

## Analytical Review on Some Fuzzy Queuing Models

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### Abstract

Our aim of this paper is to analyse FM/FM/1 queuing model with hexagonal and octagonal fuzzy numbers under  $\alpha$  cut representation through DSW algorithm. The arrival rate and service rate are fuzzy natures and also analysed the performance measure in a hexagonal and octagonal fuzzy number.

**Key words:** Fuzzy set theory, Membership Function, Hexagonal and Octagonal Fuzzy Numbers,  $\alpha$  cut, DSW Algorithm, Interval Analysis.

### I. INTRODUCTION

The study of waiting lines is an important aspect in queueing theory. We use different models and methods to analyse waiting time. Mathematical analysis of queues gives way to decrease the waiting time and waiting line. Queueing theory is a branch of applied probability. Queueing theory was introduced by A.K. Erlang in 1909. Queueing models are aids to determine the optimal number of counters so as to satisfy the customers keeping the total cost minimum. It helps the customers should get service with minimum time. The main purpose of the analysis of queueing system is to understand the behaviour of their underlying processes so that informed and intelligent decisions can be made in their organization. Queueing models have wider range of applications in service organizations as well as in manufacturing firms, where customers receive service by different kinds of servers in accordance with the queue discipline. In particular, the inter arrival times and service times are restricted to follow specific probability distributions. It is studying about such waiting line through performance measures.

Efficient methods have been developed for analysing the queueing system when its parameters such as arrival rate and service rate are known exactly. However, there are cases that these parameters may not be presented precisely due to uncontrollable factors. Specifically in many practical applications, the statistical data may be obtained subjectively i.e., arrival rate and service rate are more suitably described by linguistic terms such as fast, moderate, or slow rather by probability distribution based on statistical theory. Imprecise information of this kind will determine the system performance measure accurately.

### II. PRELIMINARIES

#### Fuzzy Set

A fuzzy set is a set where the members are allowed to have partial membership and hence the degree of membership varies from 0 to 1. It is expressed as  $A = \{ (x, \mu(x)) / x \in Z \}$  where  $Z$  is the universe of discourse and  $(x)$  is a real number,  $(x) = 0$  or  $1$ , i.e.,  $x$  is a non-member in  $A$  if  $(x) = 0$  and  $x$  is a member in  $A$  if  $(x) = 1$ .

#### OR

A fuzzy set is an ordered pair  $(A, f_A)$  where 'A' is a set and  $f: A \rightarrow [0, 1]$  is a membership function.

#### $\alpha$ -Cuts

If a fuzzy set is defined on  $X$ , for any  $\alpha \in [0, 1]$ , the  $\alpha$ -cuts of the fuzzy set is represented by  $= \{ x / \mu(x) \geq \alpha, x \in X \} = \{ (\alpha), (\alpha) \}$ , where  $(\alpha)$  and  $(\alpha)$  represent the lower bound and upper bound of the  $\alpha$ -cut of respectively.

If a fuzzy set  $A$  is defined on  $X$ , for any  $\alpha \in [0, 1]$ , the  $\alpha$ -cuts is represented by the following crisp set,

**Strong  $\alpha$ -cuts**  $\alpha_{+A} = \{ x \in X / \mu(x) > \alpha \}; \alpha \in [0, 1]$

**Weak  $\alpha$ -cuts**  $\alpha_A = \{ x \in X / \mu(x) \geq \alpha \}; \alpha \in [0, 1]$  Therefore, it is inferred that the fuzzy set  $A$  can be treated as crisp set in which all the members have their membership values greater than or at least equal to  $\alpha$ .

#### Support of a fuzzy set

The support of a fuzzy set  $A$  is the crisp set such that it is represented as  $\text{supp}A(X) = \{ x \in X / \mu(x) > 0 \}$ . Thus, support of a fuzzy set is the set of all members with a strong  $\alpha$ -cut, where  $\alpha = 0$ .

#### Height of a fuzzy set

The height of a fuzzy set,  $h\{A(x) / x \in X\}$  is the maximum value of its membership function  $\mu(x)$  such that  $= \{ x \in X / \mu(x) \geq \alpha \}$  and  $0 \neq \alpha$ .

#### Normal and Subnormal fuzzy set

A fuzzy set  $\text{Max}\{\mu(x)\} = 1$  is called as a normal fuzzy set, otherwise, it is referred as subnormal fuzzy set.

**Triangular fuzzy number Membership Function**

We define a fuzzy number M on R to be a triangular fuzzy number if its membership function  $\mu_M(x): R \rightarrow [0,1]$  is defined by

$\mu_M(x) = \frac{x-l}{m-l}$ , for  $l \leq x \leq m$  and  $\frac{x-u}{m-u}$  for  $m \leq x \leq u$ , otherwise zero. where  $l \leq m \leq u$ ,  $l$  and  $u$  stand for the lower and upper value of the support of M respectively and ‘m’ for the modal value. The triangular fuzzy number can be denoted by  $(l, m, u)$ . The support of M is the set of elements  $\{x \in R / l < x < u\}$ , when  $l = m = u$  it is a non-fuzzy number by convention.

**Trapezoidal fuzzy number Membership Function**

A fuzzy set A =  $(a_1, a_2, a_3, a_4)$  is said to be trapezoidal fuzzy number if its membership function is given by where  $a_1 \leq a_2 \leq a_3 \leq a_4$

$\mu_A(x) = 0$  for  $x < a_1$ ,  $\frac{x-a_1}{a_2-a_1}$  for  $a_1 \leq x \leq a_2$ ,  $1$  for  $a_2 \leq x \leq a_3$ ,  $\frac{a_4-x}{a_4-a_3}$  for  $a_3 \leq x \leq a_4$  and  $0$  for  $x > a_4$ .

**Pentagon Fuzzy Number with Membership Function**

A fuzzy number  $\tilde{A} = (a, b, c, d, e)$  where  $a \geq b \geq c \geq d \geq e$  is said to be a pentagon fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = 0, \text{ when } x < a, \quad \frac{x-a}{b-a} \text{ when } a \leq x \leq b, \quad \frac{x-b}{c-b} \text{ when } b \leq x \leq c, \quad 1 \text{ when } x = c, \\ \frac{d-x}{d-c} \text{ when } c \leq x \leq d, \quad \frac{e-x}{e-d} \text{ when } d \leq x \leq e, \quad 0 \text{ when } x > e.$$

**Hexagonal Fuzzy Number Membership Function**

A fuzzy number  $A_H$  is a hexagonal fuzzy number denoted by  $A_H(a_1, a_2, a_3, a_4, a_5, a_6)$  where  $a_1, a_2, a_3, a_4, a_5, a_6$  are real numbers and its membership function  $\mu_{A_H}$  is given below.

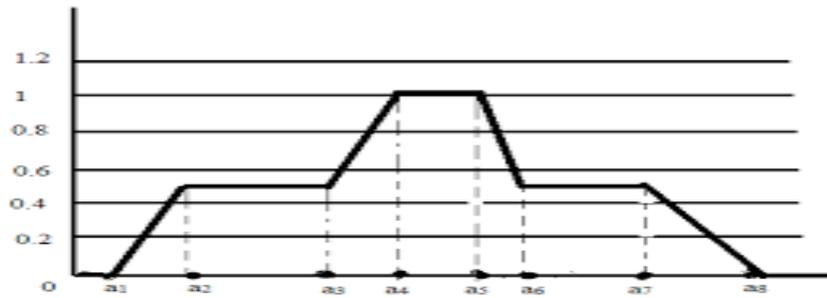
$\mu_{A_H}(x) = 0$  for  $x < a_1$ ,  $\frac{1}{2} \frac{x-a_1}{a_2-a_1}$  for  $a_1 \leq x \leq a_2$ ,  $\frac{1}{2} + \frac{1}{2} \frac{x-a_2}{a_3-a_2}$  for  $a_2 \leq x \leq a_3$ ,  $1$  for  $a_3 \leq x \leq a_4$ ,  $\frac{1}{2} - \frac{1}{2} \frac{x-a_4}{a_5-a_4}$  for  $a_4 \leq x \leq a_5$ ,  $\frac{1}{2} \frac{a_6-x}{a_6-a_5}$  for  $a_5 \leq x \leq a_6$ ,  $0$  for  $x > a_6$ .

**Octagonal Fuzzy Number Membership Function**

A fuzzy number  $\tilde{A}$  is a normal Octagonal Fuzzy Number denoted by  $(a_1, a_2, \dots, a_8)$  where  $a_i \in R, 1 \leq i \leq 8, i \in I$  and its membership function  $\mu_{\tilde{A}}(x)$  is given by

$$\mu_{\tilde{A}}(x) = 0 \quad \text{if } x < a_1, \\ \mu_{\tilde{A}}(x) = r \left( \frac{x-a_1}{a_2-a_1} \right) \quad \text{if } x \in [a_1, a_2], \\ \mu_{\tilde{A}}(x) = r \quad \text{if } x \in [a_2, a_3], \\ \mu_{\tilde{A}}(x) = r + (1-r) \left( \frac{x-a_1}{a_2-a_1} \right) \quad \text{if } x \in [a_3, a_4], \\ \mu_{\tilde{A}}(x) = 1 \quad \text{if } x \in [a_4, a_5], \\ \mu_{\tilde{A}}(x) = r + (1-r) \left( \frac{a_6-x}{a_6-a_5} \right) \quad \text{if } x \in [a_5, a_6], \\ \mu_{\tilde{A}}(x) = r \quad \text{if } x \in [a_6, a_7], \\ \mu_{\tilde{A}}(x) = r \left( \frac{a_8-x}{a_8-a_7} \right) \quad \text{if } x \in [a_7, a_8], \\ \mu_{\tilde{A}}(x) = 0 \quad \text{if } x \geq a_8,$$

**Graph of Octagonal Fuzzy Number Membership Function**



**Nonagonal Fuzzy Numbers**

A fuzzy number  $\tilde{A}$  is said to be a generalized nonagonal fuzzy number denoted by  $\tilde{A} = (a_1, a_2, \dots, a_8, a_9)$  where  $a_1, a_2, \dots, a_8, a_9$ ;  $k, w$  are real numbers such that  $a_1 \leq a_2 \leq \dots \leq a_8 \leq a_9$  and  $0 < k < w$  and its membership function is given by

$\mu_{\tilde{A}}(x) = 0$  if  $x < a_1$

$$\begin{aligned} \mu_{\bar{A}}(x) &= \frac{1}{4} \left( \frac{x - a_1}{a_2 - a_1} \right) && \text{if } x \in [a_1, a_2] \\ \mu_{\bar{A}}(x) &= \frac{1}{4} + \frac{1}{4} \left( \frac{x - a_2}{a_3 - a_2} \right) && \text{if } x \in [a_2, a_3] \\ \mu_{\bar{A}}(x) &= \frac{1}{2} + \frac{1}{4} \left( \frac{x - a_3}{a_4 - a_3} \right) && \text{if } x \in [a_3, a_4] \\ \mu_{\bar{A}}(x) &= \frac{3}{4} + \frac{1}{4} \left( \frac{x - a_4}{a_5 - a_4} \right) && \text{if } x \in [a_4, a_5] \\ \mu_{\bar{A}}(x) &= 1 - \frac{1}{4} \left( \frac{x - a_5}{a_6 - a_5} \right) && \text{if } x \in [a_5, a_6] \\ \mu_{\bar{A}}(x) &= \frac{3}{4} - \frac{1}{4} \left( \frac{x - a_6}{a_7 - a_6} \right) && \text{if } x \in [a_6, a_7] \\ \mu_{\bar{A}}(x) &= \frac{1}{2} - \frac{1}{4} \left( \frac{x - a_7}{a_8 - a_7} \right) && \text{if } x \in [a_7, a_8] \\ \mu_{\bar{A}}(x) &= \frac{1}{4} \left( \frac{a_9 - x}{a_9 - a_8} \right) && \text{if } x \in [a_8, a_9] \\ \mu_{\bar{A}}(x) &= 0 && \text{if } x > a_9 \end{aligned}$$

**Kendall’s Notation**

(a/b/c) : (d/e/f), where a = arrivals distribution, b = service time distribution, c = number of servers, d = capacity of the system, e = queue discipline, f = calling source or population.

**Intuitionistic Fuzzy Set**

Let X be a non-empty set the Intuitionistic fuzzy set  $\bar{A}^I$  of X is defined as,  $\bar{A}^I = \{ \langle X, \mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x) \rangle / x \in X \}$  where  $\mu_{\bar{A}^I}(x)$  and  $\nu_{\bar{A}^I}(x)$  are membership and non membership function such that  $\mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x) : X \rightarrow [0, 1]$  and  $0 \leq \mu_{\bar{A}^I}(x) + \nu_{\bar{A}^I}(x) \leq 1$  for all  $x \in X$

**Intuitionistic Fuzzy Number**

Intuitionistic fuzzy subset  $\bar{A}^I = \{ \langle X, \mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x) \rangle / x \in X \}$  of the real line R is called Intuitionistic fuzzy number if the following conditions are hold,

- There exists  $m \in R$  such that  $\mu_{\bar{A}^I}(m) = 1$  and  $\nu_{\bar{A}^I}(m) = 0$ .
- $\mu_{\bar{A}^I}(x)$  is a continuous function from  $R \rightarrow [0, 1]$  such that  $0 \leq \mu_{\bar{A}^I}(x) + \nu_{\bar{A}^I}(x) \leq 1$  for all  $x \in X$

**Zadeh’s Extension Principle**

Zadeh's extension principle is one of the most fundamental principles in fuzzy set theory. It provides a powerful technique in order to extend a real continuous function to a function accepting fuzzy sets as arguments. If the function is monotone, then the endpoints of the output can be determined quite easily. However, the difficulty arises when the function is non-monotone. In that case, the computation of the output is not an easy task. The purpose of this paper is to provide a new method to reduce this difficulty. The method is based on the implementation of optimisation technique over the  $\alpha$ -cuts of fuzzy set. By doing so, the endpoints of the output can be approximated. The method proposed in this paper is easy to implement and can be applied to many practical applications. Several examples are given to illustrate the effectiveness of the proposed method.

**Priority Discipline**

Queuing models considered have had the property that unit proceed to service on a first come - first served basis. This is obviously not only the manner of service and there are many alternatives such as last come - first served, selection in random order and selection by priority. In priority schemes, customers with the highest priority are selected for service ahead of those with lower priority, independent of their time of arrival into the system. There are two further refinements possible in priority situation, namely pre-emption and non-pre-emption. In pre-emptive cases the customer with the highest priority is allowed to enter service immediately even if another with lower priority is already present in service when the higher customer arrives to the 576 W. Ritha and L. Robert system. In addition, a decision has to be made whether to continue the pre-empted customers service from the point of pre-emption when resumed or to start a new. The priority discipline is said to be non-pre-emptive if there is no interruption and the highest priority customer just goes to the head of the queue to wait his turn. In practical, the priority queuing model, the input data arrival rate, service rate is uncertainly known. Uncertainty is resolved by using fuzzy set theory. Hence the classical queuing model with priority discipline will have more application if it is expanded using fuzzy models.

**DSW Algorithm**

DSW (Dong, Shah and Wong) is one of the approximate methods make use of intervals at various  $\alpha$  - cut levels in defining membership functions. It was the full  $\alpha$ -cut intervals in a standard interval analysis. Any continuous membership function can be represented by a continuous sweep of  $\alpha$ -cut n term from  $\alpha = 0$  to  $\alpha = 1$ .

The DSW algorithm consists of the following steps:

1. Select a  $\alpha$ -cut value where  $0 \leq \alpha \leq 1$ .

2. Find the intervals in the input membership functions that correspond to this  $\alpha$ .
3. Using standard binary interval operations compute the interval for the output membership functions for the selected  $\alpha$ -cut level.
4. Repeat the step 1-3 for different value of  $\alpha$ -cut representation of solution.

**2. Description of queueing model**

**(M/M/1) :( $\infty$ /FCFS) Single Server Queuing Model**

This is a queueing model, with Poisson arrival, Poisson service, single servicing channel, with infinite capacity. The service discipline is first come first serve. Here  $\lambda$  = mean arrival rate of units and  $\mu$  = mean service rate. Main characteristics of the model are given as follows

1. Poisson arrivals.
2. Arrival population is unlimited.
3. Exponential service time.
4. All arrivals wait to be served.
5.  $\lambda$  is constant.
6.  $\mu > \lambda$ .
7. Mean server utilization,  $\rho = \frac{\lambda}{\mu}$ .
8. Probability that there are n units in the model any time,  $P_n = \rho^n (1 - \rho)$ .
9. Probability (queue size  $\geq N$ ) =  $\rho^N$ .
10. Average numbers of customers  $E(L_s) = \frac{\lambda}{\mu - \lambda}$ .
11. Variance of queue length,  $\text{Var}(n) = \frac{\rho}{(1-\rho)^2}$ .
12. Average length of non-empty queue,  $E(L/L > 0) = \frac{\mu}{\mu - \lambda}$ .
13. Expected waiting time in the queue (excluded service time),  $E(W_q) = \frac{\lambda}{\mu(\mu - \lambda)}$ .
14. Expected waiting time in the system (included service time),  $E(W_q) = \frac{1}{(\mu - \lambda)}$ .
15. Average waiting time of an arrival who waits,  $E(W/W > 0) = \frac{1}{(\mu - \lambda)}$ .

**(FM/FM/1) Queuing Model with Pentagon Fuzzy Number**

It is a single server queueing system first come first served discipline. The inter arrival time A and service times S are described by the following fuzzy sets:

$$A = \{(a, \widetilde{\mu}_A(a)) / a \in X\}$$

$$S = \{(s, \widetilde{\mu}_S(s)) / s \in Y\}$$

Here X is the set of the inter arrival time and Y is the set of the service time.

$\widetilde{\mu}_A(a)$  is membership function of the inter arrival time.

$\widetilde{\mu}_S(s)$  is membership function of the service time.

The  $\alpha$  – cuts of the inter arrival time, service time are represented by

$$A(\alpha) = \{a \in X / \widetilde{\mu}_A(a) \geq \alpha\}$$

$$S(\alpha) = \{s \in Y / \widetilde{\mu}_S(s) \geq \alpha\}$$

Now we have to define membership function  $\mu_{(A,S)}(x)$  as given

$$\mu_{(A,S)}(x) = 0, \quad \text{when } x < a,$$

$$\mu_{(A,S)}(x) = \frac{x - a}{b - a} \quad \text{when } a \leq x \leq b,$$

$$\mu_{(A,S)}(x) = \frac{x - b}{c - b} \quad \text{when } b \leq x \leq c,$$

$$\mu_{(A,S)}(x) = 1 \quad \text{when } x = c,$$

$$\mu_{(A,S)}(x) = \frac{d - x}{d - c} \quad \text{when } c \leq x \leq d,$$

$$\mu_{(A,S)}(x) = \frac{e - x}{e - d} \quad \text{when } d \leq x \leq e,$$

$$\mu_{(A,S)}(x) = 0 \quad \text{when } x > e.$$

(FM/FM/1) queueing model consider an infinite source population where both the inter arrival time and the service times follows Poisson and exponential distributions with parameters  $\lambda$  and  $\mu$  respectively, where are fuzzy variables rather than crisp values.

**(M/E<sub>k</sub>/S) :( $\infty$ / FCFS) Model**

In this model, it is explained about a multiple server queueing model where both arrival rate and service rate are of fuzzy nature. In this model, arrival rate possess Poisson distribution and service rate satisfy Erlang – k distribution. It consists of ‘s’ number of service channels and a chain of ‘k’ similar stages gaining an average

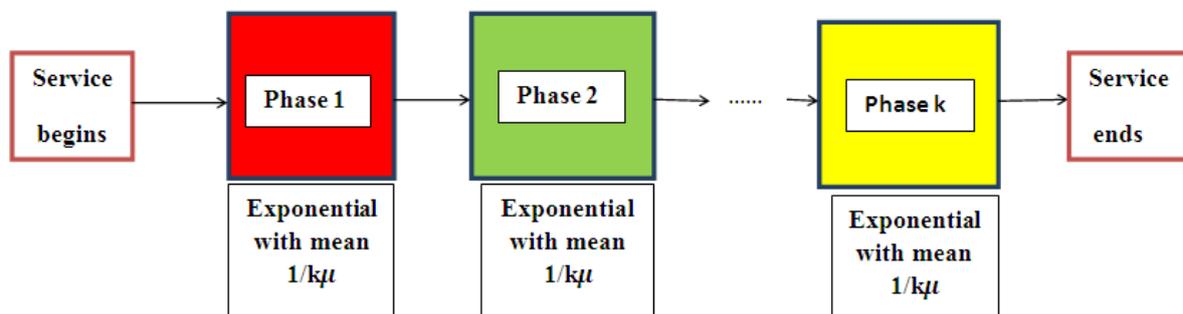
service time  $1/s\mu$  individually. The distribution of total servicing time of customers in the system is some joint distribution of time in all these stages. Each customer to be served enters the system in first phase before proceeding to the second phase up to  $k$ th phase. The assumption is that a new service does not start until a customer completes all  $k$ -phases. Moreover, the discipline followed in the queue is first come first served and it accepts the service of infinite number of customers.

**(FM/FM/C) :(FCFS/ $\infty/\infty$ ) Model**

Multi server fuzzy queue infinite calling source and first come first served discipline. In technically (FM/FM/C) :(FCFS/ $\infty/\infty$ ). Multi server queue has two or more service facility in parallel providing identical service. All the customers in the waiting line can be served by more than one station. The arrival time and the service time follow poisson and exponential distribution.

**Fuzzy Queue Model with Erlang Services**

A queuing system in  $k$  phases single server facility, denoted by  $FM/FE_k/1$  in which arrivals occur as Poisson process with fuzzy rate  $\tilde{\lambda}$  and service time according to Erlang's  $k$  distribution with fuzzy rate  $\tilde{\mu}$ . It is assumed that the service discipline is first in first out and the system capacity are infinite.



The arrival and service rates are defined as

$$\tilde{\lambda} = \{ (t, \gamma_{\tilde{\lambda}}(t)) / t \in X \}, \tilde{\mu} = \{ (s, \gamma_{\tilde{\mu}}(s)) / s \in Y \}$$

And the corresponding  $\alpha$  – cuts are

$$\tilde{\lambda}(\alpha) = \{ t \in X / \gamma_{\tilde{\lambda}}(t) \geq \alpha \}, \tilde{\mu}(\alpha) = \{ s \in Y / \gamma_{\tilde{\mu}}(s) \geq \alpha \}$$

Here X, Y are crisp rate and  $\gamma_{\tilde{\lambda}}(t), \gamma_{\tilde{\mu}}(s)$  are the corresponding function.

**III. REVIEW OF LITERATURE**

Specifically fuzzy queues have been discussed by several researchers. Buckley investigated multiple channel queueing system with finite or infinite waiting capacity and calling population. Negi and Lee formulated the  $\alpha$  cut and two variable simulation approach for analysing fuzzy queues on the basis of Zadeh extension principle. Li and Lee investigated the analytical results for M/F/1/ $\infty$  and FM/FM/1/ $\infty$  (where F represents fuzzy time and FM represents fuzzified exponential distribution) using Markov Chain. Unfortunately, their approach provided only crisp solutions. In other words, the membership functions of the performance measures are not completely described. Kao et al applied parametric programming to construct the membership functions of the performance measures for four simple fuzzy queues with one or two fuzzy variables namely M/F/1, F/M/1, F/F/1 and FM/FM/1 where F denotes fuzzy time and FM denotes fuzzified exponential time. Various researchers worked in the area of fuzzy queues. Here we are providing their contribution in short.

**R.-J. Li and E. S. Lee (1989)** investigated only M/F/1 and FM/FM/1 fuzzy queueing system and their approaches presented in this paper can be extended easily to other more complicated fuzzy queues.

**R. Srinivasan (2014)** had proposed a procedure to construct the membership functions of the performance measures in queueing systems FM/FM/1 using DSW (Dong, Shah & Wong) algorithm. They proposed a Fuzzy nature in FM/FM/1 queueing system with finite capacity and calling population are infinite. And with the help of MATLAB 7.0.4 they perform  $\alpha$  – cut of arrival rate. They have also derived  $L_q, L_s, W_q, W_s$  for above queueing system.

**S. Thamocharan(2016)** have studied on multi server fuzzy queueing Model in triangular and trapezoidal fuzzy numbers using  $\alpha$  cuts also discussed about performance measures of multi-server queue FM/FM/C by taking number of server  $C= 4$ .

**S. Shanmugasundaram and B. B. Venkatesh (2016)** studied the priority queueing model under fuzzy environment. They constructed the membership function of the performance measures of priority queueing system by using DSW algorithm which is based on  $\alpha$  cut representation of fuzzy sets.

**N. Sujatha, V. S. N. Murthy Akella and G. V. S. R. Deekshitulu(2017)** worked with fuzzy queueing models with single and multi-servers with triangular fuzzy numbers with the help of  $\alpha$  – cut method. Both arrival rate and service rate are supposed to be of fuzzy nature. Also, they have considered (M/E<sub>k</sub>/S) queue model as a numerical example and specified the method to compute L<sub>q</sub>, L<sub>s</sub>, W<sub>q</sub>, W<sub>s</sub> of the above model for various value of  $\alpha$ .

**K. Usha Madhuri, K.Chandan(2017)** studied performance measure of FM/FM/1 queueing system with pentagon fuzzy number using  $\alpha$  cut method.

**V. Suvitha, V. Visalakshi (2019)** have studied a single server queueing system where service follows an Erlang-k type with fuzzy parameter. Also, they have considered FM/FE<sub>3</sub>/1 queue model as a numerical example and specified the method to compute  $\tilde{w}, \tilde{w}_q, \tilde{L}_q, \tilde{L}$  of the model.

**M. Ragapriya, M. Kumar, C. Rajgiri, S. Jayaprakash (2019)** have proposed a procedure for construct a membership function of the performance measure using DSW algorithm in FM/FM/1 queueing model with fuzzy nature. They have also given numerical example in his study. The numerical example shows the variation of numbers  $\mu, \tilde{\lambda}$  in the figure is stationary.

**G. Kannadasan, D. Devi and N. Sathiyamoorthi(2019)** have studied the analysis of the M/M/1 queue with two vacation policies using pentagonal fuzzy numbers. Also, they have obtained the server is in the working vacation period, server is in the vacation period, the server is in the regular service period. They have obtained numerical result to all performance measures for M/M/1 queue.

**Noor HidayahMohdZaki, Aqilah NadirashSaliman, Nur Atikah Abdullah, Nur Su Ain Abu Hussain, Norani Amit(2019)** compared performance measures between Queuing theory model and Fuzzy queueing model by computing L<sub>q</sub>, L<sub>s</sub>, W<sub>q</sub>, W<sub>s</sub> at check-in counter in Airport. Based on the above result they decided that the fuzzy queueing model is much more effective and efficient to measure the performance of multi-server in a queueing system.

**V. R. Bindu Kumari, Dr. R. Govindarajan (2019)** analysed the performance measures in fuzzy numbers using DSW algorithm. they discussed the performance measures of single server queueing model in nonagonal fuzzy numbers. Numerical example also used to show the effectiveness of this approach.

**M. Shanmugasundari, S. Aarthi(2020)** suggested a different approach to solve fuzzy queueing problem which gives “N” the expected number of customers and “T” their waiting time of customers in a successful way. Two method L-R method and alpha – cut method were done in this paper. In last they found that the result obtained from above method much more optimized than the existing one.

#### IV. CONCLUSION

Zadeh introduced the concept of Fuzziness. Fuzzy queueing model was first introduced by Timothy Rose and R.J.Lie . Later many authors like J.J. Buckley, Negi, S.P.Chen , improved the above model. Recently W.Ritha , R.Srinivasan , S.Shanmuga Sundaram, S.Thamotharan, Mohammed Shapique.A analysed fuzzy queueing models using DSW algorithm. W.Ritha analysed Fuzzy N policy queues with infinite capacity. R.Srinivasan implemented DSW algorithm for the brief description of his fuzzy queueing model. S. Shanmugasundaram studied fuzzy queueing model with multiple servers using the same algorithm and also executed its performance measures. Also S. Thamotharan worked on multi server fuzzy queueing model using  $\alpha$ -cuts. In all the queueing models analysed above arrival rate and service rate exhibits Poisson distribution and exponential distribution respectively.

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