

## An Investigation on the Effects of Rigid Joints on the Weight and Load Carrying Capacity of Steel Trusses

Okonkwo V. O.<sup>a</sup>, Onodagu P. D.<sup>b</sup> and Udemba J. N.<sup>c</sup>

<sup>a</sup>Lecturer I, <sup>b</sup>Senior Lecturer

<sup>ab</sup>Department of Civil Engineering, Nnamdi Azikiwe University Awka, NIGERIA

<sup>c</sup>Department of Engineering Services, National Inland Waterways Authority Lokoja, NIGERIA

Received 28 February 2021; Accepted 13 March 2021

**ABSTRACT:** In this work an investigation was carried out on the implications of analyzing welded trusses as pin-jointed on the weight and by extension on the cost of the structure. Different trusses were studied and the effect of secondary stresses (bending moments induced by joint rigidity) on the final weight of the structure determined. The trusses were first analysed as pin-jointed and later as having rigid joints. The sizes of the truss elements were assumed to be proportional to the axial force in it. By considering the elements of the truss to be of a constant breadth  $b$ , the required depth of the section becomes representative of the size of the element. To convert it to weight the calculated depth for each element is multiplied by the corresponding length of the element. This quantity was compared for pin-jointed and rigidly connected joints. From the obtained results it was observed that the rigidly connected (welded) truss requires a lighter and hence cheaper structure than the pin-jointed truss. In conclusion it was observed that the welded joints substantially increased the load carrying capacity of the truss.

**KEYWORDS:** Rigid joints; Steel structures; Secondary stresses; Welded joints; Pinned joints; Roof weight; Roof cost.

### I. INTRODUCTION :

Trusses produce a simple structural configuration for supporting loads. They are thus prevalently used in roofing. The truss consists of an assemblage of struts and tensional elements all pinned at the joints. The pin connected joints allow for slight rotation at the joints and thus eliminate the development of bending and shear stresses in the elements of the truss. Apart from eliminating bending and shear stresses in trusses, the use of pinned joints reduces the degrees of statical indeterminacy of the structure and so eases the structural analysis of the structure. The use of trusses dates back to many years ago. They provide an economical configuration of bars that can support weight. Trusses are economical because the placement of the bars enables the structure to have a good load to weight ratio (Aswathi et al., 2015; Marusceac and Vlad, 2016). The truss has found profound use in roofs, overhead tanks, bridges, towers etc. The common types of roof trusses are the warren truss, pratt truss, fink truss and howe truss. The connection in roof trusses is either bolted, riveted or welded. Bolted connection enable the fast assembly and dismantling of the truss structure, hence pin-jointed trusses are easily deployed in the construction of temporal structures. However because many roof structures are constructed with permanence in mind, it is common to see trusses being constructed with rigid joints. This mostly occurs when the connections are welded. In principle welded connections will not allow rotation at the joints and hence its analysis as pin jointed structure does not represent the true structure. In Nigeria bolting and welding are common. The welded joint is more preferred because most fabrication are done on the site and does not require as much skill as the bolted connection. The welded roof structure does not require as much dexterity as the pin-jointed to construct. For bolted connections precise measurements are required, members are usually prepared in a workshop and then assembled at the site. Gusset plates are used to accommodate the required number of bolts and these are done by experts (Ezeagu and Onunkwo, 2015).

In the analysis of trusses the loads on the truss are normally assumed to be at the joints (Thakar and Patel 2013). The joints are also assumed to be pin-connected. In reality chords/ truss elements are continuous and so extend beyond the joints. The connections at the joints are either welded or contain multiple bolts. These joints tend to restrict relative rotation of the members at the nodes and end moments develop (Tasou, 2003; Pradeepa and Monika, 2015).

### II. METHODOLOGY

For this study five trusses were selected; the pratt-pitched roof, the pratt-flat, the Howe truss, the flat warren and the modified warren truss. The trusses were assumed to be loaded only at the joints. Secondary stresses due to eccentricities were ignored. The loadings for the sake of the study were idealized, the joints were assumed to have unit loads of 1kN. See Figure 1. The trusses were first analysed as having all joints 'pinned'

and later as having all the joints rigidly connected (welded). A relationship can be formulated between the stresses in the element of a truss and the anticipated weight of the truss. The intensity of force (force per unit area) is called stress. If the stress has a uniform distribution then (Brettle and Brown, 2011)

$$\sigma = N_p/A \tag{1}$$

where  $N_p$  is the axial force in the pin-connected truss and  $A$  is the cross sectional area. For a rectangular section with a breadth  $b$  and a depth  $d_p$  equation (1) can be rewritten as

$$\sigma = N_p/bd_p \tag{2}$$

By taking  $b$  to be equal to unity the minimum depth  $d_p$  of an element of the pinned truss can be computed from  $d_p = N_p/\sigma$  (3)

where  $\sigma$  is the grade of steel and  $N_p$  is the axial force in an element of the pin-jointed truss.

By considering a welded truss with fixed joints

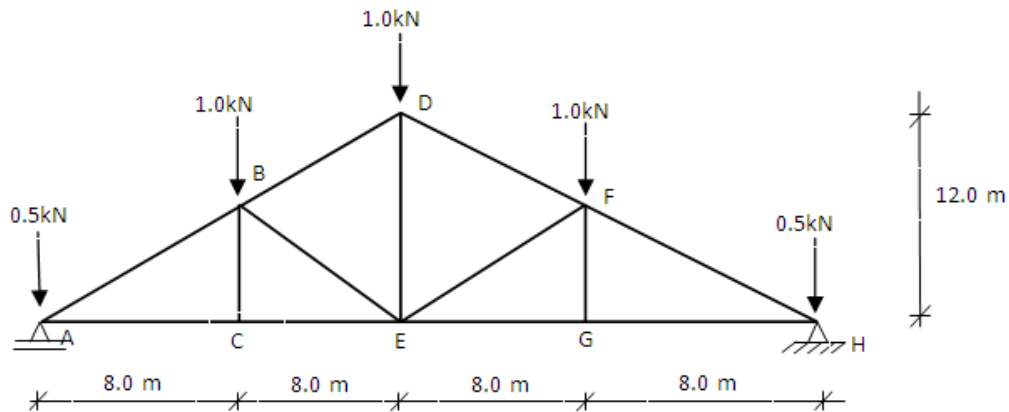
$$\sigma = N_f/bd \pm 6M/bd_f^2 \tag{4}$$

Where  $M$  is the end moment at the welded joint,  $N_f$  is the axial force in the element under consideration and  $d_f$  is the minimum depth of the element of the welded truss. By taking  $b$  to be equal to unity and rearranging equation (4) we obtain a quadratic expression for finding the minimum value of  $d_f$  for any value of end moment  $M$  and axial force  $N_f$ .

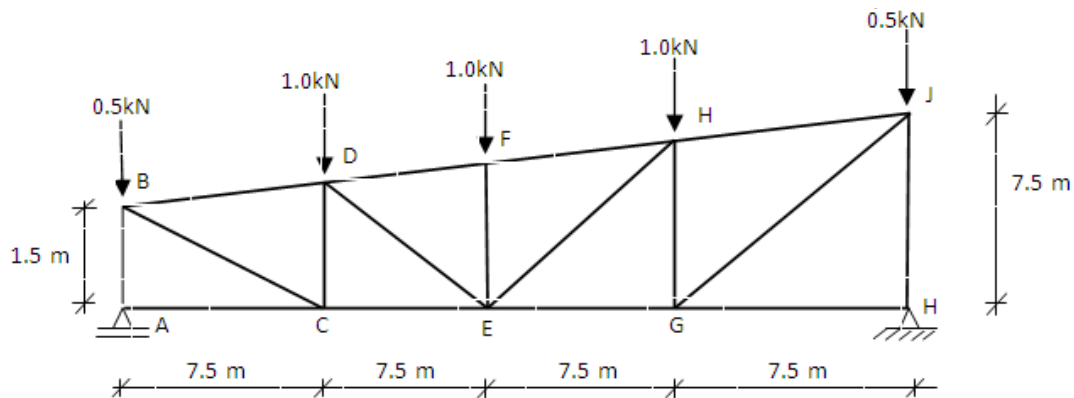
$$\sigma(d_f)^2 - d_f N_f - 6M = 0 \tag{5}$$

Only the absolute values of  $N_f$  and  $M$  are used in equation (5).

From equations (3) and (5) the depth of an element of a truss is computed at a constant breadth  $b = 1$ . To get an expression for volume the calculated depth is multiplied with the length  $L$  of the element. The sum total of the volume of the elements of a truss gives the volume of the truss. At constant density of steel, the cost of steel is proportional to the volume of steel. Hence roof trusses fabricated with less volume of material can be assumed to be cheaper.



(a) Truss 1: Pratt-pitched truss



(b) Truss 2: Pratt-flat truss

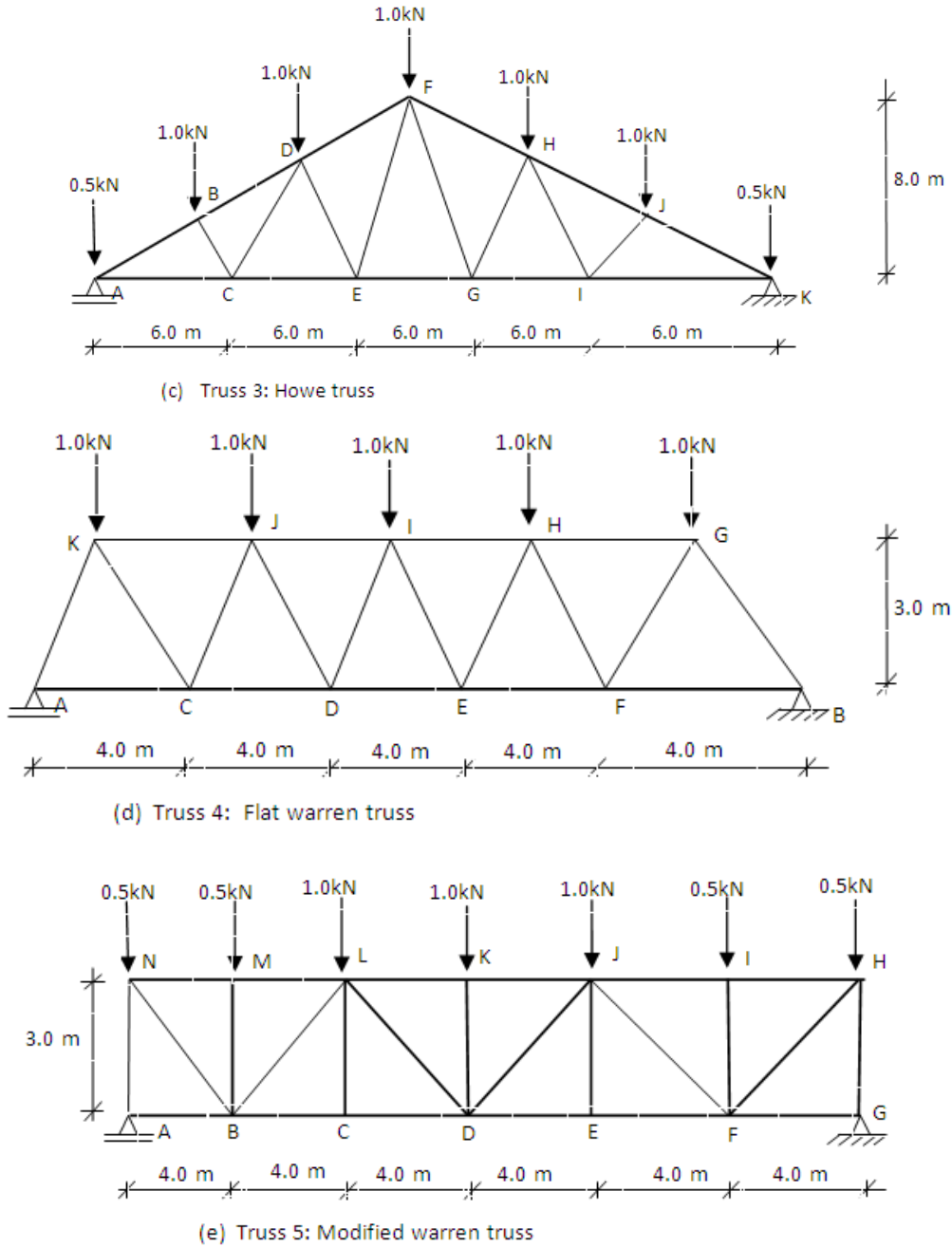


Figure 1: Some selected steel roof trusses

### III. RESULTS AND DISCUSSIONS

From the analysis of truss 1 both as pin-connected and as rigidly connected we obtained the axial forces in the members when pinned  $N_p$  and the axial forces in the elements when rigidly connected (welded)  $N_f$ . The corresponding depth of elements required  $d_p$  and  $d_f$  were calculated from equations (3) and (5). These values are presented in table 1 below.

Table 1: Calculated axial forces, depth of elements and volume per unit breadth for truss 1

Member	Length (m)	$N_p$ (kN)	$N_f$ (kN)	M (kNm)	$d_p$ (mm)	$d_f$ (mm)	$d_p L$ (mm <sup>2</sup> )	$d_f L$ (mm <sup>2</sup> )
AC	8	2	1.456	1.748	7.2727	6.1783	58181.6	49426.4
CE	8	2	1.525	1.2	7.2727	5.1196	58181.6	40956.8
EG	8	2	1.525	1.2	7.2727	5.1196	58181.6	40956.8
GH	8	2	1.456	1.748	7.2727	6.1783	58181.6	49426.4
HF	10	2.5	1.922	1.514	9.0909	5.7509	90909.0	57509.0

FD	10	1.667	1.519	1.426	6.0618	5.5806	60618.0	55806.0
DB	10	1.667	1.519	1.426	6.0618	5.5806	60618.0	55806.0
BA	10	2.5	1.922	1.514	9.0909	5.7509	90909.0	57509.0
BC	6	0	0.371	0.544	0	3.4458	0	20674.8
BE	10	0.883	0.206	0.452	3.2190	2.8107	32109.0	28107.0
DE	12	0	0	0	0	0	0	0
EF	10	0.883	0.206	0.452	3.2109	2.8107	32109.0	28107.0
FG	6	0	0.371	0.544	0	3.4458	0	20674.8
						$\Sigma$	599998.40	504960.00

From table 1 above we see that the required total volume of steel per unit breadth for the pinned connected truss is 599998.4mm<sup>2</sup> while the one for the equivalent welded truss is 504960.0mm<sup>2</sup>. The volume of steel required for the welded truss is 15.8% lower than the one required for an equivalent pin connected truss.

Truss 2 was also analysed both as pin-connected and as rigidly connected and the axial forces in the members when pinned  $N_p$  and the axial forces in the elements when rigidly connected (welded)  $N_f$  obtained. The corresponding depth of elements required  $d_p$  and  $d_f$  were calculated from equations (3) and (5) . These values are presented in table 2 below.

**Table 2:** Calculated axial forces, depth of elements and volume per unit breadth for truss 2

Member	Length (m)	$N_p$ (kN)	$N_f$ (kN)	M (kNm)	$d_p$ (mm)	$d_f$ (mm)	$d_p L$ (mm <sup>2</sup> )	$d_f L$ (mm <sup>2</sup> )
AC	7.0	0	0.548	1.228	0	5.1772	0	36240.40
CE	7.0	3.5	1.72	1.224	12.7273	5.1709	89091.10	36196.20
EG	7.0	1.75	0.50	1.183	6.3636	5.0814	44545.20	35569.80
GI	7.0	0	0.07	1.200	0	5.1169	0	35818.30
IJ	7.51	2.0	2.28	1.43	7.2727	5.5898	54619.98	41979.40
JH	7.159	3.06	0.351	1.33	11.1273	5.3874	79659.23	38567.86
HF	7.159	3.29	0.853	0.864	11.9636	4.3402	85646.22	31071.06
FD	7.159	3.22	1.007	1.174	11.7091	6.5270	83824.28	46726.14
DB	7.159	3.58	0.956	1.718	13.0182	6.1241	93196.00	43841.82
BA	1.50	2.00	1.197	0.471	7.2727	3.2079	10909.05	4811.85
BC	7.159	3.58	0.3754	0.897	13.0182	4.4246	93196.00	31675.27
DC	3.0	0.75	0.4868	0.901	2.7273	4.4346	8181.90	13303.80
DE	7.616	0.698	0.127	0.959	2.5382	4.5744	19330.22	34837.35
FE	4.5	1.33	0.968	1.056	4.8473	4.2002	21812.85	18900.90
EH	9.22	1.79	0.876	2.375	6.520	7.2000	60114.40	66384.00
HG	6.0	1.875	1.143	1.52	6.8182	5.7609	40909.20	34565.40
GJ	10.26	2.656	1.122	1.899	9.6582	6.4389	99093.13	66063.11
							884128.76	616552.66

From table 2 above we observe that the required total volume of steel per unit breadth for the pinned connected truss is 884128.76mm<sup>2</sup> while the one for the equivalent welded truss is 616552.66mm<sup>2</sup>. This shows that the volume of steel required for the welded truss is 30.26% lower than the one required for an equivalent pin connected truss.

Trusses 3, 4 and 5 were also analysed both as pinned connected and as rigidly connected (welded) and their values of axial forces  $N_p$  and  $N_f$  and the volume of steel required obtained. They are presented in tables 3, 4 and 5 below.

**Table 3:** Calculated axial forces, depth of elements and volume per unit breadth for truss 3

Member	Length (m)	$N_p$ (kN)	$N_f$ (kN)	M (kNm)	$d_p$ (mm)	$d_f$ (mm)	$d_p L$ (mm <sup>2</sup> )	$d_f L$ (mm <sup>2</sup> )
AC	6.000	4.687	1.851	4.226	17.0436	9.6056	102261.60	57633.60
CE	6.000	3.750	3.077	2.065	13.6364	6.7179	81818.40	40307.40
EG	6.000	2.813	2.839	1.033	3.7564	4.7526	22538.40	28515.60
GI	6.000	3.75	3.077	4.404	14.6909	9.3942	88145.40	56365.20
IK	6.000	4.687	1.851	4.226	17.0436	9.6056	102261.60	57633.60
KJ	5.667	5.313	2.476	4.042	19.3200	9.3954	109486.44	53243.73
JH	5.667	4.973	2.874	2.386	18.0836	7.2204	102479.76	40918.19
HF	5.667	3.261	2.846	2.627	12.0332	7.5760	68192.14	42933.19

FD	5.667	3.261	2.846	2.576	12.0332	7.5021	68192.14	42514.40
DB	5.667	4.973	2.874	2.836	18.0836	7.8714	102479.76	44607.22
BA	5.667	5.313	2.846	4.040	19.3200	9.3938	109486.40	53234.66
BC	2.848	1.000	0.041	1.656	3.6364	6.0110	10356.47	17119.33
CD	6.666	1.170	0.681	1.344	4.2545	5.4164	28362.20	36107.89
DE	5.696	1.390	0.155	0.746	5.0545	4.0347	28790.43	22981.65
EF	8.544	1.390	0.223	0.763	5.0545	4.2876	43185.65	36633.25
FG	8.544	1.390	0.223	0.763	5.0545	4.2876	43185.65	36633.25
GH	5.696	1.390	0.041	0.746	5.0545	4.0344	28790.43	22979.94
HI	6.666	1.170	0.681	1.344	4.2545	5.4164	28362.20	36107.89
IJ	2.848	1.000	0.041	1.656	3.6364	6.0110	10356.47	17119.33
						Σ	1178731..58	743589.14

**Table 4:** Calculated axial forces, depth of elements and volume per unit breadth for truss 4

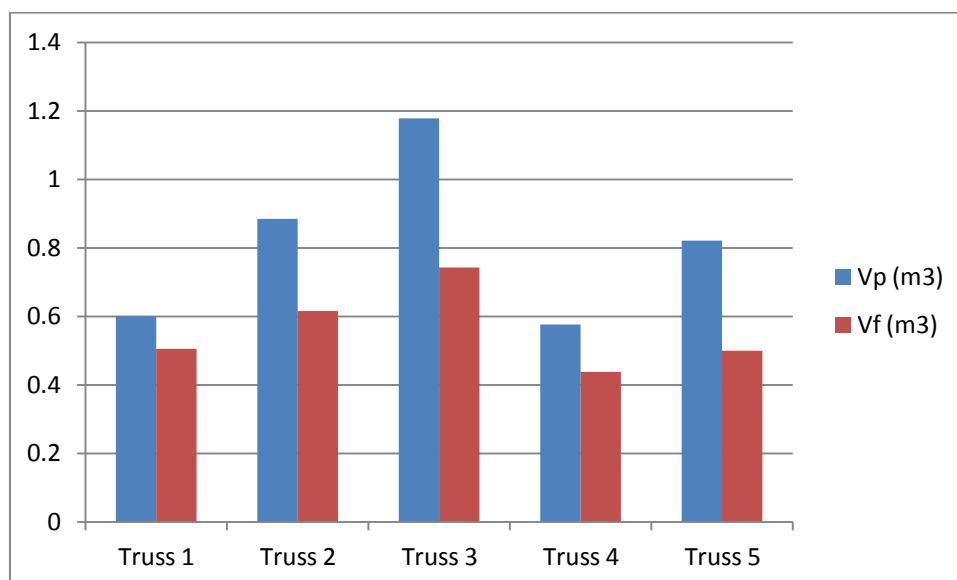
Member	Length (m)	N <sub>p</sub> (kN)	N <sub>f</sub> (kN)	M (kNm)	d <sub>p</sub> (mm)	d <sub>f</sub> (mm)	d <sub>p</sub> L (mm <sup>2</sup> )	d <sub>f</sub> L (mm <sup>2</sup> )
AC	4.0	1.33	1.052	2.637	4.8364	7.587	19345.60	30348.00
CD	4.0	3.33	2.181	2.318	12.1091	7.111	48436.40	28444.00
DE	4.0	4.0	2.595	1.857	14.5455	6.371	58182.00	25484.00
EF	4.0	3.33	2.181	2.317	12.1091	7.111	48436.40	28444.00
FB	4.0	1.33	1.052	2.637	4.8364	7.587	19345.60	30348.00
BG	3.606	2.4	1.975	0.862	8.7273	4.340	31470.64	15640.04
GH	4.0	2.667	1.585	2.382	9.6982	7.212	38792.80	28848.00
HI	4.0	4.0	2.485	2.200	14.5455	6.933	58182.00	27732.00
IJ	4.0	4.0	2.485	2.200	14.5455	6.933	58182.00	27732.00
JK	4.0	2.667	1.598	2.382	9.6982	7.212	38792.80	28848.00
KA	3.606	2.4	1.975	0.862	8.7273	4.340	31470.64	15640.04
KC	3.606	2.4	0.311	1.136	8.7273	4.978	31470.64	17950.67
CJ	3.606	1.2	0.728	1.231	4.3636	5.183	15735.14	18689.90
JD	3.606	1.2	0.122	1.464	4.3636	5.652	15735.14	20381.11
DI	3.606	0	0.306	1.158	0	5.027	0	18127.36
IE	3.606	0	0.306	1.158	0	5.027	0	18127.36
EH	3.606	1.2	0.122	1.464	4.364	5.652	15735.14	20381.11
HF	3.606	1.2	0.728	1.231	4.364	5.184	15735.14	18689.90
FG	3.606	2.4	0.311	1.136	8.7273	4.979	31470.64	17950.67
						Σ	576518.72	437806.1

**Table 5:** Calculated forces, depth of elements and volume for truss 5

Member	Length (m)	N <sub>p</sub> (kN)	N <sub>f</sub> (kN)	M (kNm)	d <sub>p</sub> (mm)	d <sub>f</sub> (mm)	d <sub>p</sub> L (mm <sup>2</sup> )	d <sub>f</sub> L (mm <sup>2</sup> )
AB	2.0	1.25	0.751	1.191	4.5455	5.0990	9090.90	10198.00
BC	4.0	1.25	1.341	1.94	4.5455	6.5084	18181.80	26033.06
CD	4.0	3.25	2.268	1.53	11.8182	5.7818	47272.72	23127.20
DE	4.0	3.25	2.268	1.53	11.8182	5.7818	47272.72	23127.20
EF	4.0	1.25	1.341	1.94	4.5455	6.5084	18181.80	26033.06
FG	2.0	1.25	0.751	1.191	4.5455	2.0824	9090.90	4164.80
IH	4.0	0	0.361	0.6	0	3.6187	0	14474.80
IJ	4.0	2.75	1.622	1.53	10.000	5.7806	40000	23121.60
KJ	4.0	2.75	2.066	1.36	10.000	5.4510	40000	21804.00
LK	4.0	2.75	2.066	1.36	10.000	5.4510	40000	21804.00
ML	4.0	2.75	1.622	2.00	10.000	6.6087	40000	26434.80
NM	2.0	0	0.316	0.63	0	1.5141	0	3028.20
AM	4.472	2.795	0.39	0.218	10.1636	2.1816	45451.62	9756.12
MB	4.0	0	0.582	1.362	0	5.4523	0	21809.20
MC	5.656	2.121	0.422	0.565	7.7127	3.5118	43623.03	19862.74
CL	4.0	1.0	0.405	0.896	3.6362	3.6370	14545.20	14548.00
CK	5.656	7.07	0.346	1.036	25.709	9.6695	145410.10	54690.69
DK	4.0	0	0.303	0	0	0.0013	0	5.20
KE	5.656	7.07	0.346	1.036	25.709	9.6685	145410.10	54690.69

JE	4.0	1	0.405	0.896	3.6363	4.4222	14545.20	17688.80
EI	5.656	2.121	0.422	0.564	7.7127	3.5087	43623.03	19845.21
IF	4.0	0	0.582	1.3768	0	5.4819	0	21927.60
IG	4.472	2.795	0.956	0.218	10.1636	2.1826	45451.62	9760.59
GH	4.0	0.5	1.039	0.811	1.8182	4.2084	7272.72	16833.60
AN	4.0	0.5	1.039	0.811	1.8182	4.2084	7272.72	16833.60
						$\Sigma$	821696.18	500800.76

From table 3, 4 and 5 above we also observed a similar trend. The computed volumes of steel per unit breadth were all found to be less for the welded truss than for the pin connected truss. These are summarized in Figure 1 below. It is also observed that the presence of rigid joints ( provided by welding) alters the axial forces in the trusses and very often reduces them.



**Figure 1:** Chart showing the Volume of steel required for a pinned truss and the corresponding welded truss.

Figure 1 is a bar chart showing the volume of steel for the pin connected truss and the corresponding volume for the welded truss. From the chart it is seen that the volume of steel required for the welded truss was always lower than that required for the corresponding pin connected truss. Steel is sold in tonnes (1000kg). Mass is the product of volume and density. Since the cost of steel is proportional to the mass of steel we have

$$\text{cost} \propto \rho \times V \quad (6)$$

Where  $\rho$  is the density of steel and  $V$  the volume of steel. If the density is constant then we can establish that the cost of steel is proportion to the volume.

$$\text{cost} \propto V \quad (7)$$

From equation (7) can infer that trusses that require less volume of steel will be cheaper than those requiring more volume of steel. This implies that the truss with the minimum weight of material is considered as an economic truss (Andrzej, 1987; Murali et al, 2014). When truss 1 was welded, volume dropped by 15.84% thus representing a cost reduction of 15.84% in steel by adopting a welded connection. When truss 2 was welded, the required volume of dropped by 30.26% thus representing a cost reduction of 30.26% in steel by adopting a welded connection. Likewise when truss 3 was welded, volume dropped by 36.92% thus representing a cost reduction of 36.92% in steel by adopting a welded connection. For truss 4 and 5 the cost reduction were 24.07% and 39.05% respectively.

#### IV. CONCLUSION AND RECOMMENDATION:

From the results we can make the following conclusions

1. The calculated axial forces in the rigidly connected trusses were generally less than that obtained for the corresponding pin-connected truss.
2. The presence of flexural effect (bending moments) due to joint rigidity redistributed the axial forces in the truss system.

3. The use of rigid joints produced a lighter structure. If weight of the structure is considered proportional to the cost. Then the use of rigid joints produced a cheaper structure.
4. From 3 above we can infer that for truss structures of same weight the one with rigidly connected joints will have a higher load carrying capacity.  
For permanent steel structures we need to consider their fabrication with rigid joint as a money saving measure. The structure analysed as having rigid joints will be lighter in weight. Put in a different perspective, a welded steel truss will be able to support significantly more load than the equivalent pin connected truss.

#### **REFERENCES:**

- [1]. Andrzej M. B., 1987. Criteria and Methods of Structural Optimization, Springer Sciences and Business Media pp 130
- [2]. Aswathi D. K., Preetha P., Neethu S., 2105. Analytical Study on fatigue Behaviour of steel truss girder joints, International Journal of Innovative Science, Engineering and Technology, Vol. 2, Issue 10, pp 574 - 578
- [3]. Brett M. E. and Brown D. G. 2011. Steel Building Design: Worked example for students In accordance with Eurocode and the UK National Annexes, The Steel Construction Institute Berkshire. Pp 257
- [4]. Tasou P. 2003. Trusses In: Steel Designers Manual 6<sup>th</sup> Edition. The Steel Construction Institute.UK
- [5]. Ezeagu C. A and Onunkwo R. C.2015, Complex Truss Analogy Using Plastic and Elastic Analysis. International Journal of Latest Research in Engineering and Technology, vol.1 Iss. 7. Pp 49 – 90
- [6]. Marusceac V, Vlad M., 2016. Economic Solution for short span bridges using reinforced glue laminated timber and steel, Procedia Engineering, Vol. 156, pp 227 – 232. Elsevier
- [7]. Murali S. V. S, Markandeya R. P., Majj S. P., Gopi S., 2014. Study on Economic Configuration of flat roofed steel trusses, International Journal of Scientific Research and Development (IJSRD), Vol. 2, Issue 2, pp 688 - 692
- [8]. Pradeepa S. and Monika N. R., 2015. Design and Comparison of Various Types of Long Span Roof Trusses, International Journal of Science and Engineering Research, Vol 3. Iss. 10, pp 5687-5690
- [9]. Thakar J. D and Patel P. G., 2013. Comparative Study of Pre-engineered Steel Structure by Varying the width of Structure, International Jour Figure 1: Some selected steel roof trussesnal of Advanced Engineering Technology, Vol 4, Iss. 3. Pp. 56 - 62

Okonkwo V. O, et. al. "An Investigation on the Effects of Rigid Joints on the Weight and Load Carrying Capacity of Steel Trusses." *IOSR Journal of Engineering (IOSRJEN)*, 11(03), 2021, pp. 33-39.