio $(\frac{A_e}{A})$ of 8 and throat area, A = 0.00397503904 m^2,with divergent length 0.068 m also the contact point of X_{n2} = (0.01207, 0.04254 ^ 0.01207, - 0.04254) with small circleand the two circles contact point on X_{n1} = (X_{n1} = (0, 0.035 ^ 0,

- 0.035)and other analysis parameter: Mach number =2.574, Exit temperature =2,294.72 K, Exit Velocity=2,225.3 m/sec , Exit pressure = 308,603.95pascal.

REFERENCE

[1]. http://webserver.dmt.upm.es/~isidoro/bk3/c17/Nozzles.pdf
[2]. https://www.narom.no/undervisningsressurser/sarepta/rocket-theory/rocket-propellants-2/solid-propellants/
[3]. https://www.sciencedirect.com/science/article/pii/S2352146518300279
[4]. http://maenas.eng.usu.edu/MAE_5540_Web/propulsion_systems/subpages/Rocket_Propulsion_Elements.pdf
[5]. https://www.researchgate.net/publication/328086005_Design_of_Nozzle_for_High Powered_Solid_Rocket_Propellant/fulltext/5bb6cabe4585159e8d8686a6/328086005_Design_ofNozzle_for_High-Powered_Solid_Rocket_Propellant.pdf?origin=publication_detail
[6]. http://edge.rit.edu/edge/P18102/public/Engine%20Specifications/Propulsion/Nozzle/Nozzle
[7]. https://core.ac.uk/download/pdf/154060575.pdf
[8]. https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19660015713.pdf

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