

Analysis and Optimization of Fast Channel Estimation In Millimeter Wave Vehicular Communication

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Received 10 August 2021; Accepted 05 August 2021

Abstract: A system that has moving vehicles and infrastructural roadside units communicating with each other to provide live information about the road, such as traffic congestion and warnings, thus acting as communication nodes is called as a Vehicular Communication system. To realize high data rate waves with large spectral channels are required and mm Wave band have large spectral band, hence it is considered among other waves. Being a communication system with the general structure of Transmitter-Channel-Receiver, the efficiency and usability of the system heavily depends upon the Channel constraints and factors. Hence with respect to designing the system, detailed understanding of channel estimation through factual simulations and literature review becomes mandatory. This paper focuses on modelling the channel and simulating the required channel estimation parameters through MATLAB and MATLAB's LTE Toolbox. In this paper, two different algorithms were used, including Maximum Likelihood Estimate and Least Squares. The result has shown that Least Squares estimate provides a much better signal estimation.

Keywords: Millimeter Wave Communication, Vehicular Communication, Maximum Likelihood Estimate, Least Square ML Estimate

I. INTRODUCTION

Millimeter waves are electromagnetic wave that lies in the range of frequencies 30 – 300 GHz and the wavelength ranges from 10 to 1 mm. Millimeter waves are also known as extremely high frequency (EHF) waves. The MM wave band makes its presence in the electromagnetic spectrum in between the super high frequency band and the far infrared band. The millimeter wave frequency band has various applications such as in telecommunications, weapons systems, scientific research, security screening, medicine, police radar, thickness gauging etc. The wide bandwidth of millimeter wave provides high transmission rate which makes it attractive for ultrahigh speed wireless and satellite communications. The millimeter waves are transmitted in narrow focused beams. The hardware size can be reduced for millimeter waves because of its high frequency. It's extremely high frequencies enable multiple short-distance usages at same frequency without interfering each other and can penetrate fog/cloud/rain which enables various applications like all-weather radar and sensing. The atmospheric attenuation caused by this band is much higher than lower placed bands, meaning that gases present in the atmosphere are capable of absorbing it. This affects the range usability scenario of the band where the range is near to 1km in terrestrial conditions. The short wavelength enables antennas of average size to present a small beam width, which increases the potential for frequency reuse. In 5G networks, the frequency ranges that are present at the bottom of the band are used as an application.

Vehicular communication is a major application of mm wave which includes Vehicle to vehicle communication (V2V), Vehicle to infrastructure (V2I) communication, Vehicle to network communication (V2N) and Vehicle to pedestrian communication (V2P). Computer networks in which vehicles and roadside units are the communicating nodes is called vehicular communication system. The main purpose of vehicular communication is to increase road safety and eliminate the damage due to vehicular collisions. The use of mm Wave for vehicular communications is not a new concept but it still faces several open challenges. In our project we are focusing on V2I communication. This is a communication framework that is aimed at enabling information exchange between multiple vehicles and a set of devices that work in accordance with systems followed for highways in that country. They are enabled by a network of hardware, software, and firmware, the V2I technology is typically wireless and bi- directional network that exchange information from infrastructure devices to the vehicle through a necessary network and vice versa.

Wireless communication channels have some undesirable effects on the signals transmitted through it that results in the receiver end of the communication system having to deal with imperfect information that is affected by attenuation, phase shift of the signals, delays, and distortion to due to expected and unexpected element hindrances. This mandates the need for efficient and accurate channel estimation Thus, channel estimation plays a very important role on the performance of wireless communication. Channel estimation is the process of finding correlation between the array of complex numbers on the left and the array of complex numbers on the right. In wireless communication link, channel state information (CSI) provides the known channel properties of the link. It should be estimated by the receiver and fed back to the transmitter and hence the transmitter and receiver has different channel state information. Depending upon the rate at which the channel changes due to several governing factors, limits get set on the channel state information acquisition.

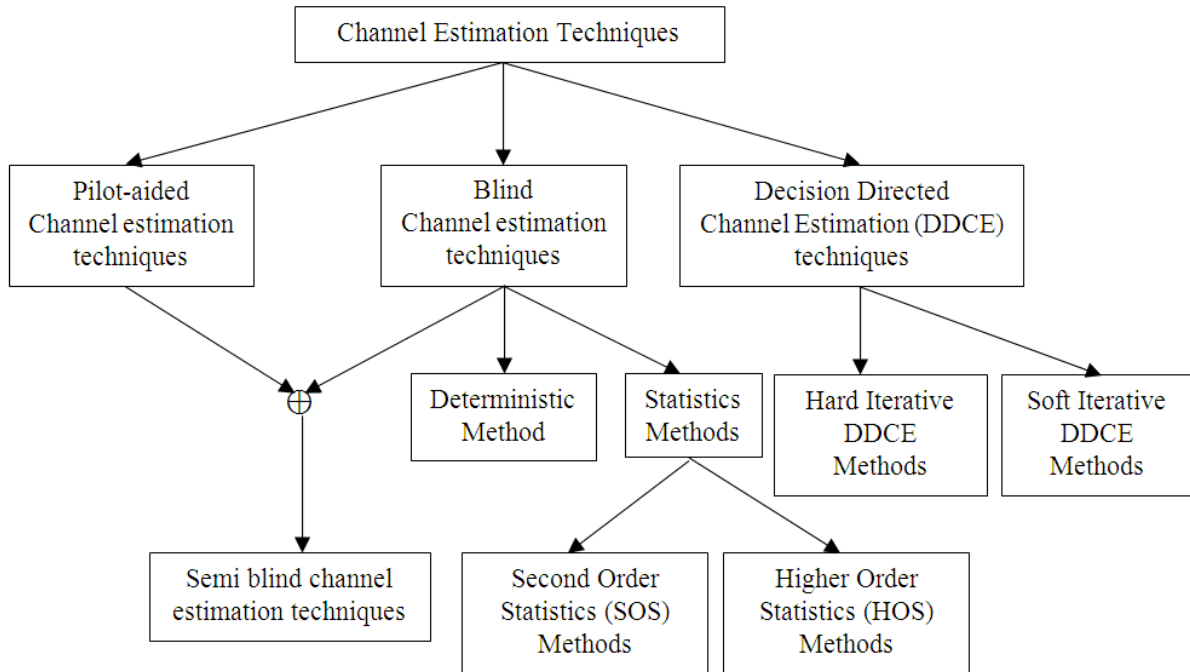


Figure 1: Classification of Channel Estimation

II. THEORETICAL REVIEW

A. Maximum Likelihood Estimate

Maximum likelihood is a relatively simple method of constructing an estimator for an unknown parameter h . In this paper, a simple communication system that has a single sensor that is used for the measurement of a particular parameter of interest such as temperature, pressure, position etc. The sensors are affected by noise providing noisy observations. The observation can be expressed as

$$y = h + v \tag{1}$$

where h represents the true parameter and v is a Gaussian noise ($E\{v\} = 0$ and $\sigma^2 = E\{v^2\}$). Since the noise is Gaussian in nature, the observation also is of Gaussian nature and can be seen as a Gaussian Random Variable ($E\{y\} = h$, where h is a constant true parameter). The Probability Density Function (PDF) of the Gaussian with mean h and σ^2 is,

$$F_y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y-h)^2} \tag{2}$$

where h is the unknown parameter that has to be estimated. Hence the process is called as parameter estimation. Assume that there are N number of sensor nodes that make a measurement at a particular instant in time.

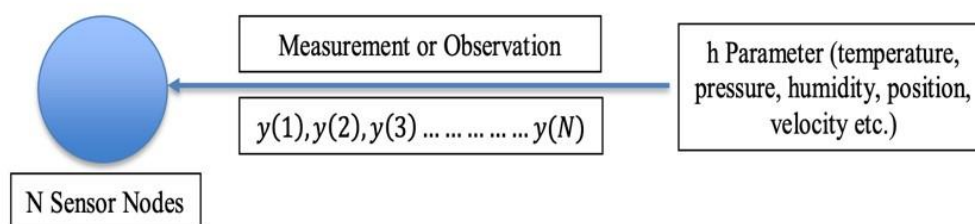


Figure 2: Multiple Node Communication System

Here,

$$\begin{aligned} y(1) &= h + v(1) \\ y(2) &= h + v(2) \\ &\vdots \end{aligned} \tag{3}$$

$$y(N) = h + v(N) \Rightarrow y(k) = h + v(k)$$

Thus, the PDF of each individual observation $y(k)$ is given as:

$$F_y(y(k)) = \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{1}{2\sigma^2}(y(k)-h)^2} \tag{4}$$

Assuming that initially the noise elements are Independent Identically Distributed (IID), we get that all the observations are also independent in nature and thus we can compute the Joint PDF as:

$$F_{Y(1)Y(2)...Y(N)}(y(1), y(2) \dots \dots y(N)) = \left[\frac{1}{\sqrt{2\pi\sigma^2}} \right]^N e^{-\frac{1}{2\sigma^2} \sum_{k=1}^N (y(k)-h)^2} = P(\bar{y}; h) \tag{5}$$

This equation implies that the joint PDF is a function of the unknown parameter h known as its Likelihood

Function where $P = \text{Likelihood}$ and $\bar{y} = \text{observation vector} = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix}$.

Now, maximizing $P(\bar{y}; h) \equiv \text{Maximizing } \ln(P(\bar{y}; h))$, because natural logarithm is a monotonically increasing function.

$$\begin{aligned} \Rightarrow \ln P(\bar{y}; h) &= \left(\left[\frac{1}{\sqrt{2\pi\sigma^2}} \right]^N e^{-\frac{1}{2\sigma^2} \sum_{k=1}^N (y(k)-h)^2} \right) \\ &= -\frac{N}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{k=1}^N \mathcal{L}(\bar{y}; h) = \text{Log Likelihood} \end{aligned} \tag{6}$$

Here, $-\frac{N}{2} \ln(2\pi\sigma^2)$ and $\frac{1}{\sigma^2}$ is a constant thus the negative term and the basic constraint in the above equation is minimised by differentiating and equating it to 0. Then we get

$$\hat{h} = \frac{1}{N} \sum_{k=1}^N y(k) \tag{7}$$

Here, \hat{h} is the estimate of the unknown parameter h for which the likelihood function maximizes. This is also called as the ML estimate of h . It shows that the ML estimate is the average of the observations $y(1), y(2) \dots \dots y(N)$. The estimation lies in the understanding of how close \hat{h} is to h .

Mean and variance of the estimate is

$$E\{\hat{h}\} = h \tag{8}$$

$$E\{(\hat{h} - h)^2\} = \frac{\sigma^2}{N} \tag{9}$$

From this equation we can say that the variance of IID Noise samples decreases be a factor of N. Hence \hat{h} can be represented as,

$$\hat{h} \sim N\left(h, \frac{\sigma^2}{N}\right) \tag{10}$$

This shows that as we increase N the estimate becomes more and more accurate. As N tends to infinity, the variance tends to 0 i.e., the estimate coincides with the true parameter value. To test the reliability of the estimate we focused on the number of observations required so that the probability that the estimate \hat{h} lies within $\frac{\sigma}{2}$ of the true parameter is greater than 0.9999 or that it provides 99.99% reliability.

$$Pr(|\hat{h} - h|) \leq \frac{\sigma}{2} \geq 0.9999 \tag{11}$$

Let $w = \hat{h} - h$, be the estimation error. Hence the above equation computes the Probability of Estimation Error (PEE) which is also a very important parameter in the process of Channel Estimation.

$$Pr\left(|w| \geq \frac{\sigma}{2}\right) \leq 0.0001 \tag{12}$$

From (10),

$$w = \hat{h} - h \sim \left(0, \frac{\sigma^2}{N}\right) \text{ and } F_w(w) = \frac{1}{\sqrt{(2\pi\frac{\sigma^2}{N})}} e^{-\frac{N}{2\sigma^2}w^2} \tag{13}$$

Equation (12) becomes,

$$Pr\left(w \geq \frac{\sigma}{2}\right) + Pr\left(w \leq -\frac{\sigma}{2}\right) \leq 0.0001 \tag{14}$$

$$\int_{\frac{\sigma}{2}}^{\infty} F_w(w)dw + \int_{-\infty}^{-\frac{\sigma}{2}} F_w(w)dw \leq 0.0001 \tag{15}$$

As $F(w)$ is an even function we get,

$$\int_{\frac{\sigma}{2}}^{\infty} \frac{1}{\sqrt{2\pi\frac{\sigma^2}{N}}} e^{-\frac{N}{2\sigma^2}w^2} dw \leq 0.00005 \tag{16}$$

Taking, $\frac{w^2}{\sigma^2/N} = t^2 \Rightarrow dw = \frac{\sigma}{\sqrt{N}} dt$

$$\frac{1}{\sqrt{2\pi}} \int_{\frac{\sigma}{2}}^{\infty} \frac{1}{\sqrt{N}} e^{-\frac{t^2}{2}} dt \leq 0.00005 \tag{17}$$

Above equation gives the PDF of standard gaussian random variable with mean equal to 0 and variance equal to 1. The general representation of Q function is:

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \tag{18}$$

$$Q\left(\frac{\sqrt{N}}{2}\right) \leq 0.00005 \Rightarrow N \geq 61 \text{ samples}$$

Taking $Q(x) = \frac{1}{2} e^{-\frac{x^2}{2}}$,

$$N \geq 74 \text{ samples}$$

This implies that when we have 74 or number of observations, the reliability provided by ML estimate is 99.99%.

B. Least Squares ML Estimate

Least squares is a time-honoured estimation procedure. It is used for channel estimation in multi-antenna system. We considered a system with multiple transmit antennas and Single receiver antenna.

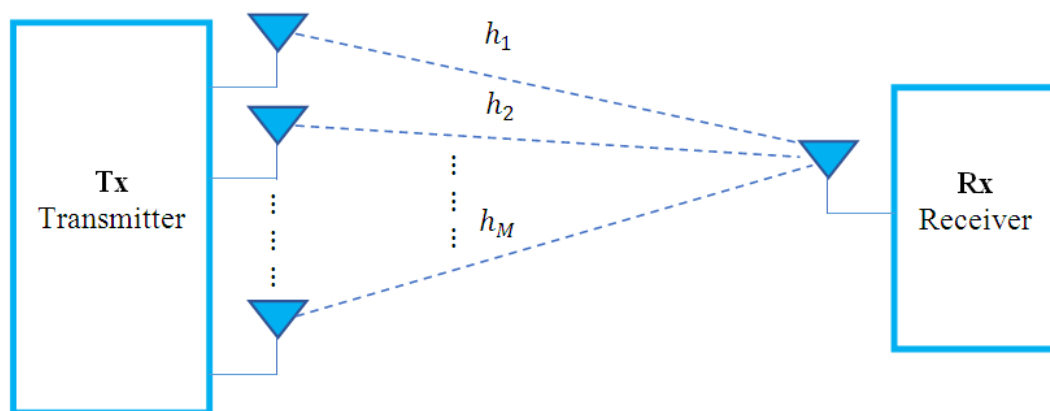


Figure 3: Multiple Transmitter Single Receiver System

Vector representation is taken due to the presence of multiple antennas.

$$x_i(k) = \text{Pilot symbol transmitted from } i^{\text{th}} \text{ antenna at time } k.$$

Thus received symbol becomes:

$$y(k) = h_1x_1(k) + h_2x_2(k) + \dots + h_Mx_M(k) + v(k)$$

The model for multiple antenna channel estimation;

$$\begin{aligned} \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix} &= \begin{bmatrix} x_1(1) & \cdots & x_M(1) \\ \vdots & \ddots & \vdots \\ x_1(N) & \cdots & x_M(N) \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} + \begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(N) \end{bmatrix} \\ \Rightarrow \bar{y} &= X\bar{h} + \bar{v} \end{aligned} \quad (1)$$

Where \bar{y} is the observation vector of matrix size $N \times 1$, X is the pilot matrix of size $N \times M$, \bar{h} is parameter vector of matrix size $M \times 1$ and \bar{v} is the noise vector of matrix size $N \times 1$.

To derive the Likelihood function, consider $v(1), v(2), \dots, v(N)$ to be IID Gaussian random variables with mean 0 and variance σ^2 . Then the probability density function of each noise sample,

$$F(v(k)) = \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{1}{2\sigma^2}v(k)^2} \quad (2)$$

Assuming that initially the noise elements are Independent Identically Distributed (IID), we get that all the observations are independent in nature and thus we can compute the Joint PDF as:

$$\begin{aligned} F_{v(1)v(2)\dots v(N)}(v(1), v(2) \dots \dots v(N)) &= \text{Product of the individual PDFs} \\ &= F_{v(1)}v(1).F_{v(2)}v(2) \dots \dots F_{v(N)}v(N) \end{aligned} \quad (3)$$

From (2) and the above equation,

$$\text{Joint PDF} = \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{1}{2\sigma^2}v(1)^2} * \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{1}{2\sigma^2}v(2)^2} * \dots * \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{1}{2\sigma^2}v(N)^2} \quad (4)$$

$$\begin{aligned} \text{PDF of noise vector } \bar{v} &= \begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(N) \end{bmatrix} \\ \bar{v}^T \bar{v} &= [v(1)v(2) \dots \dots v(N)] \begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(N) \end{bmatrix} \\ \bar{v}^T \bar{v} &= \sum_{k=1}^N v(k)^2 = \|\bar{v}\|^2, \text{ which is the norm vector.} \end{aligned} \quad (5)$$

Rearranging (1),

$$\begin{aligned} \bar{v} &= \bar{y} - X\bar{h} \\ \text{Mean } E\{\bar{y}\} &= X\bar{h} \\ P(\bar{y}; \bar{h}) &= \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{1}{2\sigma^2}\|\bar{y} - X\bar{h}\|^2} \end{aligned} \quad (6)$$

Where P is the likelihood function of channel vector \bar{h} (unknown parameter) and \bar{y} is the observation vector. When the probability density function is viewed as the function of an unknown parameter \bar{h} , it becomes the likelihood function for parameter vector \bar{h} .

Now for Log Likelihood function, take natural log of likelihood function.

$$\Rightarrow \ln(P(\bar{y}; \bar{h})) = -\frac{N}{2} \ln 2\pi\sigma^2 - \frac{1}{2\sigma^2} \|\bar{y} - X\bar{h}\|^2 \quad (7)$$

To find the value of H , maximize $\ln(\bar{y}; \bar{h})$ since $-\frac{N}{2} \ln 2\pi\sigma^2$ and $\frac{1}{2\sigma^2}$ are constant. ML estimate of parameter vector is found by minimizing the norm of voice vector ($\|\bar{y} - X\bar{h}\|^2$) which is a Gaussian vector and also the square of the norm error is least square.

i.e.; Considering (5),

$$\begin{aligned} \|\bar{y} - X\bar{h}\|^2 &= (\bar{y} - X\bar{h})^T (\bar{y} - X\bar{h}) \\ &= \bar{y}^T \bar{y} - \bar{h}^T X^T \bar{y} - \bar{y}^T X \bar{h} + \bar{h}^T X^T X \bar{h} \end{aligned} \quad (8)$$

By applying symmetric property to simplify this lost function that we have develop for the least square cost function (Equation 8), we get

$$\bar{y}^T (X\bar{h}) = (\bar{y}^T X\bar{h})^T = \bar{h}^T X^T \bar{y} \quad (9)$$

From (8) and (9),

$$\|\bar{y} - X\bar{h}\|^2 = \bar{y}^T \bar{y} - 2\bar{h}^T X^T \bar{y} + \bar{h}^T X^T X \bar{h} \quad (10)$$

The maximum likelihood estimate of the channel vector h is the one which minimizes the cost functions. For finding the value for \bar{h} for which the cost function will be minimum, we have to differentiate the cost function with respect to channel vector \bar{h} and equate it to zero. Let us consider a simple function,

$$F(\bar{h}) = \bar{c}^T \bar{h} = [C_1 \quad C_2 \quad \dots \quad C_M] \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} \quad (11)$$

Thus the vector derivative becomes,

$$\begin{aligned} \frac{dF}{d\bar{h}} &= \begin{bmatrix} \partial F / \partial h_1 \\ \vdots \\ \partial F / \partial h_M \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_M \end{bmatrix} = \bar{c} \\ \frac{d}{d\bar{h}} (\bar{c}^T \bar{h}) &= \frac{d}{d\bar{h}} (\bar{h}^T \bar{c}) = \bar{c} \end{aligned} \quad (12)$$

The quadratic form is defined as,

$$F(\bar{h}) = \bar{h}^T P \bar{h} \quad (13)$$

Where P is a symmetric matrix ($P = P^T$). Using product rule for computing the derivative we get,

$$\frac{d}{d\bar{h}} (\bar{h}^T P \bar{h}) = 2P\bar{h} \quad (14)$$

On differentiating equation (8),

$$\frac{dF(\bar{h})}{d\bar{h}} = -2X^T \bar{y} + 2(X^T X)\bar{h} \quad (15)$$

Setting equation (15) to zero, we get

$$\bar{h} = (X^T X)^{-1} X^T \bar{y}, \text{ this is an invertible equation.}$$

This is the value of \bar{h} where the derivative of the least square cost function is zero. Therefore this is the minima of the least cost function hence it corresponds to the maximum likelihood estimate.

Therefore the maximum likelihood estimate \hat{h} of the parameter vector \bar{h} is given as:

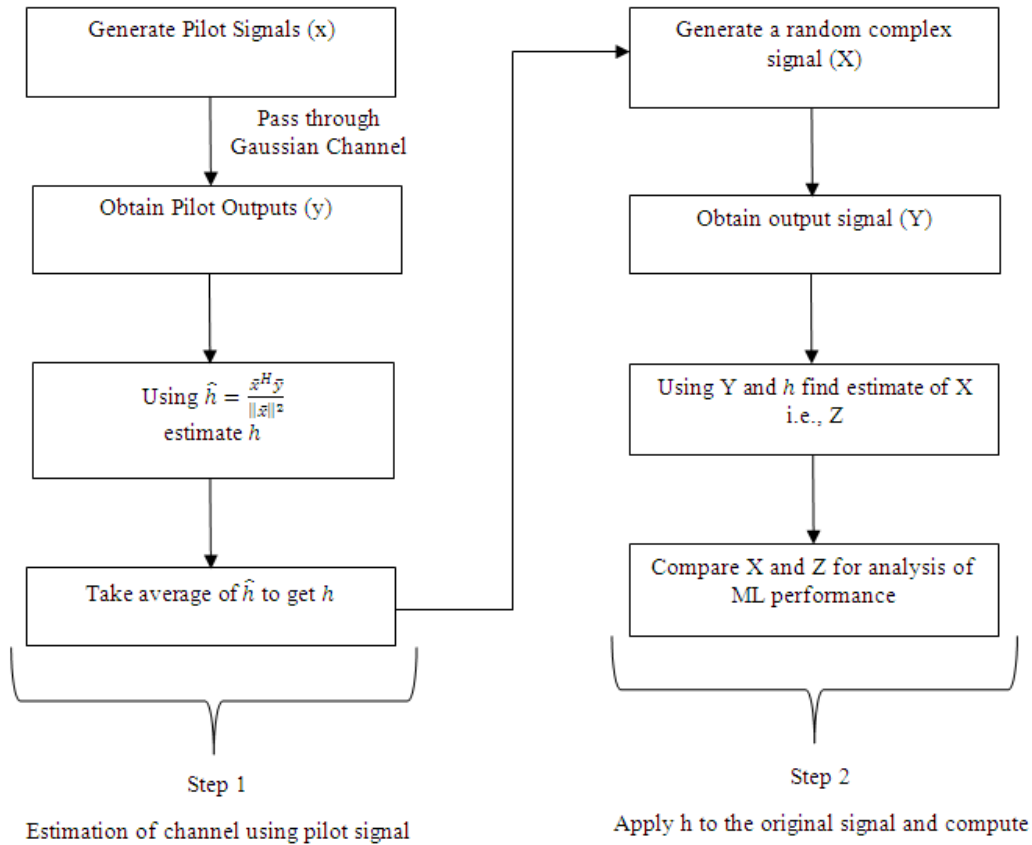
$$\hat{h} = (X^T X)^{-1} X^T \bar{y} \quad (16)$$

Where, $(X^T X)^{-1} X^T$ is the pseudo inverse of X and the average of the estimate (\hat{h}) yields the parameter vector (\bar{h}).

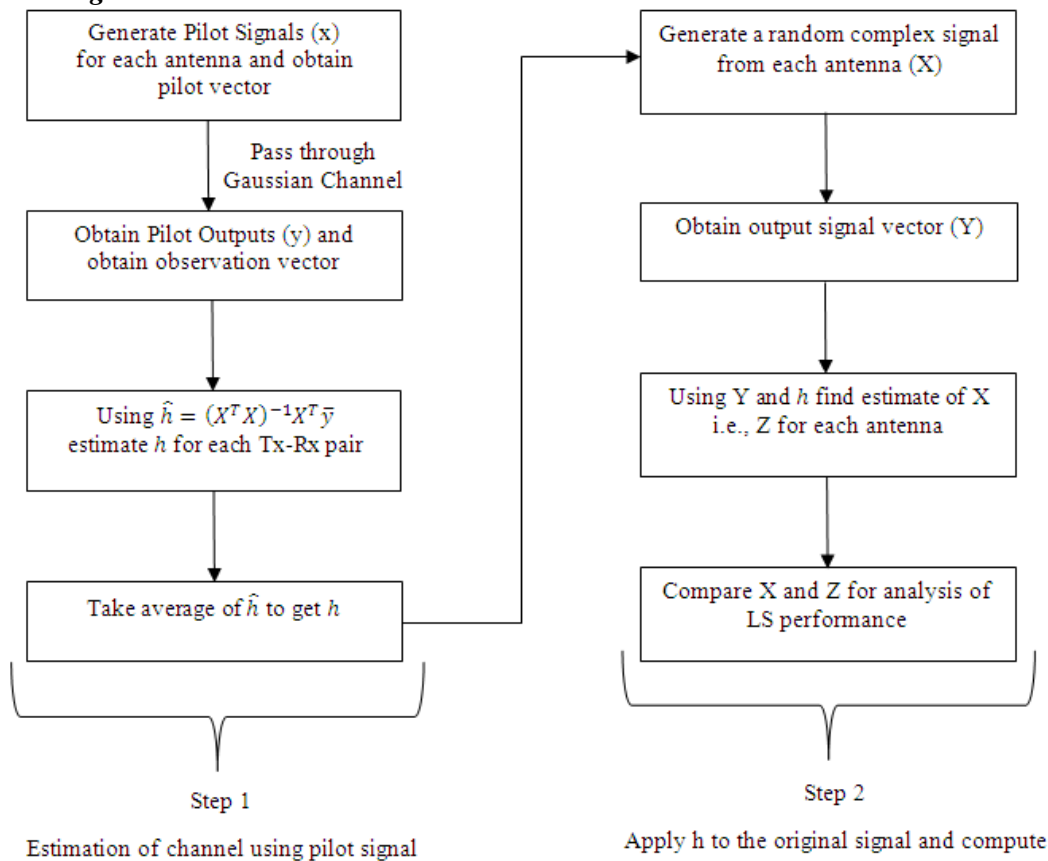
III. METHODOLOGY

During V2I communication, the received signal is assumed to have undergone interference due to fading. The transmitted signal can be found from the received signal by finding the channel in V2I communication, for that a pilot signal which is usually a single frequency signal (reference signal) is first generated, which is represented by 'x', and received signal 'y'. To find the channel, channel estimate 'h', is the parameter considered. Here, the channel is assumed to remain the same, and does not change during the transmission process, i.e., slow fading process and the channel is Gaussian channel. The noise is assumed to be additive white Gaussian noise (AWGN). After finding the channel estimate, Maximum Likelihood and Least Squares algorithms are used to find the estimated signal from the received signal and compare it with the original signal and compute the estimation error.

A. ML Algorithm



B. LS Algorithm



IV. SIMULATIONS AND CALCULATIONS

In order to validate the results, computer simulations were carried with system parameters: $N = 61$ (pilot signals) for obtaining channel coefficient estimate and we assumed the channel as slow fading channel with noise as white and gaussian. We considered 10 iterations to obtain average estimation error for both Maximum Likelihood estimate and Least Square ML estimate.

A. Maximum Likelihood Estimate

The output for Maximum Likelihood Channel Estimation algorithm for a single transmitter single receiver model through MATLAB simulation was observed as shown below.

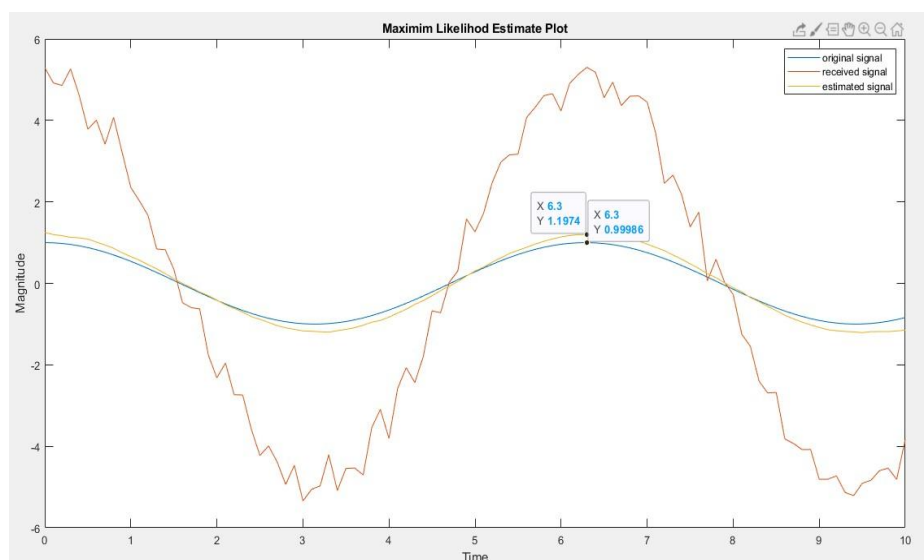


Figure 4: MATLAB Simulation for ML Estimate (using cosine wave)

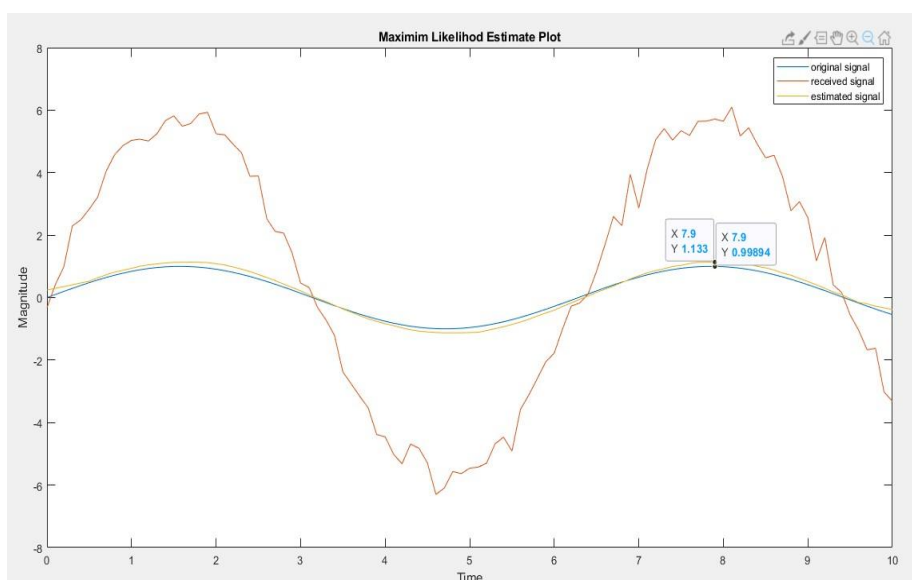


Figure 5: MATLAB Simulation for ML Estimate (using sine wave)

Calculations:

Considering the peaks of the original signal and observed signal after implementation of maximum likelihood algorithm the estimation error can be obtained.

Table 1: ML estimate: 1Tx and 1Rx

S No	Peak of Original Signal (A)	Peak of Observed Signal (B)	Estimation Error = B-A
01	0.99986	1.2583	0.25844

02	0.99986	1.0429	0.04304
03	0.99986	1.2515	0.25164
04	0.99986	1.2761	0.27624
05	0.99986	1.3046	0.30474
06	0.99986	1.0629	0.06304
07	0.99986	1.1898	0.18994
08	0.99986	1.2623	0.26244
09	0.99986	1.1098	0.10994
10	0.99986	1.0679	0.06804

Average Estimation error (ML Estimate: 1Tx-1Rx) = 0.18275

B. Least Squares ML Estimate

The output of Least Square maximum likelihood channel estimation for a single transmitter single receiver model through MATLAB simulation was observed as shown below.

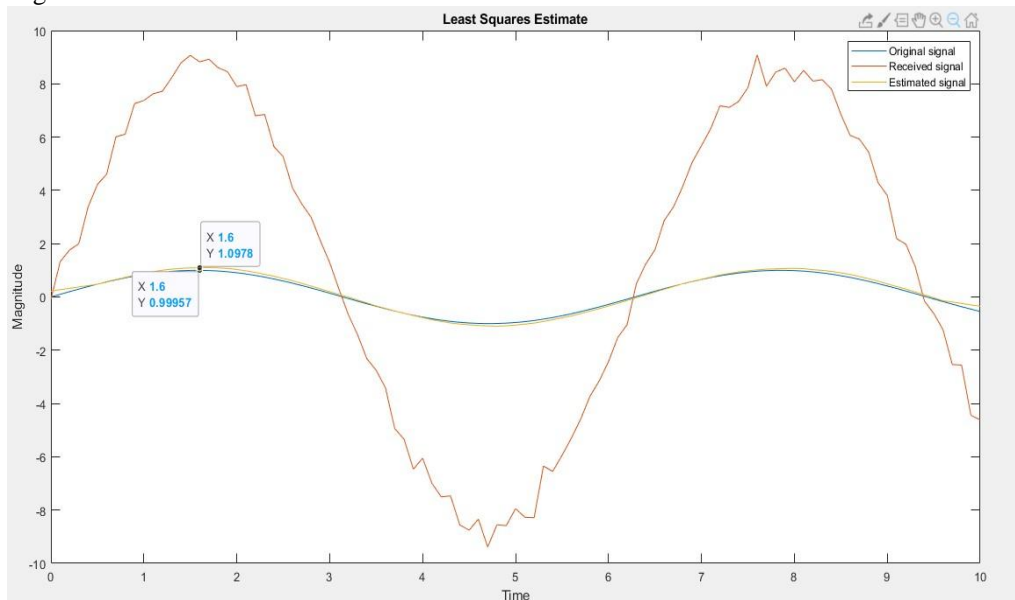


Figure 6: MATLAB Simulation for LS Estimate for 1 Tx and 1 Rx

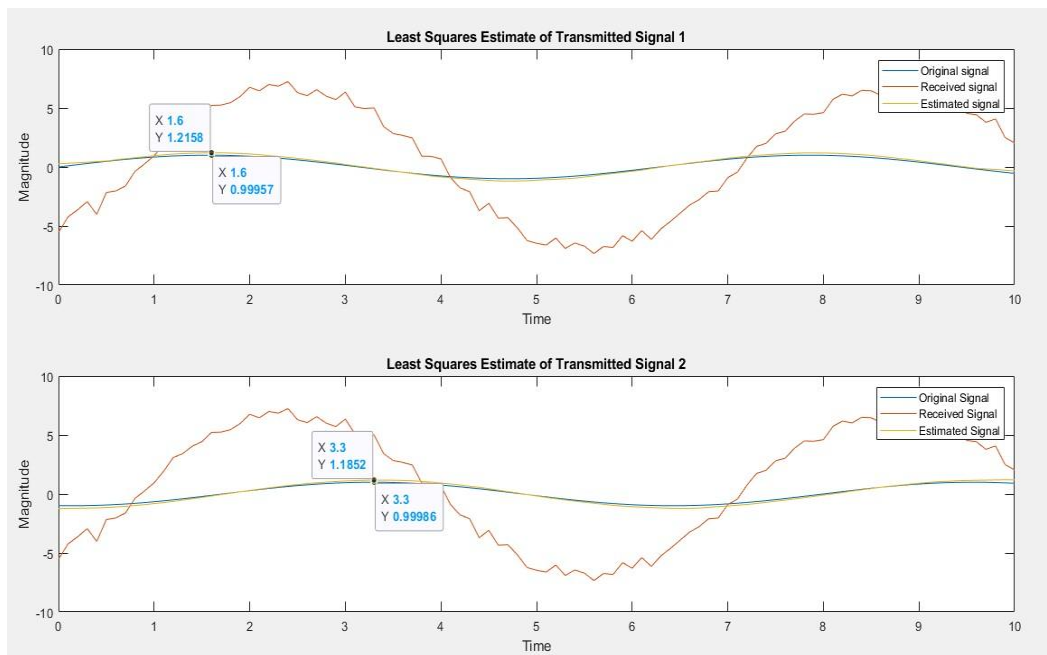


Figure 7: MATLAB Simulation for LS Estimate for 2 Tx and 1 Rx

Calculations:

Considering the peaks of the original signal and observed signal after implementation of Least Square algorithm the estimation error can be obtained.

Table 2: LS estimate: 1Tx and 1Rx

S No	Peak of Original Signal (A)	Peak of Observed Signal (B)	Estimation Error = B-A
01	0.99894	1.0899	0.09096
02	0.99894	1.0785	0.07956
03	0.99894	1.093	0.09406
04	0.99894	1.0933	0.09436
05	0.99894	1.0812	0.08226
06	0.99894	1.0871	0.08816
07	0.99894	1.0975	0.09856
08	0.99894	1.0872	0.08826
09	0.99894	1.0721	0.07316
10	0.99894	1.096	0.09706

Average Estimation error (LS Estimate: 1Tx-1Rx) = 0.08864

Table 3: LS estimate: 2Tx and 1Rx

S No	Peak of Original Signal (A)	Peak of Observed Signal (B)	Estimation Error = B-A
01	0.99894	1.1812	0.18226
02	0.99986	1.2058	0.20594
03	0.99894	1.1699	0.17096
04	0.99986	1.2343	0.23444
05	0.99894	1.1656	0.16666
06	0.99986	1.1607	0.16084
07	0.99894	1.142	0.14306
08	0.99986	1.2284	0.22854
09	0.99894	1.1897	0.19076
10	0.99986	1.1966	0.19674

Average Estimation error (LS Estimate: 2Tx-1Tx) = 0.18802

V. CONCLUSION

With the potential to offer orders of magnitude of greater capacity over current communication systems, Millimeter Wave is a promising candidate for future wireless networks. MM Wave Communication is used in many applications among which vehicular communication is one we have focused on. The technology of vehicular communication is one of the boons to the automobile industry and the people. Vehicular Communications opens new possibilities to develop advanced traffic monitoring solutions. Our objective was to attain a best possible estimate under realistic channel conditions. In this paper we considered Maximum Likelihood Channel Estimation and Least Squares Maximum Likelihood Channel Estimation to estimate the channel coefficient and compare the estimation error of the signals received. It was observed that Maximum Likelihood algorithm provides a good channel estimation for a single transmitter receiver system with an average estimation error of 0.18275. Unlike maximum likelihood, least squares ml estimate can be applied to any problem. The least squares estimation problem can be solved in closed form, and it is relatively straightforward to derive the statistical properties for the resulting parameter estimates. Thus, Least Square ML Estimate is shown to be more practical for vehicular to infrastructure communications.

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