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# Analysis of Conjugate Heat Flow in a Domestic Model Room

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## I. INTRODUCTION

Analysis of conjugate convective flows and heat transfer performance of built environment has been an interesting research subject because of its technical applications in building design.

In this study, we shall investigate the temperature distribution of air in a heated room as well as the conjugate conduction heat flow through the walls and convective heat loss to the surroundings.

Conduction occurs in a solid when its ends are at different temperatures. The rate of conduction depends on the thermal conductivity of the solid material and the temperature gradient [1] [2].

A typical heated, non-ventilated domestic room is modelled as a 3D box with an installed heater. The floor and roof are considered adiabatic while heat loss takes place through the walls which are in interaction with the external ambient temperature conditions.

We simplify our analysis by assuming that the temperature of the room is spatially uniform at any instant (lumped capacitance). Also, considering symmetry in the $y$ and $z$-directions, the problem is further simplified to a 1D transient conjugate heat transfer.

The transient temperature distributions in the indoor air and solid wall domains are analysed and compared for different wall material specifications. These results provide some correlation between material selection, indoor thermal comfort level and energy consumption.

## PROBLEM STATEMENT

To investigate the temperature distribution of air in a heated room as well as the conjugate conduction heat flow through the walls and convective heat loss to the surroundings for different material specifications in order to provide correlation between material selection on indoor thermal comfort level and energy consumption.

## ASSUMPTIONS

The following simplifying assumptions are made:

1. A typical heated, non-ventilated domestic room is modelled as a 3D box with internal heat generation
2. The floor and roof of the room are considered adiabatic
3. Heat loss takes place through the external walls which are in interaction with fixed outside temperature conditions
4. The temperature in the room is spatially uniform at any instant of time (lumped capacitance) since the room is not very large
5. Considering symmetry in the $y$ and $z$-directions, the analysis is simplified to a 1D transient conjugate heat transfer problem
6. The thermal conductivities of the wall material and the air in the room do not vary with space or time, i.e. materials are isotropic
7. The convective heat transfer coefficient ($h$) of air remains constant throughout

## SCHEMATIC

The room is modelled as a 3D box with dimensions, $L$ x $B$ x $H$ and wall thickness, $W$ as shown in fig. 1.

Considering symmetry in all directions, we simplify the problem to a 1D transient problem by taking a section $AA$ through the centre of the room along the $z$-direction as shown in fig. 2.

A steam heater which is turned off at the initial time is installed in the room and serves as a source of heat generation.

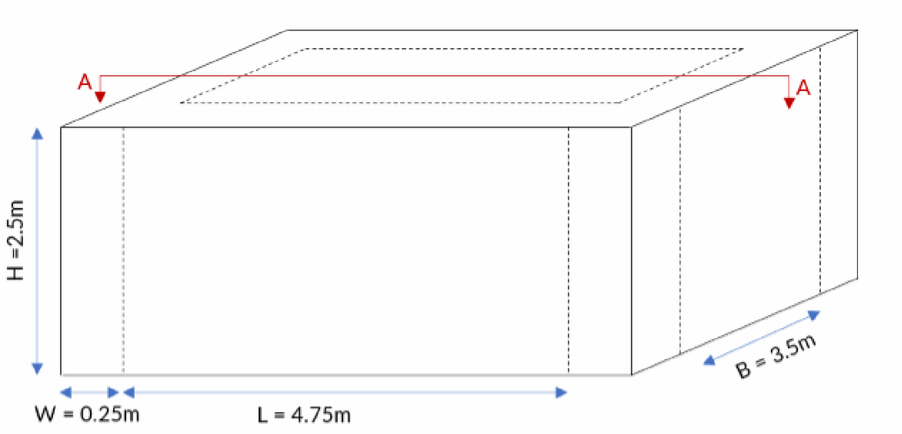



*Fig 1: Model showing basic features of the Room*

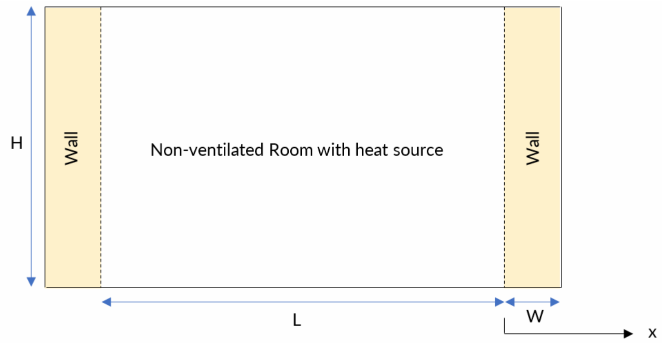



*Fig 2: Simplified schematic for 1D transient analysis*

## MATHEMATICAL MODELLING

Let us define the following parameters for our analysis:

*External ambient temperature,* $T_\infty = 5°C\ (278K)$
*Convective heat coefficient for air,* $h = 5\ W/m^2K$
*Heat generation rate of the room heater,* $E_g = 7\ kW$
*Specific heat capacity of air,* $c_a = 500\ J/kgK$
*Density of air,* $\rho_a = 1.225\ kg/m^3$
*Mass of air in the room,* $m = \rho LHB$
*Total internal surface area of room,* $A_s = 2(LB+LH+BH)$
*External surface area of walls,* $A_e = 2H(L+B+4W)$

We also define the following properties for some selected wall construction materials [3] which shall be used for our analysis:

***Table 1:*** *Properties of selected Wall Construction Materials*

| Material | Thermal Conductivity, $k$ ($W/mK$) | Density, $\rho$ ($kg/m^3$) | Specific heat Capacity, $c_p$ ($J/kgK$) |
|---|---|---|---|
| Aircrete Block | 0.15 | 600 | 1000 |
| Brick | 0.73 | 1700 | 800 |
| Concrete | 1.13 | 2000 | 1000 |
| Steel | 50 | 8000 | 500 |

At the initial time, the heater is off and the temperature in the room and across the walls are the same as ambient conditions.

$$T_R(t = 0) = T_W(t = 0) = T_\infty$$

At times $t > 0$, the heater is on and generates heat at a constant rate. We assume that the temperature in the room is spatially uniform at any instant of time (lumped capacitance). Thus, the temperature inside the room varies with time only:

$$T_R = T_R(t)$$

Considering conjugate conduction through the walls, the temperature distribution across the external walls varies with time and the x-direction:

$$T_W = T_W(x, t)$$

Since the room is modelled as a transient isothermal object with internal heat generation, conservation of energy in the room applies as follows [1]:

$$E_{in} + E_g - E_{out} = E_{st}$$

$$E_g - hA_s[T_R(t) - T_\infty] = mc_a \frac{dT_R}{dt}$$

At steady state conditions, i.e, $t \to \infty$, we can obtain an expression for the steady state temperature as follows:

$$E_g - hA_s[T_R(\infty) - T_\infty] = 0$$

$$E_g = hA_s[T_R(\infty) - T_\infty]$$

Where $T_R(\infty)$ is the steady state temperature, $T_{RSS}$ given by;

$$T_{RSS} = T_\infty + \frac{E_g}{hA_s}$$

Substituting for $E_g$ in the governing equation, we obtain;

$$hA_s[T_{RSS} - T_\infty] - hA_s[T_R - T_\infty] = mc_a \frac{dT_R}{dt}$$

$$T_R - T_{RSS} = -\frac{mc_a}{hA_s}\frac{dT_R}{dt}$$

$$\frac{dT_R}{dt} = -\frac{hA_s}{mc_a}[T_R - T_{RSS}]$$

With initial condition:

At $t = 0$, $T_R = T_\infty$

The governing equation for transient 1D conduction through the walls without heat generation is [1];

$$\frac{d^2T_W}{dx^2} = \frac{1}{\alpha}\frac{dT_W}{dt}$$

Boundary Conditions:

At $x = 0$,

$$\frac{dT_W}{dx} = -\frac{h}{k}[T_W - T_R(t)]$$

At $x = W$,

$$\frac{dT_W}{dx} = -\frac{h}{k}[T_W - T_\infty]$$

Initial Condition:

At $t = 0$, $T_W = T_\infty$

To obtain $T_R$ and $T_W$ respectively, we solve the governing equation using Finite Difference method in the following steps [2]:

1. Discretize the room domain into 1 x 1 grid and the wall domain into an $m$ x 1 grid for analysis
2. Consider a number of time steps $p$
3. Replace the partial derivatives with finite difference approximations
4. Replace the time derivative with first order forward difference approximation
5. Replace the space derivative with second order centred-difference approximation

The governing equations become;

$$\left(\frac{T_R^{p+1} - T_R^p}{\Delta t}\right) = -\frac{hA_s}{mc_a}\left(T_R^p - T_{RSS}\right)$$

$$\left(\frac{T_W^{p+1} - T_W^p}{\Delta t}\right) = \alpha\left(\frac{T_{W-1}^p - 2T_W^p + T_{W+1}^p}{\Delta x^2}\right)$$

Where $i$ represents the node location along the $x$ direction and $p$ represents the time step.

The finite difference approximation for energy balance in the room becomes;

Where; $Ao = \left(\frac{hA_s \Delta t}{mc_a}\right)$; is an arbitrary constant.

The finite difference approximation of the conduction equation through the wall becomes;

$$T_W^{p+1} = T_W^p + Fo\left(T_{W-1}^p + T_{W+1}^p - 2T_W^p\right)$$

Where, $Fo = \left(\frac{\alpha \Delta t}{\Delta x^2}\right)$; is the Fourier number.

The convective boundary condition equations for conduction through the walls becomes;

$$T_1^{p+1} = Fo\left[2Bi\,T_R^p + 2T_2^p + \left(\frac{1}{Fo} - 2\text{Bi} - 2\right)T_1^p\right]$$

$$T_m^{p+1} = Fo\left[2Bi\,T_\infty + 2T_{m-1}^p + \left(\frac{1}{Fo} - 2\text{Bi} - 2\right)T_m^p\right]$$

Where, $Bi = \left(\frac{h\Delta x}{k}\right)$; is the Biot number.

1. The above method is an explicit method where temperatures at future times ($p + 1$) are directly obtained based on temperatures at present times as shown in the final equations above
2. Explicit methods are conditionally stable. The stability criteria is given as $Fo \leq 0.25$
3. Stability of the end nodes requires that $Fo(1 + Bi) \leq 0.5$
4. Time step ($\Delta t$) needs to be small for more accuracy
5. The error is of the order, $O(\Delta t) + O(\Delta x^2)$

## II. RESULTS AND DISCUSSIONS

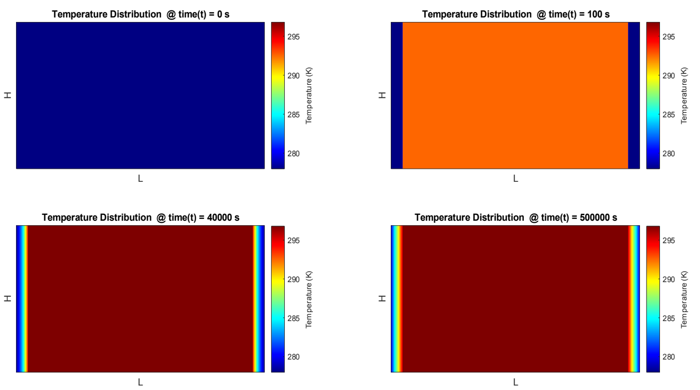
Using MATLAB, we model the heat flow in the room by choosing appropriate grid size/time step and checking for stability of the solution. Once a stable solution is obtained, we plot the temperature distribution in the room and across the wall at different time intervals for the different materials.

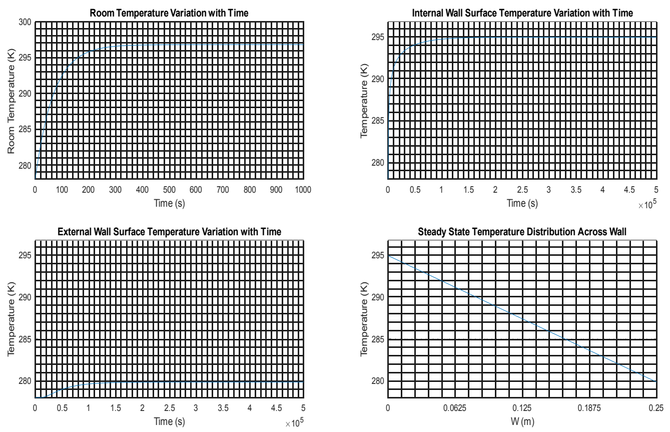
We also plot the transient temperature distribution for the room as well as the internal and external surfaces of the wall. This gives a graphical depiction of the time it takes for the temperature in the room as well as conduction through the walls to attain steady state conditions.

Finally, at steady state, we plot the temperature distribution across the wall. This gives an indication of the rate of heat loss from the room to the surroundings.

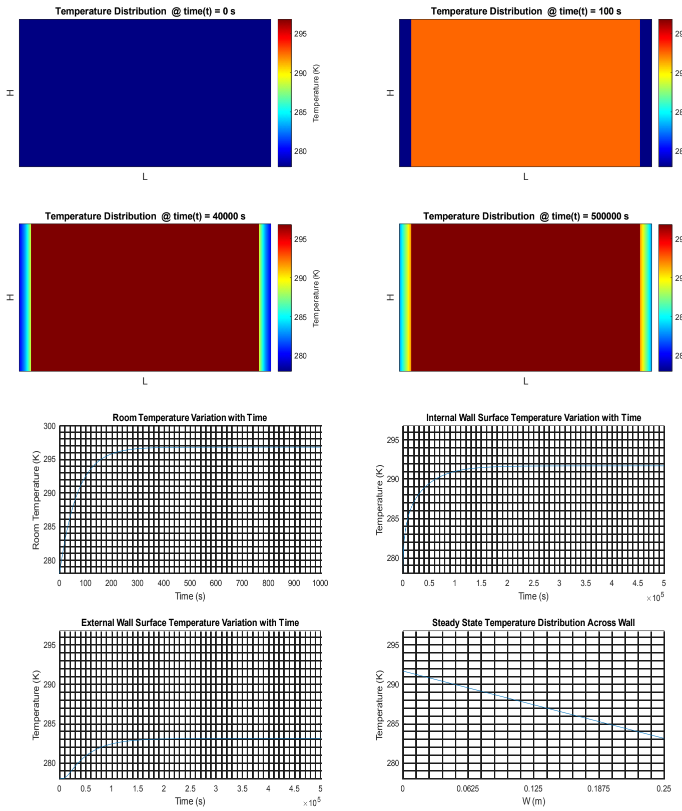
The results are shown in the plots below:

(a) AIRCRETE BLOCK

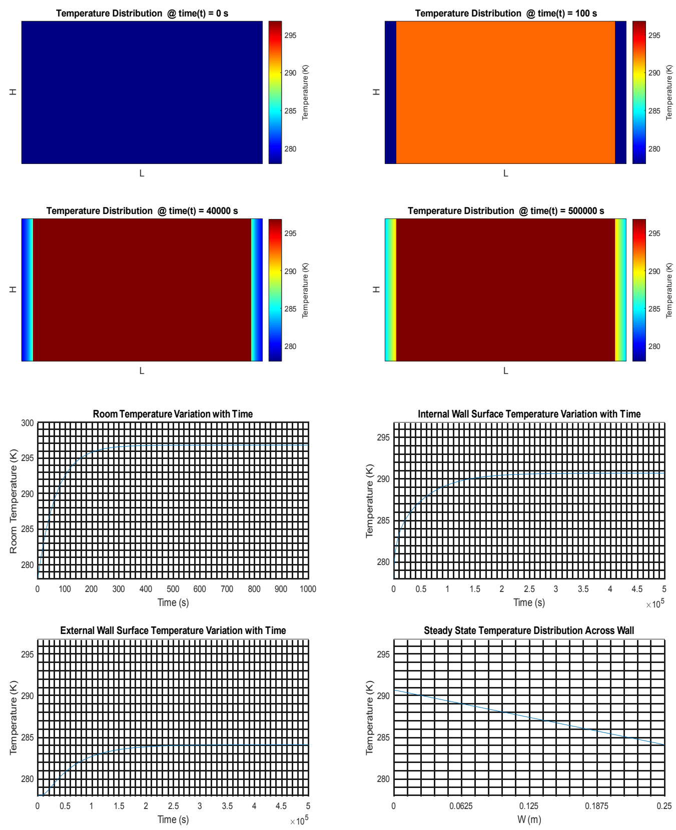


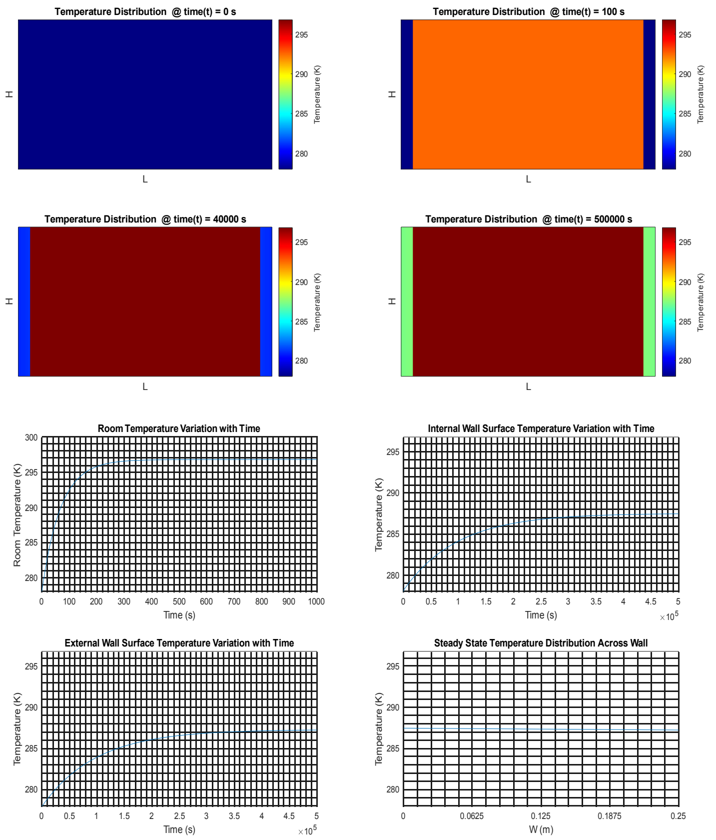

(b) BRICK




(c) CONCRETE




(d) STEEL




From the above plots, we make the following observations:

1. A steady state temperature of 296.8$K$ is reached in the room approximately 520$s$ after the heater is switched on.

2. It takes a longer time for the wall to attain a steady state temperature distribution. This depends on the material of construction.

3. The mid-wall temperature at steady state conditions is approximately 287.5$K$, irrespective of material of construction.

4. Comparison of the temperature distribution for the different wall materials shows that Aircrete block maintains the least external wall temperature gradient with the surroundings as shown in fig. 3 below. This indicates that Aircrete block provides the greatest insulation and minimizes heat loss through the walls.

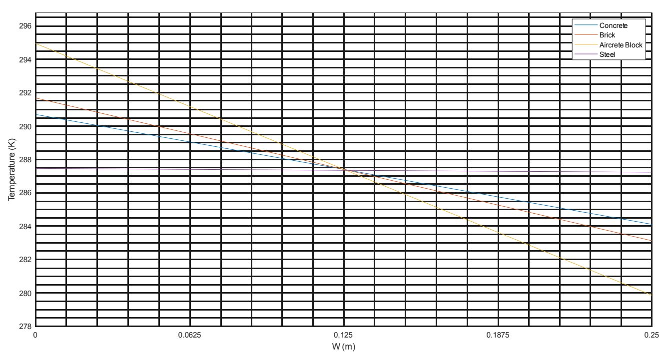



*Fig 3: Steady State Wall Temperature Distribution*

5. In contrast, the temperature distribution across the steel wall is almost uniform at steady state conditions, indicating maximum Temperature gradient with the surroundings, and thus, higher rate of heat loss.

6. The rate of heat loss by convection from the walls to the surroundings is given by:

$$q = hA_e(T_{we} - T_\infty)$$

   It is thus obvious that a higher temperature gradient at the edge of the wall ($T_{we}$ - $T_\infty$) would result in maximum rate of heat loss. The rate of heat loss for the selected wall materials is presented in Table 2 below.

***Table 2**: Comparison of the Rates of Heat Loss through different Wall Materials*

| Material | Steady State External Wall Temperature, $T_{we}$ ($K$) | Rate of Heat Loss, $q$ ($W$) |
|---|---|---|
| Aircrete Block | 279.86 | 429.2 |
| Brick | 283.12 | 1,184.2 |
| Concrete | 284.10 | 1,411.3 |
| Steel | 287.23 | 2,133.6 |

7. The rate of heat loss for an Aircrete Block wall is approximately 6% of the heat supplied by the room heater, whereas a steel wall could be losing up to 30%

## III. CONCLUSIONS

From the foregoing analysis, we can conclude that the use of wall construction materials with low thermal conductivity minimizes the rate of heat loss to the surroundings. This is a key consideration in Building Engineering, especially in locations that require heating or cooling of residential buildings.

Such materials with low thermal conductivity allow intermittent switching on and off of the heat or cooling source to minimize energy consumption whilst maintaining indoor thermal comfort.

Aircrete block is recommended over solid brick and concrete walls as it is able to maintain the room and surroundings at highly disparate temperatures under steady state conditions due to its ability to sustain very wide temperature distribution across a given wall thickness.

Ongoing research on the use of composite materials with even lower thermal conductivities is set to transform the Building Industry.

## REFERENCES

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