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Heat Transfer effects in MHD Micropolar Dusty Fluid over an Exponentially Porous Stretching Sheet

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Abstract: The purpose of this study is to examine how an MHD dusty fluid transfers heat over an exponentially porous sheet. By transforming the partial differential equations governing the flow into nonlinear ordinary differential equations, similarity transformations can solve them. Numerous parameters, including the magnetic field, mass concentration, Prandtl number, Eckert number, and porosity, among others, have been explored for their effects on the microrotation, velocity, and temperature profiles of both the fluid phase and the dust phase. The MATLAB BVP4C package was used to draw the graphic findings.

Keywords: Jeffrey fluid, exponentially porous sheet, dusty fluid, Eckert number

I. INTRODUCTION

Micropolar fluids were one of the kinds in Non-Newtonian fluids where it has a microstructure that comprises of rigid particles. The theory of micropolar fluid given by **Eringen**[1]was used wisely in the industrial and engineering processes including paints, lubricants, animal blood, polymers, colloidal and suspension fluids etc.

First, **Sakiadis**[2] investigated boundary layer flow of an incompressible viscous fluid over moving continuous solid surfaces. Then **Crane**[3] carried out his work and studied two dimensional flow past a linearly stretching sheet. His experiment played remarkable role in the polymer industry. **Chakrabarti** and **Gupta**[4] studied MHD flow and heat transfer on a stretching surface.However, **Hayat et al**.[5]studied the method (named Homotopy analysis method) to solve the MHD flow a micropolar fluid near a stagnation point towards a non-linear stretching surface. **Anand et.al** [6]had made an investigation to study the structure of an unsteady MHD flow of Dusty fluid over exponentially porous sheet accompanied by heat transfer through parallel plate.Later,the micropolar fluid flow and transfer due to a stretching/shrinking sheet was investigated by **Aurangzaib et al**. [7]. **Hossain et.al**[8] discussed about boundary layer flow and heat transfer in a micropolar fluid past a permeable flat plate.

Recently, the applications of a flow gas particles mixture in the fluid have a great deal of attention in the industrial field. It is because that it is rarely impossible to find an impurity-free fluid .Fluid contain foreign objects or dust particles in it. The presence of these micro or millimetre sized dust particles in the base fluids cause to improve the properties of the heat transfer of the fluids. Research and study on these fluids has been useful in many practical applications such as combustion, petroleum, casting, hot rolling, drawing wires of copper, power plant piping ,glass blowing, and also polymer industries .Considering the increasing applications involving dusty fluid, the modelling of dusty fluid become attracted to researchers. **Marble**[9]examined the effects of dusty gases on fluid dynamics. **Ramesh and Gireesha**[10] investigated dusty fluid flow past a stretching surface in the presence of radiation effects. MHD free convection of a dusty fluid over a vertical permeable surface was examined by **Hazarika and Konch**[11] numerically. **Roopa et.al**[12]also investigated unsteady flow and heat transfer of a dusty fluid between two parallel plates with variable physical properties. **Siddiqa et al.**[13] investigated the effect of dusty fluid on a natural convection flow due to a heated vertical surface with the two-phase boundary layer model of fluid and dust phases. Further, **Arifin et al.**[14]studied the aligned magnetic field of two phases mixed convection flow in dusty Casson fluid over a stretching sheet numerically.**Izani and Ali**[15] alsostudied the hydromagnetic mixed convection flow over an exponentially stretching sheet with fluid particle suspension. **G.MPavithra and B.J.Gireesha**[16] investigated the effect of Internal Heat Generation/Absorption on dusty fluid flow over an exponentially stretching sheet with viscous dissipation. Their findings were excellent match with earlier studies and are of physical and engineering interest. **B.J.Gireesha et.al**[17]analysed the boundary layer flow and heat transfer of a dusty fluid over an exponentially stretching sheet. Very recently, **A Dasman et.al**[18]studied the formulation of dusty micropolar fluid Mathematical model.They compared the numerical solutions with previous studies to check the validity of their model. Their findings give advance understanding on dusty micropolar fluid to researchers. They also discussed the boundary layer flow of a dusty micropolar fluid due to a stretching sheet with constant wall temperature numerically[19]. **Lou et.al**[20] in their work, examined the momentum and thermal transportation of rotating dusty micropolar fluid flux with suspension of conducting dust particles across the stretched sheet .

Motivating with all mentioned researches, the aim of this study is to analysethe heat transfer of MHD dusty fluid over an exponentially porous sheet .The partial differential equations governing the flow are solved by similarity transformations by converting them to nonlinear ordinary differential equations. The effect of various parameters like magnetic field, mass concentration, Prandtl number, Eckert number and porosity etc. on the microrotation, velocity and temperature profiles of both fluid and dust phase have been studied. The graphical results were drawn using BVP4C package of MATLAB software.

MATHEMATICAL FORMULATION

On two dimensional coordinate system (x, y), an incompressible micropolar dusty fluid moves towards porous exponentially stretching sheet with velocity components (u,v) for fluid phase and (u_p,v_p) for the dust phase along x-axis and y-axis respectively with velocity u=u_w= $ae^{x/L}$ at the surface of the sheet. A uniform magnetic field B_0 is applied perpendicular to the stretching surface and induced magnetic field is assumed to be negligible.T is the temperature of the fluid and its value at surface is assumed to be $T_w$and its ambient value is T_∞

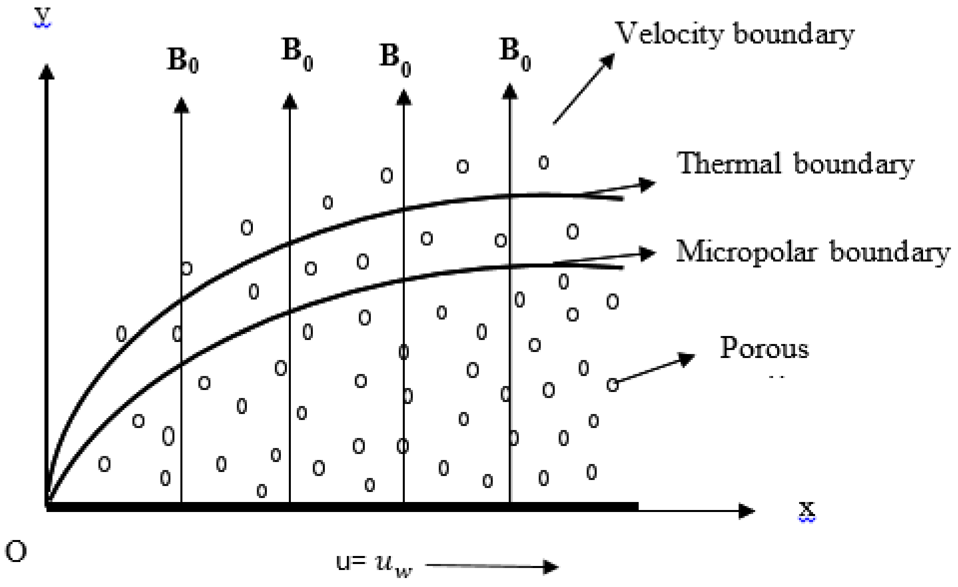

Figure 1. Geometry of the problem and coordinate system

Under these assumptions, using the regular Boundary layer and Boussinesq Approximations the Governing equations for the fluid and dust phase therefore are as under:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(\nu+\frac{K^*}{\rho})\frac{\partial^2 u}{\partial y^2}+\frac{K^*}{\rho}\frac{\partial w}{\partial y}+\frac{K_0 N}{\rho}(u_p-u)-\frac{\sigma B_0^2 u}{\rho}-\frac{\nu}{K_1}u \qquad (2)$$

(2)

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\frac{\gamma_0}{\rho j}\frac{\partial^2 w}{\partial y^2}-\frac{K^*}{\rho j}\left(2w+\frac{\partial u}{\partial y}\right) \qquad (3)$$

For Dusty phase

$$\frac{\partial u_p}{\partial x}+\frac{\partial v_p}{\partial y}=0 \qquad (4)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{K_0 N}{\rho_p}(u - u_p) \tag{5}$$

$$C_p \rho(\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}) = K_T \frac{\partial^2 T}{\partial y^2} + \frac{\rho_p \rho C_p}{\tau_T}(T_p - T) + \frac{\rho_p}{T_p}(u_p - u) \tag{6}$$

$$u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = -\frac{C_p}{\tau_T C_m}(T_p - T) \tag{7}$$

Where w is the component of micro-rotation vector normal to the x and y axes, τ_T is called thermal relaxation time of particle phase. j denotes micro inertia density, K* is micropolar coefficient and γ_0 is spin gradient viscosity, T and Tp are the temperature of fluid and dust phase, respectively. K_0 signifies the Stoke's drag constant, K_T represents thermal conductivity andN represents the number density of dust particles and micro rotation. ν , ρ , ρ_p and σ represents the coefficient of kinematic viscosity, the density of the fluid phase, density of dust phase and electrical conductivity respectively. C_p and C_m denotes specific heat at constant pressure and specific heat of dust particle respectively together with the following relevant boundary conditions

$u = u_w$, v=0, w=0, $T = T_w$ at y=0

$$u \to 0\,, u_p \to 0\,, v_p \to v\,, w \to 0\ ,\ T \to T_\infty\ ,\ T_p \to T_\infty \text{ at } y \to \infty \tag{8}$$

Where $u_w = ae^{x/L}$ is the sheet velocity, a represents the reference velocity and L is the reference length. Hereby introducing the following similarity variables

$$u = ae^{xL} f'(\eta), \qquad v = -\sqrt{\frac{\nu a}{2L}} e^{x/2L}\left(f(\eta) + \eta f'(\eta)\right), \qquad w = a\sqrt{\frac{a}{2\nu L}} e^{3x/2L} g(\eta)$$

$$\eta = \sqrt{\frac{a}{2\nu L}} e^{x/2L} y,\ u_p = ae^{x/L} F'(\eta), \qquad v_p = -\sqrt{\frac{\nu a}{2L}} e^{x/2L}(F(\eta) + \eta F'(\eta))$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\ ,\quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty} \tag{9}$$

Applying Similarity Variables in Equations (1) - (7) using (9)

Equation (2) and (3) becomes

$$(1 + A) f''' + ff'' - 2f'^2 + Ag' + lB_v(F' - f') - (M + K_p) f' = 0 \tag{10}$$

$$(1 + \frac{A}{2}) g'' + fg' - 3gf' + A(2g + f'') = 0 \tag{11}$$

Where

$A = \dfrac{K^*}{\mu}$ (Micropolar parameter) $\qquad l = \dfrac{mN}{\rho}$ (Mass concentration parameter)

$B_v = \dfrac{L}{ae^{x/L}\tau_v}$ (Fluid particle interaction parameter) $M = \dfrac{\sigma B_0^2 L}{\rho a e^{x/L}}$ (Magnetic field parameter)

$T_v = \dfrac{m}{K_0}$ (Relaxation time of the dust particle) $\qquad K_p = \dfrac{\nu L}{aK_1^{x/L}}$ (Porosity parameter)

According to Aurangzaib et.al.[7] γ_0 is the spin gradient viscosity with

$$\gamma_0 = (\mu + \frac{K^*}{\nu}) j = \mu(1+\frac{A}{2}) j$$

Where $j = \frac{2L\nu}{ae^{x/L}}$ represents the micro inertia per unit mass and $A = \frac{K^*}{\mu}$ is the material parameter.

Similarly, using (9) the equations (5-7) of Dusty fluid flow transformed in the following manner:

$$FF'' - 2F'^2 + 2B_v(f' - F') = 0 \tag{12}$$

$$\theta'' + Prf\theta' + 2lB_T Pr(\theta_p - \theta) + 2lB_v EcPr(F' - f')^2 = 0 \tag{13}$$

where $Pr = \frac{\mu C_p}{K_T}$ is the Prandtl number, $B_T = \frac{L}{ae^{x/L}\tau_T}$ is the fluid particle interaction parameter for temperature and $Ec = \frac{U_w^2}{C_p(T_w - T_\infty}$ is the Eckert number.

$$F'\theta_p' - 2\gamma B_T(\theta_p - \theta) = 0 \tag{14}$$

Where $\gamma = \frac{C_p}{C_m}$ signifies the specific heat ratio.

Boundary conditions (8) in similarity variables now reduced to

at $\eta = 0,\ f(0) = 0,\ f'(0) = 1,\ g(0) = 0,\ \ \theta(0) = 1$

at $\eta \to \infty,\ f'(\infty) \to 0,\ F'(\infty) \to 0,\ F(\infty) \to f(\infty),\ g(\infty) \to 0,\ \theta(\infty) \to 0, \theta_p(\infty) \to 0$

(15)

SOLUTION ALGORITHM

Above system of nonlinear ordinary differential equations (10)-(14) with boundary constraints (15) were solved using the robust numerical computing of bvp4c approach of MATLAB by transforming into first order nonlinear system of differential equations as follows:

$$f = y_1, f' = y_2, f'' = y_3, g = y_4, g' = y_5, \theta = y_6, \theta' = y_7, \theta_p = y_8\ ,\ \ F = y_9, F' = y_{10}$$

$$y_1' = y_2 \tag{16}$$

$$y_2' = y_3 \tag{17}$$

$$y_3' = \frac{1}{1+A}\left[2y_2^2 - y_1y_3 - Ay_5 - lB_v(y_{10} - y_2) + (M + K_p)y_2)\right] \tag{18}$$

$$y_4' = y_5 \tag{19}$$

$$y_5' = \frac{1}{\left(1+\frac{A}{2}\right)}\left[3y_4y_2 - y_1y_5A(2y_4 + y_3)\right] \tag{20}$$

$$y_6' = y_7 \tag{21}$$

$$y_7' = -Prfy_1y_7 - 2lB_T Pr(y_8 - y_6) - 2lB_v EcPr(y_{10} - y_2)^2 \tag{22}$$

$$y_8' = \frac{1}{y_{10}}\left[2\gamma B_T(y_8 - y_6)\right] \tag{23}$$

$$y_{10}' = \frac{1}{y_9}\left[2y_{10}^2 - 2B_v(y_2 - Y_{10})\right] \quad (24)$$

The boundary conditions so formed are as follows:

at $\eta = 0$ $y_1(0) = 0$, $y_2(0) = 1$, $y_4(0) = 0$, $y_6(0) = 1$

at $\eta \to \infty$ $y_2(\eta) \to 0$, $y_{10}(\eta) \to y_1(\eta)$, $y_4(\eta) \to 0$, $y_6(\eta) \to 0$, $y_8(\eta) \to 0$ (25)

II. RESULT & DISCUSSION

Graphical outcomes of the impacts of pertinent parameters like fluid particle interaction parameter, porosity, mass concentration, Prandtl number, Eckert number and magnetic field etc. on velocity and temperature and a micro-rotation distribution profile for both fluid and dusty phases were acquired.

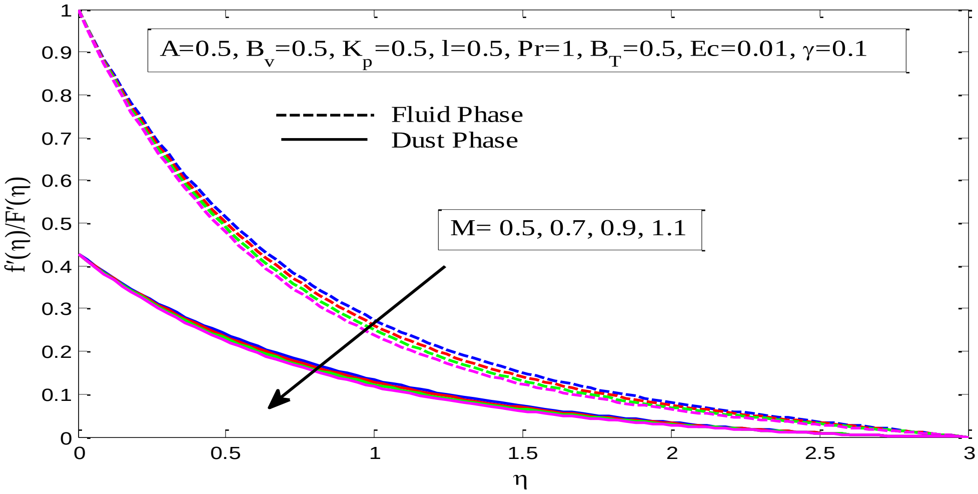

Figure 2. Effect of M on velocity profile

Figure 2 depicts the effect of Magnetic field (M) on velocity profile for both fluid and dust phase. It is been clearly observed from the figure that velocity profiles of both the phases decreases with the increasing values of M.

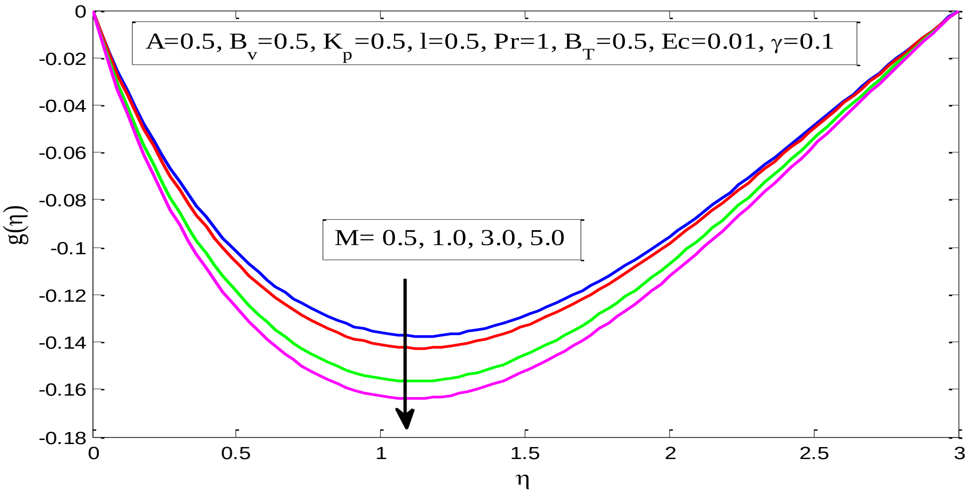

Figure 3. Effect of M on Micro-rotation

It is been demonstrated from Figure 3 that with increase values of M , there is a decrease in microrotation profile which infer that magnetic field settles down the fluid particles near the sheet.

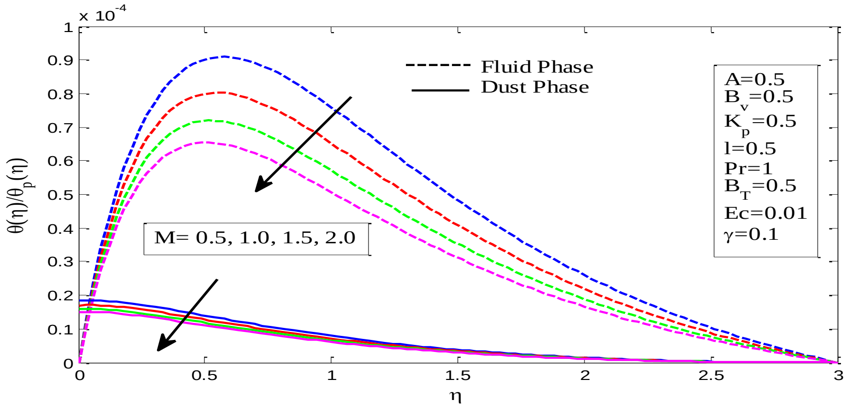

Figure 4. Effect of M on Temperature distribution

Figure 4 explains that temperature profile for both phases decrease with increasing values of M

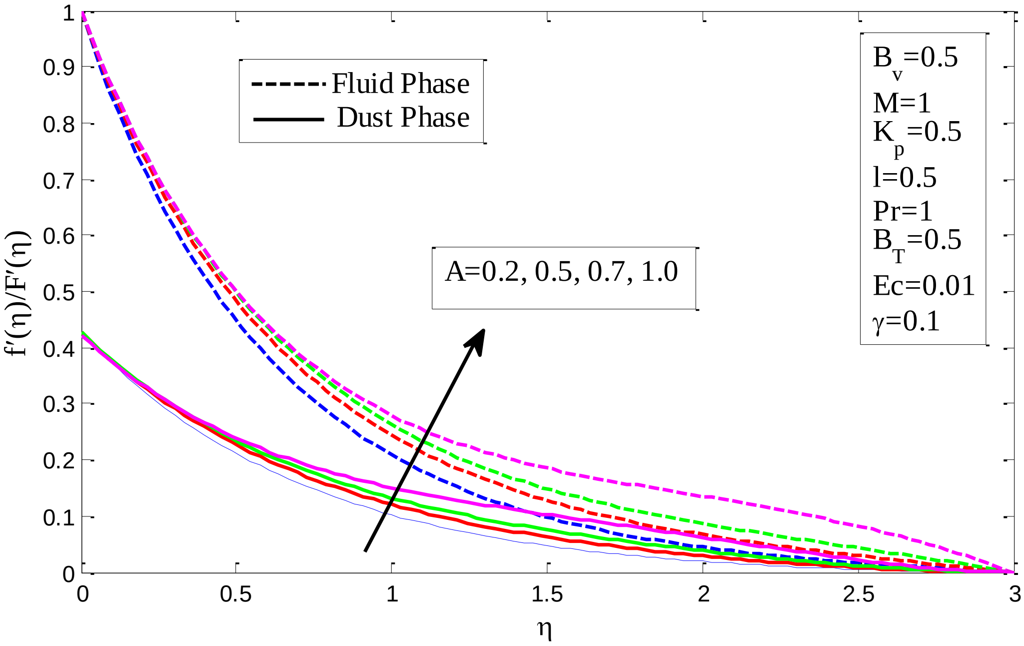

Figure 5. Effect of A on velocity profile

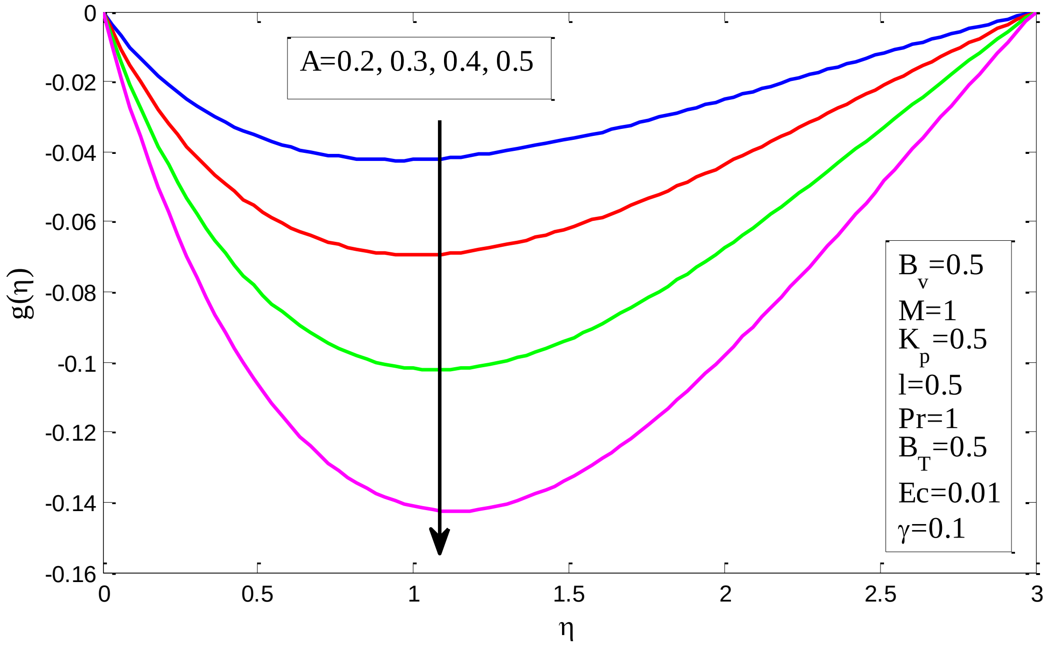

Figure 6. Effect of A on micro-rotation

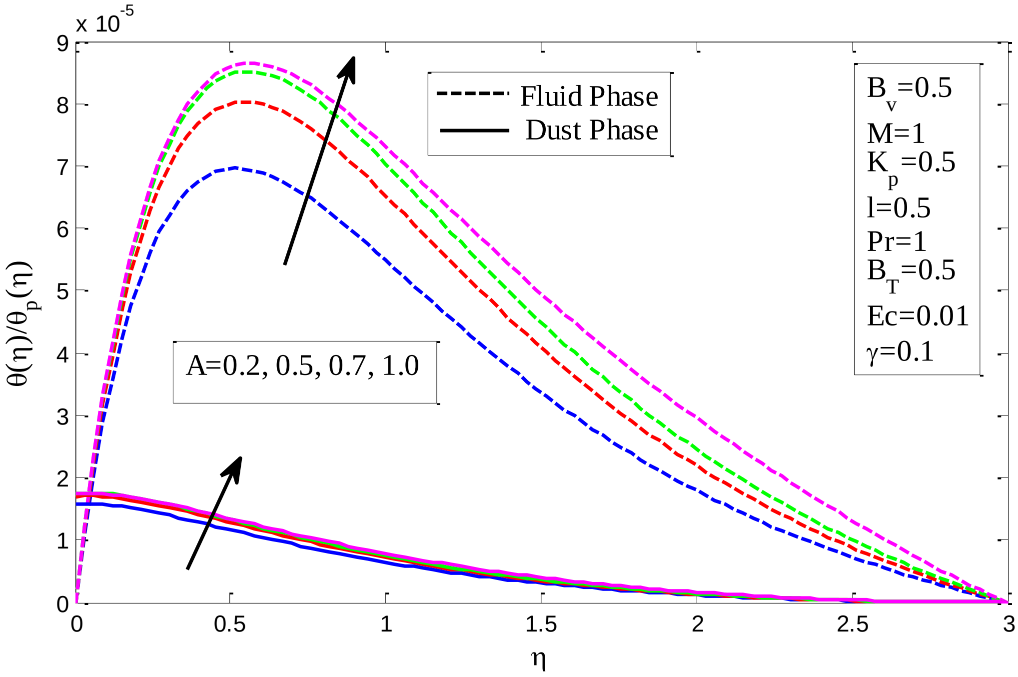

Figure 7. Effect of A on temperature distribution

It can be noticed from Figure 5 that velocity profiles of both fluid phase and dust phase increases as there is an increase in the values of micropolar parameter(A) while Figure 6 illustrated that increasing values of A, decreases microrotation profile. The temperature profile for both fluid and dust phase is depicted in Figure 7.It is been observed that temperature profile increases with the increase of A

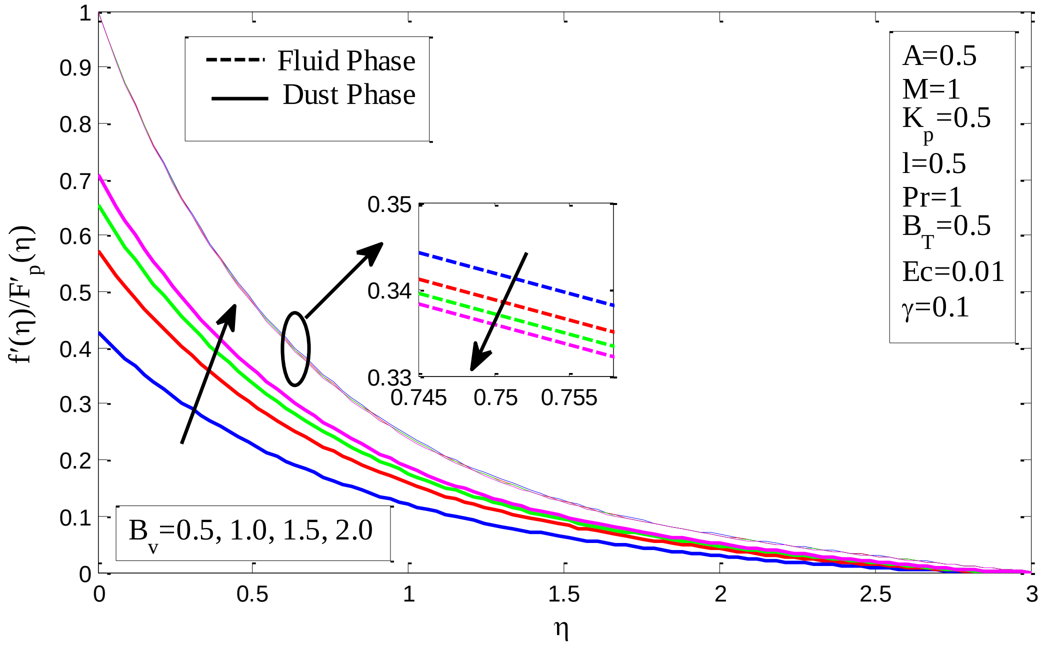

Figure 8. Effect of $B_v$on velocity profile

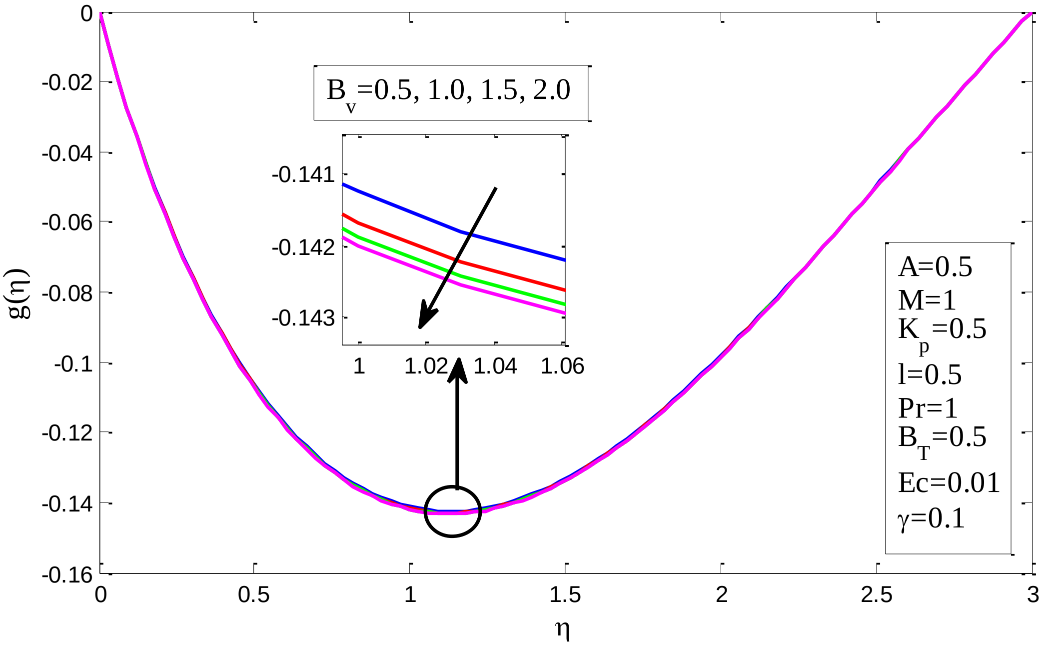

Figure 9. Effect of B_v on micro-rotation

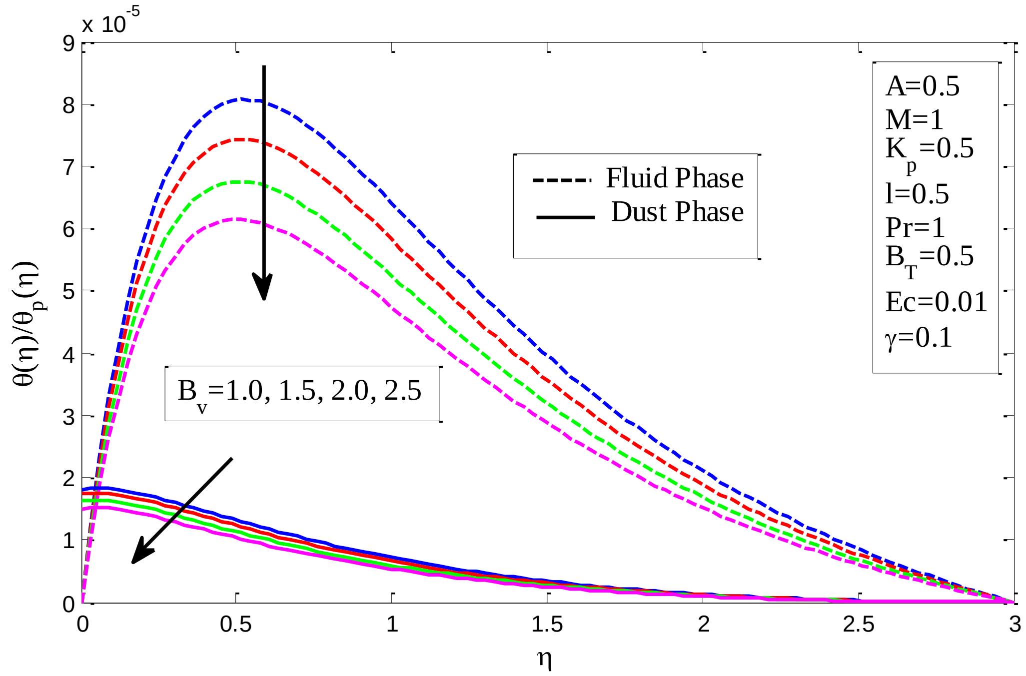

Figure 10. Effect of B_v on temperature distribution

Figure 8,9 and 10 demonstrates the effect of fluid particle interaction parameter (B_v) on velocity, microrotation and temperature profiles respectively.It is been inferred from Figure 8 that the fluid particle interaction parameter (B_v) decreases the velocity components in the fluid phase but trend is opposite in case of dust phase. It is clearly shown from the figure that there is an increase in dust phase with increase of $B_{v.}$ Figure 9 demonstrated that microrotation decreases with the increase in B_v. Figure 10 depicted that temperature distribution decreases both in the fluid and dust phase as fluid particle interaction parameter increases.

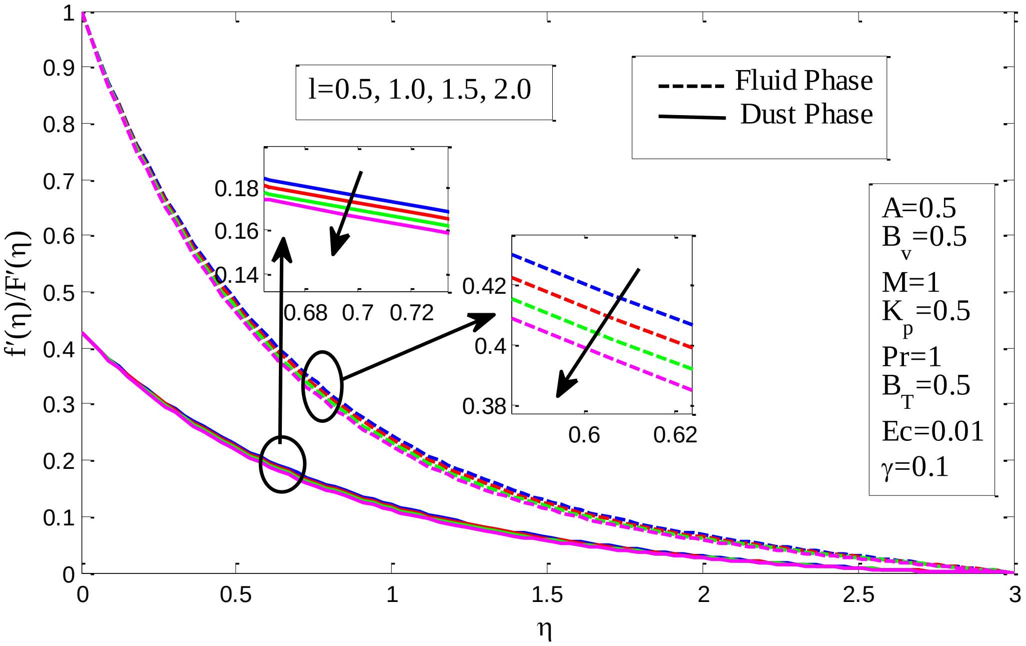

Figure 11. Effect of *l* on velocity profile

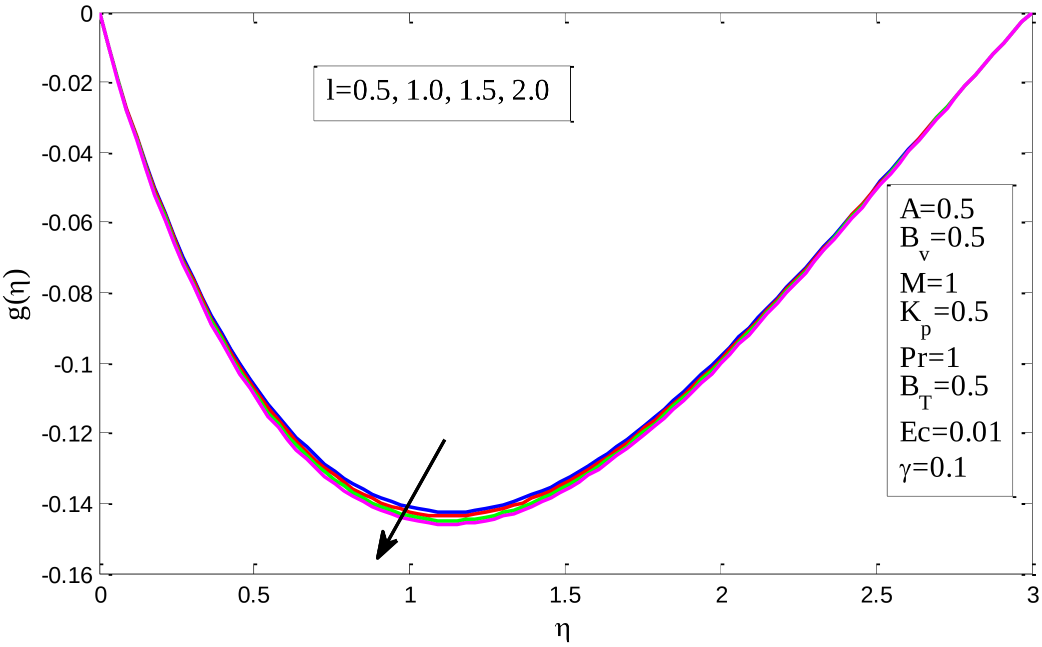

Figure 12. Effect of *l* on micro-rotation

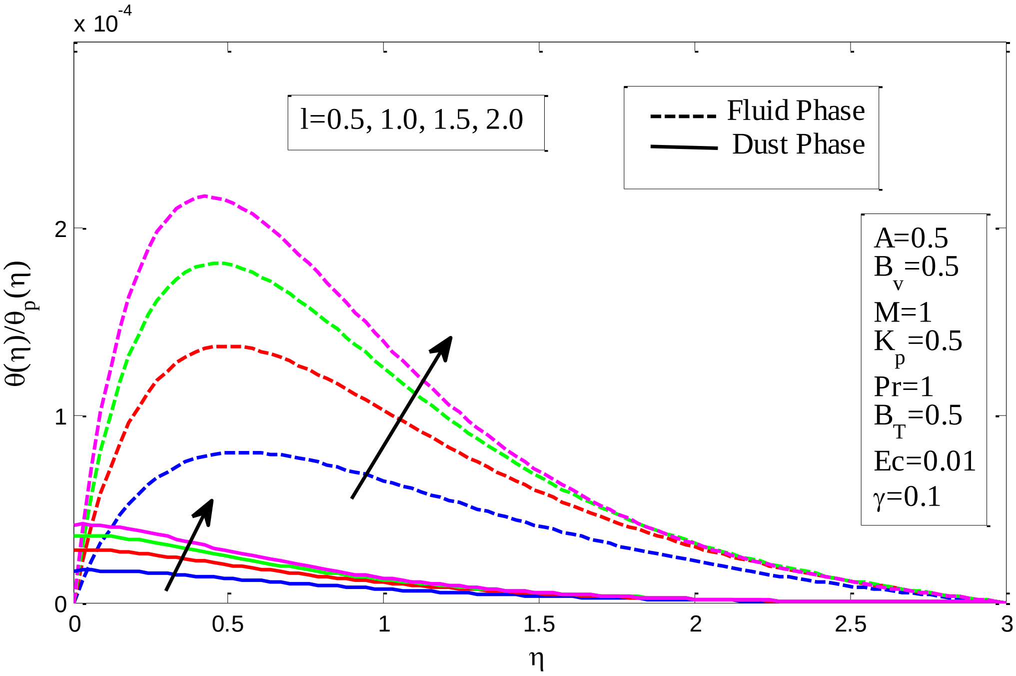

Figure 13. Effect of *l* on temperature distribution

Velocity profile of the fluid phase and dust phase decreases with the increase in mass concentration as can be clearly seen in Figure 11. Figure 12 reveals that microrotation profile of fluid decrease as there is increase in mass concentration.Figure 13 illustrated the increase in temperature distribution with the increase in mass concentration (l)

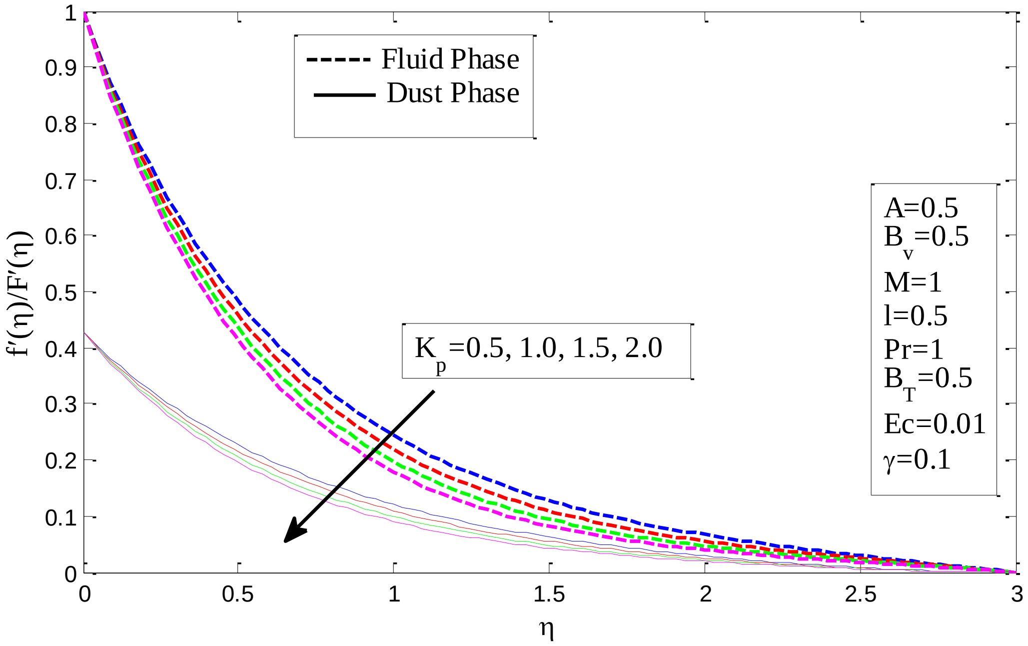

Figure 14. Effect of K_p on velocity profile

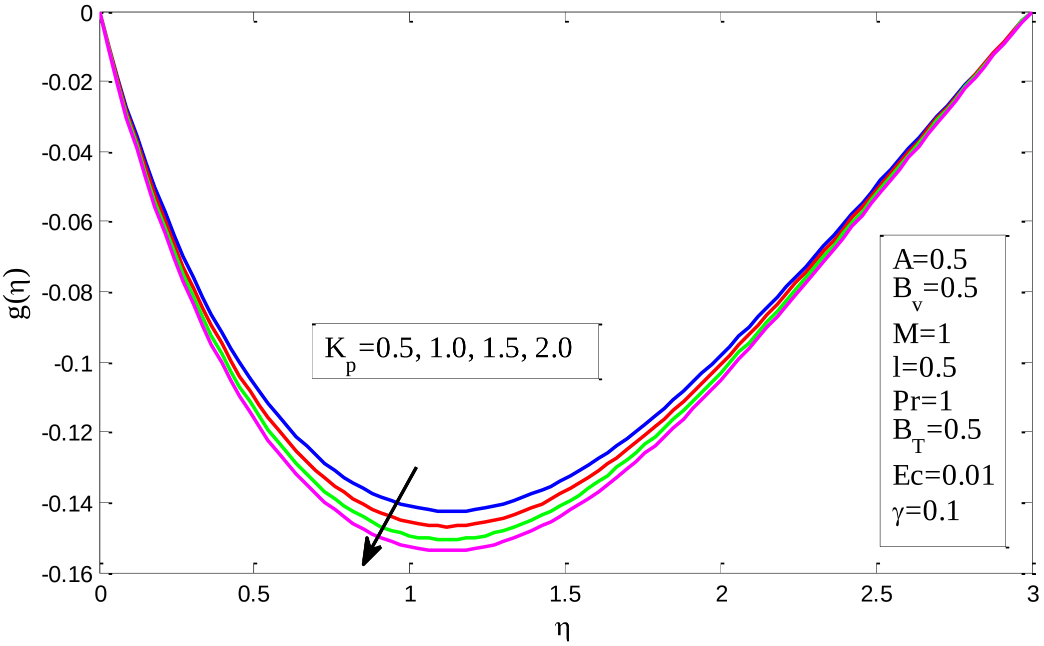

Figure 15. Effect of K_p on micro-rotation

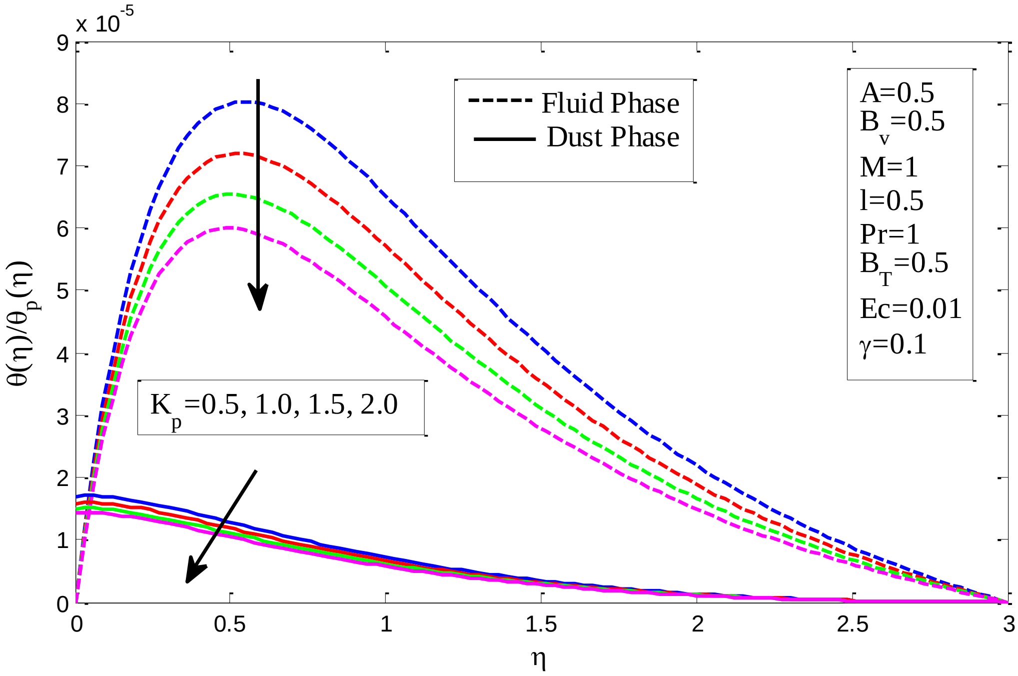

Figure 16. Effect of K_p on temperature distribution

The observations from Figure 14, 15 and 16 showed that the increase in porosity parameter (K_p) decreases the velocity profile as well as temperature distribution of both fluid and dust phase. Also, there is a decrease in microrotation profile with increase in K_p

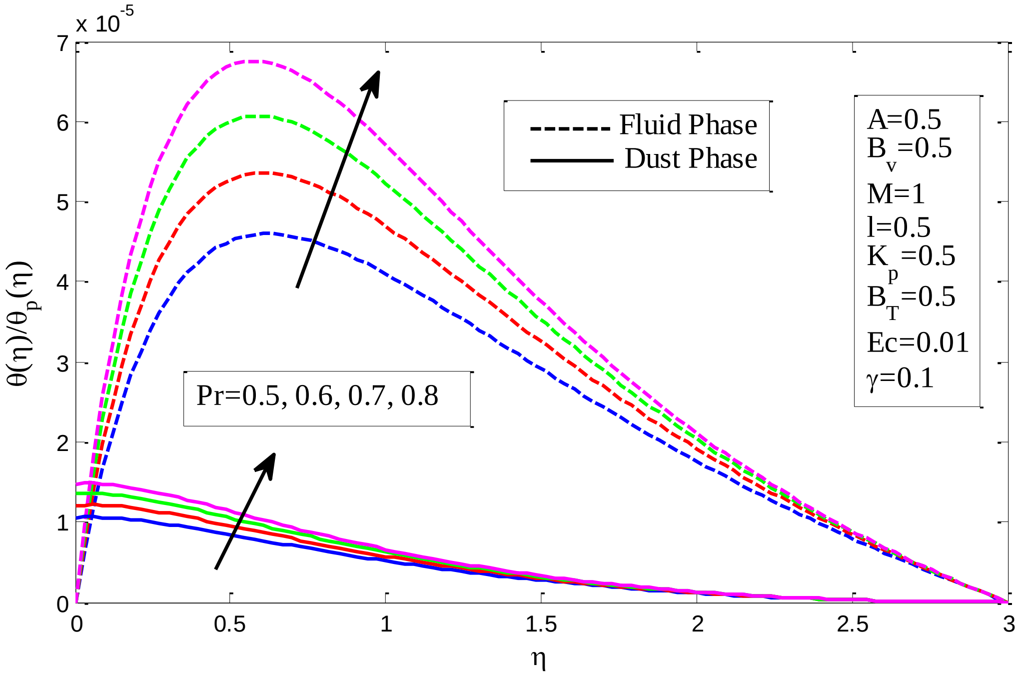

Figure 17. Effect of Pr on temperature distribution

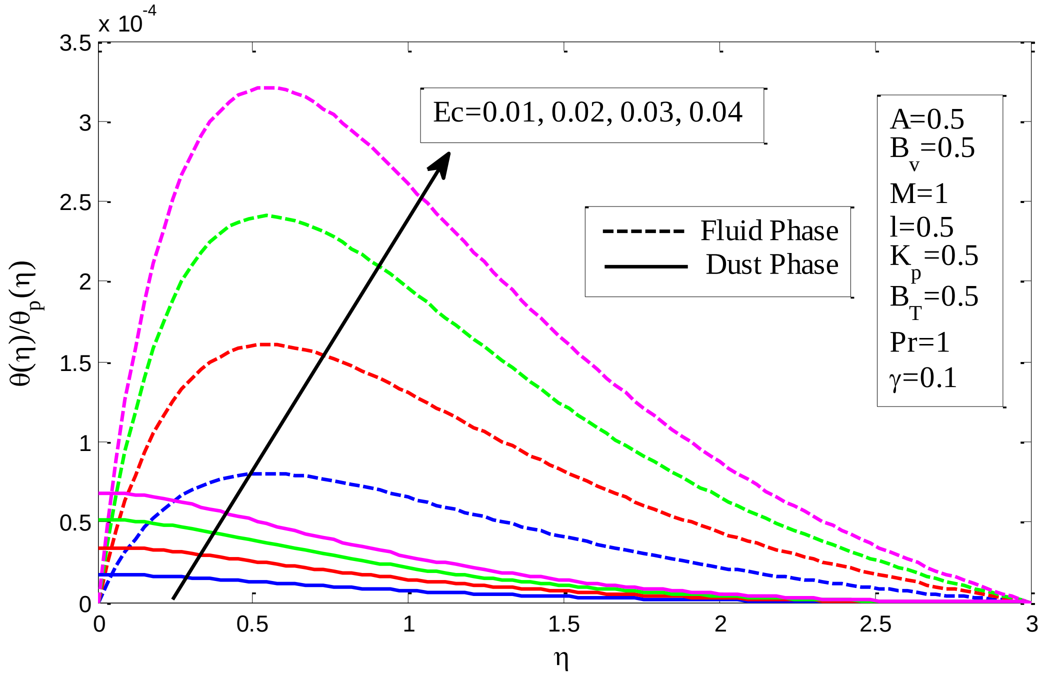

Figure 18. Effect of Ec on temperature distribution

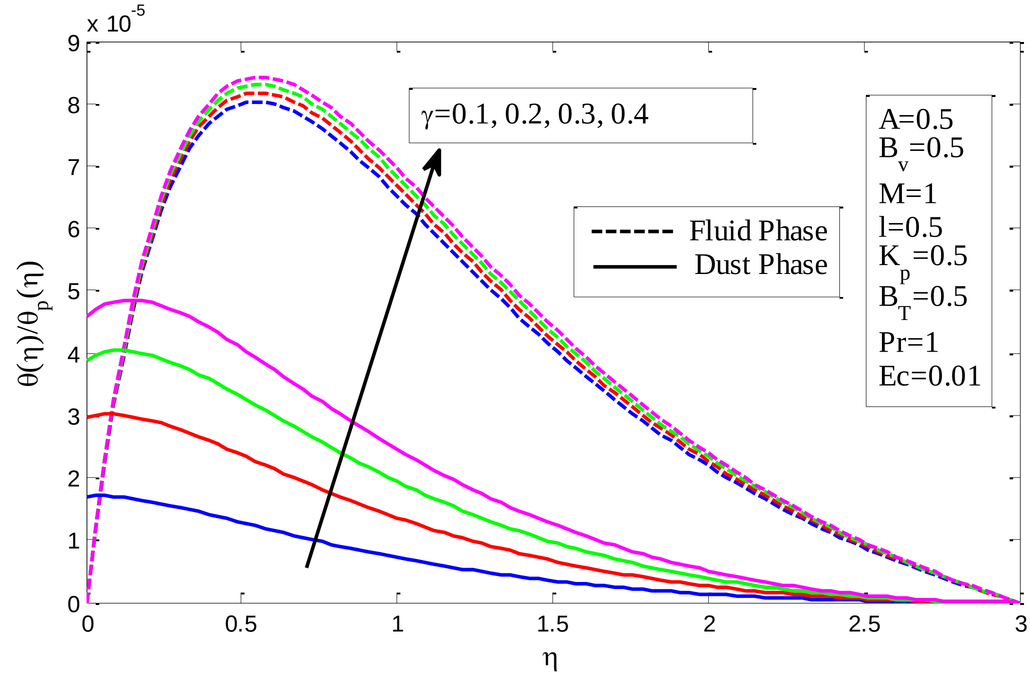

Figure 19. Effect of γ on temperature distribution

Figure 17,18 and 19 depicts that increase in the value of prandtl number (Pr), Eckert number (Ec) and specific heat ratio (γ), increases the temperature of both fluid and the dust phase. Increasing the value of Ec, temperature increases for both phases.

III. CONCLUSION

In this study, efforts have been made to find significant role of heat transfer in dusty micropolar fluid over exponentially porous stretching sheet. The governing partial differential equations were converted to nonlinear ordinary differential equations using similarity transformation and solved with the help of bvp4c function of MATLAB software.The major key points of the findings can be summarised as follows:

Fluid Phase

- Higher inputs of magnetic field, porosity, mass concentration and fluid particle interaction parameter lessens the fluid velocity.
- Presence of Micropolar parameter increases the fluid velocity.

Dust Phase

- Higher the value of Magnetic field, porosityand mass concentration parameter, the lower the velocity profile of dust particle
- The higher inputs of Micropolar parameter and the fluid particle interaction parameter boost up the velocity profile in the dust phase.

Microrotation

- Magnetic field settles down the fluid particles near the sheet.
- The escalating values of micropolar parameter lessens the microrotation
- The micro-rotation distribution profile depreciates by higher inputs of fluid particle interaction parameter and mass concentration parameter.

Temperature distribution amplified with larger inputs of Prandtl number and Eckert number both in fluid phase and dust phase.

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