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Golden Ratio Prime Numbers With Three Prime Digits

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Abstract
After defining, the golden ratio prime numbers with three prime digits will be presented . How many golden ratio prime numbers with three prime digits are there in the interval (10^{n-1}, 10^n) (where n≥2 integer number)? On the one hand, it has been counted by computer among the prime numbers with three prime digits. On the other hand, the function (1), (2), (3), (4) gives the approximate number of golden ratio prime numbers with three prime digits in the interval (10^{n-1},10^n). Near-proof reasonig has emerged from the conformity of Mills' prime numbers with golden ratio prime numbers with three prime digits. The sets of golden ratio prime numbers with three prime digits are probably infinite.

I. INTRODUCTION

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ($F_0=0$, $F_1=1$, $F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form 4n+3), Leyland-primes (in the form x^y+y^x, where $1\leq x\leq y$), Pell-primes ($P_0=0$, $P_1=1$, $P_n=2P_{n-1}+P_{n-2}$), Bölcsföldi-Birkás primes (all digits are prime, the number of digits is prime and the sum of digits is prime),, etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of golden ratio prime numbers with three prime digits…

The definition of the golden ratio: the numbers m and n (where m>n) are in golden ratio, if (m+n)/m=m/n=c
$c = 1{,}618\ldots$ irrational

1. Golden ratio prime numbers with prime digits 2, 3, 5

Definition: the prime number p is golden ratio prime number with prime digits 2, 3, 5, if
all digits are 2 or 3 or 5 and
$|p/c - q| < k$, where q is a positive integer number and
$|p/c^2 - r| < k$, where r is a positive integer number,
and k=0,05.
The positive integer numbers p, q, r are in golden ratio.

If k=0,05:

p	**p/c**	**p/c^2**	**Golden ratio**
233	144,00	89,00	233, 144, 89
5333	3296,04	1259,03	5333, 3294, 1259

etc…

The golden ratio prime numbers with digits 2, 3, 5 are as follows:

{233}, {5333}, {53353}, {222533, 252533, 255253, 323233, 335323, \
522523, 525353, 535333, 552523}, {2225533, 2232353, 2355233, 2535223, \
2555353, 3555353, 5225533, 5332333, 5332553}, {22223323, 22323523, \
22335223, 22353523, 22355353, 23323523, 23355523, 23525533, 23555533, \
25232533, 32253553, 32325253, 32352253, 32352533, 32522323, 32552323, \
33222223, 33253223, 33253333, 35223553, 35253553, 35325253, 52233323, \
52525553, 52535533, 52552333, 53522333, 55332253, 55552223, \
55555553}, {222555323, 223233323, etc.

G(n) is the factual frequency of golden ratio prime numbers with prime digits 2, 3, 5 in the interval (10^{n-1}, 10^n), where n≥2 integer number.

G(2)=0, G(3)=1, G(4)=1 , G(5)=1, G(6)=1 , G(7)=9, G(8)=9. G(9)=30, G(10)=95, G(11)=228, G(12)=638, G(13)=1600, G(14)=4479, G(15)=12155, G(16)=26633, etc.
H(n) function gives the number of golden ratio prime numbers with prime digits 2, 3, 5 in the interval $(10^{n-1}, 10^n)$, where n≥11 integer number.
The function H(n) is

H(n)= 0,4x2,1^{n-1} - 615 where n≥11 integer **(1)**

The factual number of golden ratio primes with prime digits 2,3,5 (G(n)) and the number of golden ratio prime numbers calculated according
to function H(n) are as follows:

Number of digits	The factual number of golden ratio primes with digits 2,3,5 in the interval $(10^{n-1}, 10^n)$	The number of golden ratio primes with digits 2, 3, 5 calculated according to function $H(n)=0,4x2,1^{n-1} - 615$ in the interval $(10^{n-1}, 10^n)$	
n	G(n)	H(n)	G(n)/H(n)
11	228	52,1952	4.3682
12	638	785,9110	0,8118
13	1600	2327,3310	0.6875
14	4479	5563,8920	0,8050
15	12155	12360,0680	0.9834
16	26633	26633,9274	0.9999
17		56607,7476	
18		119552,7700	
19		251737,3170	
		etc.	

2. Golden ratio prime numbers with prime digits 2, 3, 7

Definition: the prime number p is golden ratio prime number with prime digits 2, 3, 7, if all digits are 2 or 3 or 7 and
$|p/c - q| < k$, where q is a positive integer number and
$|p/c^2 – r| < k$, where r is a positive integer number,
and k=0,05.
The positive integer numbers p, q, r are in golden ratio.

If k=0,05:

p	**p/c**	**p/c^2**	**Golden ratio**
233	144,00	89,00	233,144,89
733	453,03	279,99	733,453,280 etc.

The golden ratio prime numbers with prime digits 2, 3, 7 are as follows:

{233, 733}, {7333}, {22777, 37273, 73237}, {233327, 273773, 322327, \
323233, 323377, 332273, 722237, 722737, 772273, 772333, 773273}, \
{2227727, 2272223, 2273273, 2323733, 2723737, 2727727, 2773273, \
3233327, 3237233, 3272723, 3273773, 3333773, 3727727, 3737233, \
3773773, 7222373, 7223737, 7233277, 7273723, 7722373, 7737733, \
7772333}, etc.
L(n) is the factual frequency of golden ratio prime numbers with prime digits 2, 3,7 in the interval $(10^{n-1}, 10^n)$, where n≥2 integer number.
L(2)=0, L(3)=2, L(4)=1 , L(5)=3, L(6)=12 , L(7)=22, L(8)=51, L(9)=171, L(10)=445. L(11)=1239, L(13)=3367, L(14)=9110, L(15)=24673, L(16)=51259
M(n) function gives the number of golden ratio prime numbers with prime digits 2, 3, 7 in the interval $(10^{n-1}, 10^n)$, where n≥9 integer number.
The function M(n) is

$M(n)=0,3x2,2^{n-1}$ - 110 where n≥9 integer **(2)**

The factual numbers of golden ratio primes with prime digits 2,3,7 (L(n)) and the number of golden ratio prime numbers with prime digits 2,3,7 calculated according to function M(n) are as follows:

Number of digits	The factual number of golden ratio primes in the interval $(10^{n-1}, 10^n)$	The number of golden ratio primes calculated according to function $M(n)=0,3x2,2^{n-1} - 110$ in the interval $(10^{n-1}, 10^n)$	
n	L(n)	M(n)	L(n)/M(n)
9	51	54,7600	0,9313
10	171	252,1308	0,6782
11	445	686,7977	0,6479
12	1239	1642,9550	0,7541
13	3367	3746,5008	0,8987
14	9110	8374,3017	1,0944
15		18665,4637	
16		41064,0203	
		etc.	

3. **Golden ratio prime numbers with prime digits 2, 5, 7**

Definition: the prime number p is golden ratio prime number with prime digits 2, 5, 7 if all digits are 2 or 5 or 7 and
$| p/c - q | < k$, where q is a positive integer number and
$| p/c^2 – r | < k$, where r is a positive integer number,
and k=0,05.
The positive integer numbers p, q, r are in golden ratio.

If k=0,05:

The golden ratio prime numbers with prime digits 2, 5, 7 are as follows:

{}, {}, {22777}, {277577, 522227, 757727}, {2227727, 2522227, \
2522557, 2552777, 2727727, 2775527, 5227727, 5257727, 5525777, \
5555777, 5757727, 5775527, 7257227, 7272257}, {22255577, 22277257, \
22557727, 22722527, 25277257, 25572257, 27255577, 27572257, 27725527, \
27727577, 27757577, 52257757, 55275557, 55277777, 55775227, 57522757, \
57557527, 57777557, 72225277, 72252277, 72725777, 72752557, 72772577, \
75752227, 75772577, 77255557, 77577557}, etc.

P(n) is the factual frequency of golden ratio prime numbers with prime digits 2, 5, 7 in the interval $(10^{n-1}, 10^n)$, where n≥2 integer number.

P(2)=0, P(3)=0, P(4)=0 , P(5)=1, P(6)=3 , P(7)=14, P(8)=27, P(9)=101, P(10)=233, P(11)=435, P(12)=1636, P(13)=4537, P(14)=12165, P(15)=20734, etc.

Q(n) function gives the number of golden ratio prime numbers with prime digits 2, 5, 7 in the interval $(10^{n-1}, 10^n)$, where n≥8 integer number.

The function Q(n) is **$Q(n)=0,33x2,22^{n-1}$ - 73** where n≥8 integer (3)

The factual numbers of golden ratio prime numbers with prime digits 2, 5, 7 is P(n) and the number of golden ratio prime numbers with prime digits 2, 5, 7 calculated according to function Q(n) are as follows:

Number of digits The factual number of golden ratio primes in the interval $(10^{n-1}, 10^n)$ The number of golden ratio primes calculated according to function $Q(n)=0,33x2,22^{n-1} - 73$

n	P(n)	Q(n) in the interval $(10^{n-1}, 10^n)$	P(n)/Q(n)
8	27	14,6970	1,8371
9	101	121,6874	0.8299
10	233	359,2058	0,6486
11	601	886,4971	0,6779
12	1636	2057,0837	0,7953
13	4537	4655,7857	0,9745
14	12165	10424,9043	1,1669
15	20734	23232,3475	0,8925
		51575,8115	
		114498,3014	
		254186,2292	
		etc.	

4. **Golden ratio prime numbers with prime digits 3, 5, 7**

Definition: the prime number p is golden ratio prime number with prime digits 3, 5, 7, if all digits are 3 or 5 or 7 and
$| p/c - q | < k$, where q is a positive integer number and
$| p/c^2 - r | < k$, where r is a positive integer number,
and k=0,05.

If k=0,05:

p	**p/c**	**p/c^2**	**Golden ratio**
733	453,03	279,99	733,453,280
etc.			

The golden ratio prime numbers with prime digits 3, 5, 7 are as follows:
733}, {5333, 7333}, {33757, 53353, 53777, 75533}, {377353, 535333, \
535757, 537773, 573737, 735733}, {3333773, 3337577, 3373753, 3377557, \
3537773, 3553777, 3555353, 3573373, 3755753, 3773773, 5355733, \
5357537, 5553337, 5555777, 5755753, 5773553, 7337357, 7533377, \
7537333, 7733573, 7735573, 7737733}, {33553577, 33555373, 33555737, \
33753553, 33775733, 35357557, 35377577, 37335733, 37373773, 37753553, \
37773353, 53333557, 53337353, 53353357, 53355373, 53375537, 53575733, \
53737373, 55333337, 55335337, 55335557, 55357373, 55375757, 55537753, \
55555553, 55733773, 55735577, 57333337, 57335557, 57335777, 57375757, \
57553757, 57573353, 57733553, 57753733, 57755377, 57773753, 57777557, \
73335533, 73337753, 73353757, 73377377, 73537577, 73553377, 73773373, \
75337753, 75355333, 75357773, 75377377, 75535357, 75555377, 75775373, \
77353537, 77357773, 77577337, 77577557, 77733377}, etc.

R(n) is the factual frequency of golden ratio prime numbers with prime digits 3, 5, 7 in the interval $(10^{n-1}, 10^n)$, where $n \geq 2$ integer.

R(2)=0, R(3)=1, R(4)=2 , R(5)=4, R(6)=6 , R(7)=22, R(8)=57. R(9)=173, R(10)=435, R(11))=1125, R(12)=3157, R(13)=9065, R(14)=23196, R(15)=39767

S(n) function gives the number of golden ratio prime numbers with prime digits 3, 5, 7 in the interval $(10^{n-1}, 10^n)$, where $n \geq 8$ integer number.

The function S(n) is **$S(n)=0,33x2,3^{n-1}$** where $n \geq 8$ integer **(4)**

The factual number of golden ratio primes with prime digits 3, 5, 7 (R(n)) and the number of golden ratio prime numbers with prime digits 3, 5, 7 calculated according to function S(n) are as follows:

Number of digits	The factual number of golden ratio primes in the interval $(10^{n-1}, 10^n)$	The number of golden ratio primes calculated according to function $S(n)=0,33x2,3^{n-1}$ in the interval $(10^{n-1}, 10^n)$

n	R(n)	S(n)	R(n)/S(n)
8	57	112,3593	0,5073
9	173	258,4263	0,6694
10	435	594,3804	0,7319
11	1125	1367,0749	0,8229
12	3157	3144,2722	1,0040
13	9065	7231,8261	1,2535
14	23196	16633,1999	1,3946
15	39767	38256,3599	1,0395
		87989,6278	
		202376,1439	
		465465,1310	
	etc.		

If k=0,001 is, are the golden ratio prime numbers with three prime digits:
with prime digits 2, 3, 5:

{5332553}, {}, {322353323}, {3253535533, 5222335253}}, etc.,

with prime digits 2, 3, 7:

{23772733, 37333327}, {272772223, 273222373, 332227733, 377227373, \
723327733, 727232227}, etc.,

with prime digits 2, 5, 7:

{5525777, 7272257}, {}, {525755557}, {2727525257, 5572572557,
5757557257, 7222575527, 7222725577, 7275772577, 7577557727}}, etc.,

with prime digits 3, 5, 7:

{735733}, {}, {57573353}, {537755753, 575757773,
775733333}, {3337737737, 3353735773, 3357335753, 3377375557,
3557353733, 3775555577, 5755577573, 7335335573}}, etc.

5. Number of the elements of the set of golden ratio prime numbers with prime digits 3, 5, 7 [3], [9],[10], [11], [12].

Let's take the set of Mills' prime numbers!

Definition: The number $m=[M \text{ ad } 3^n]$ is a prime number, where M=1,3063778838630806904686144926O2 is the Mills' constant, and n=1,2,3,… is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2, 11, 1361, 2521008887,…

The connection n→m is the following:1→2, 2→11, 3→1361, 4→2521008887,… The Mills' prime number $m=[M \text{ ad } 3^n]$ corresponds with the interval $(10^{m-1},10^m)$ and vice versa. For instance:2→$(10, 10^2)$, 11→$(10^{10},10^{11})$, 1361→$(10^{1360},10^{1361})$, etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals $(10^{m-1}, 10^m)$ that contain at least one Mills' prime number is infinite. The number of golden ratio primes with prime digits 3, 5, 7 in the interval $(10^{m-1},10^m)$ is

$S(m)=0,33x2,3^{m-1}$. The number of golden ratio prime numbers with prime digits 3, 5, 7 is probably infinite:

$\lim_{n\to\infty} R(n)=\infty$ and $\lim_{n\to\infty} S(n)=\infty$ are probably where n is positive integer and

II. CONCLUSION

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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