

**CONSTRUCTION OF A SUMMATION FORMULA INTERLACED WITH CONTIGUOUS RELATION**

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**Abstract**

The main aim of this paper is to develop a summation formula interlaced with Hypergeometric function and contiguous relation.

2010 MSC NO : 33C05 , 33C20 , 33D15 , 33D50 , 33D60

**Key words:** Contiguous relation, Recurrence relation, Legendre’s duplication formula.

**A Introduction**

**Generalized Gaussian Hypergeometric function of one variable**

$${}_A F_B(a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!} \tag{1}$$

where the parameters  $b_1, b_2, \dots, b_B$  are neither zero nor negative integers and  $A, B$  are non negative integers.

**Contiguous Relation :**

[Abramowitz p.558(15.2.19)]

$$(a-b)(1-z) {}_2F_1(a, b; c; z) = (c-b) {}_2F_1(a, b-1; c; z) + (a-c) {}_2F_1(a-1, b; c; z) \tag{2}$$

**Legendre’s duplication formula :**

$$\Gamma(2z) = 2^{2z-1} \frac{\Gamma(z)\Gamma(z+\frac{1}{2})}{\sqrt{\pi}}, z \neq 0, -1, -2, \dots \tag{3}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = 2^{b-1} \frac{\Gamma\left(\frac{b+1}{2}\right)\Gamma\left(\frac{b}{2}\right)}{\Gamma(b)} \tag{4}$$

**A New Summation Formula**

[Asish et al. p.337(10)]

$${}_2F_1\left(a, b; \frac{a+b-1}{2}; \frac{1}{2}\right) = 2^{b-1} \frac{\Gamma\left(\frac{a+b-1}{2}\right)}{\Gamma(b)} \left[ \frac{\Gamma\left(\frac{b}{2}\right)}{\Gamma\left(\frac{a-1}{2}\right)} \left\{ \frac{b+a-1}{a-1} \right\} + 2 \frac{\Gamma\left(\frac{b+1}{2}\right)}{\Gamma\left(\frac{a}{2}\right)} \right] \tag{5}$$

**Recurrence relation**

$$\Gamma(\zeta+1) = \zeta \Gamma(\zeta) \tag{6}$$

**B MAIN SUMMATION FORMULAE**

For the formula  $a \neq b$

$${}_2F_1\left(a, b; \frac{a+b-37}{2}; \frac{1}{2}\right) = 2^{b-1} \frac{\Gamma\left(\frac{a+b-37}{2}\right)}{(a-b)\Gamma(b)} \left[ \frac{\Gamma\left(\frac{b}{2}\right)}{\Gamma\left(\frac{a-37}{2}\right)} \left\{ \frac{-8200794532637891559375a}{\prod_{\Psi=1}^{19} \{a-(2\Psi-1)\}} \right\} \right]$$

$$\begin{aligned}
 & \frac{20125013723397976152375a^2 - 19688993487602867898225a^3 + 10792700030471840300745a^4}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-3824294822931302783964a^5 + 946995223404049011324a^6 - 171930790626988570804a^7}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{23615262213846406804a^8 - 2505874787291646498a^9 + 208251057899323218a^{10} - 13663776163658478a^{11}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{710084079834558a^{12} - 29186718196012a^{13} + 942715036492a^{14} - 23625216132a^{15} + 449681892a^{16}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-6278151a^{17} + 60591a^{18} - 361a^{19} + a^{20} + 8200794532637891559375b - 29248961707347868727325a^2b}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{37851190175224311447870a^3b - 21940789525204753208628a^4b + 8502383149162038952848a^5b}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-2040060627807052600148a^6b + 399455532659455476552a^7b - 51162010304921997390a^8b}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{5906444348989582320a^9b - 442904964889302630a^{10}b + 32180938804379508a^{11}b - 1452739207922756a^{12}b}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{67865112823536a^{13}b - 1806276624516a^{14}b + 53481117768a^{15}b - 767573001a^{16}b + 13527696a^{17}b}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-79781a^{18}b + 702a^{19}b - 20125013723397976152375b^2 + 29248961707347868727325ab^2}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-19800075528365005357848a^3b^2 + 16622611834183848417132a^4b^2 - 6266048334316896142212a^5b^2}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{1836520912680391333656a^6b^2 - 307201532349114442312a^7b^2 + 48921372698387829926a^8b^2}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-4437363066377882498a^9b^2 + 434677894485480360a^{10}b^2 - 22693604020866792a^{11}b^2}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{1437536850798412a^{12}b^2 - 42900139812068a^{13}b^2 + 1777083009832a^{14}b^2 - 27936753208a^{15}b^2}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{738751065a^{16}b^2 - 4688307a^{17}b^2 + 73112a^{18}b^2 + 19688993487602867898225b^3}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-37851190175224311447870ab^3 + 19800075528365005357848a^2b^3 - 4503750265045373716452a^4b^3}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{2709175596139093913000a^5b^3 - 690717883494142312536a^6b^3 + 159806170940925510560a^7b^3}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-18693533365324290330a^8b^3 + 2474362256642528252a^9b^3 - 155733021266823000a^{10}b^3}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{13083648862273600a^{11}b^3 - 452888078707428a^{12}b^3 + 25089085659400a^{13}b^3 - 445972598760a^{14}b^3}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{16321815328a^{15}b^3 - 115123983a^{16}b^3 + 2686866a^{17}b^3 - 10792700030471840300745b^4}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{21940789525204753208628ab^4 - 16622611834183848417132a^2b^4 + 4503750265045373716452a^3b^4}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-435288313904230106684a^5b^4 + 199265968138644164212a^6b^4 - 34736511039266478156a^7b^4}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{6539600328616295798a^8b^4 - 525114022520478820a^9b^4 + 58775067343921500a^{10}b^4}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-2442419156864596a^{11}b^4 + 177688704115832a^{12}b^4 - 3665316008340a^{13}b^4 + 178310115100a^{14}b^4}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-1428718148a^{15}b^4 + 46142811a^{16}b^4 + 3824294822931302783964b^5 - 8502383149162038952848ab^5}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{6266048334316896142212a^2b^5 - 2709175596139093913000a^3b^5 + 435288313904230106684a^4b^5}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-20099620842189428540a^6b^5 + 7294542947922180808a^7b^5 - 858122871970174668a^8b^5}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{134499059719895728a^9 b^5 - 7068114134989908a^{10}b^5 + 678335525811944a^{11}b^5 - 16734032518476a^{12}b^5}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{1062990012896a^{13}b^5 - 9882951156a^{14}b^5 + 423830264a^{15}b^5 - 946995223404049011324b^6}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{2040060627807052600148ab^6 - 1836520912680391333656a^2b^6 + 690717883494142312536a^3b^6}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-199265968138644164212a^4b^6 + 20099620842189428540a^5b^6 - 467892286162441712a^7b^6}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{138686382323023964a^8b^6 - 10538184531582708a^9b^6 + 1399987726233096a^{10}b^6 - 43367983002344a^{11}b^6}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{3609666033428a^{12}b^6 - 40010466076a^{13}b^6 + 2234741392a^{14}b^6 + 171930790626988570804b^7}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-399455532659455476552ab^7 + 307201532349114442312a^2b^7 - 159806170940925510560a^3b^7}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{34736511039266478156a^4b^7 - 7294542947922180808a^5b^7 + 467892286162441712a^6b^7}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-5502741133253364a^8b^7 + 1362724070798376a^9b^7 - 60417876724920a^{10}b^7 + 6902195972256a^{11}b^7}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-95535194508a^{12}b^7 + 6962078952a^{13}b^7 - 23615262213846406804b^8 + 51162010304921997390ab^8}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-48921372698387829926a^2b^8 + 18693533365324290330a^3b^8 - 6539600328616295798a^4b^8}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{858122871970174668a^5b^8 - 138686382323023964a^6b^8 + 5502741133253364a^7b^8 - 30542424219450a^9b^8}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{6433836899490a^{10}b^8 - 126443639790a^{11}b^8 + 12570420330a^{12}b^8 + 2505874787291646498b^9}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-5906444348989582320ab^9 + 4437363066377882498a^2b^9 - 2474362256642528252a^3b^9}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{525114022520478820a^4b^9 - 134499059719895728a^5b^9 + 10538184531582708a^6b^9 - 1362724070798376a^7b^9}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{30542424219450a^8b^9 - 62359143990a^{10}b^9 + 11338026180a^{11}b^9 - 208251057899323218b^{10}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{442904964889302630ab^{10} - 434677894485480360a^2b^{10} + 155733021266823000a^3b^{10}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-58775067343921500a^4b^{10} + 7068114134989908a^5b^{10} - 1399987726233096a^6b^{10} + 60417876724920a^7b^{10}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-6433836899490a^8b^{10} + 62359143990a^9b^{10} + 13663776163658478b^{11} - 32180938804379508ab^{11}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{22693604020866792a^2b^{11} - 13083648862273600a^3b^{11} + 2442419156864596a^4b^{11} - 678335525811944a^5b^{11}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{43367983002344a^6b^{11} - 6902195972256a^7b^{11} + 126443639790a^8b^{11} - 11338026180a^9b^{12}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-710084079834558b^{12} + 1452739207922756ab^{12} - 1437536850798412a^2b^{12} + 452888078707428a^3b^{12}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-177688704115832a^4b^{12} + 16734032518476a^5b^{12} - 3609666033428a^6b^{12} + 95535194508a^7b^{12}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-12570420330a^8b^{12} + 29186718196012b^{13} - 67865112823536ab^{13} + 42900139812068a^2b^{13}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-25089085659400a^3b^{13} + 3665316008340a^4b^{13} - 1062990012896a^5b^{13} + 40010466076a^6b^{13}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-6962078952a^7b^{13} - 942715036492b^{14} + 1806276624516ab^{14} - 1777083009832a^2b^{14} + 445972598760a^3b^{14}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{-178310115100a^4b^{14} + 9882951156a^5b^{14} - 2234741392a^6b^{14} + 23625216132b^{15} - 53481117768ab^{15}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{27936753208a^2b^{15} - 16321815328a^3b^{15} + 1428718148a^4b^{15} - 423830264a^5b^{15} - 449681892b^{16}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{767573001ab^{16} - 738751065a^2b^{16} + 115123983a^3b^{16} - 46142811a^4b^{16} + 6278151b^{17} - 13527696ab^{17}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \\
 & + \frac{4688307a^2b^{17} - 2686866a^3b^{17} - 60591b^{18} + 79781ab^{18} - 73112a^2b^{18} + 361b^{19} - 702ab^{19} - b^{20}}{\prod_{\Psi=1}^{19}\{a-(2\Psi-1)\}} \} \\
 & + \frac{\Gamma\left(\frac{b+1}{2}\right)}{\Gamma\left(\frac{a-36}{2}\right)} \left\{ \frac{13097439371123680592250a - 22153940827063831976400a^2 + 17925525907364074526550a^3}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \right. \\
 & + \frac{-7567822489717658616192a^4 + 2458271220624423404328a^5 - 479271213946622028480a^6}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{84654065201507029112a^7 - 9092335240513010304a^8 + 982146241369204236a^9 - 62317491573823200a^{10}}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{4336214993731764a^{11} - 165231363762816a^{12} + 7512309428744a^{13} - 166874823360a^{14} + 4857736536a^{15}}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{4336214993731764a^{11} - 165231363762816a^{12} + 7512309428744a^{13} - 166874823360a^{14} + 4857736536a^{15}}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-56930688a^{16} + 986442a^{17} - 4560a^{18} + 38a^{19} - 13097439371123680592250b + 23139738814018920063390a^2b}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-21516156363100184047872a^3b + 11827389742977955137336a^4b - 3325546317323479116032a^5b}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{832659328882303559224a^6b - 114754994980386645248a^7b + 16598022165655290516a^8b}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-1276900600798194944a^9b + 116930668507644420a^{10}b - 5224908392674048a^{11}b}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{314648876615320a^{12}b - 8027640199936a^{13}b + 319151847768a^{14}b - 4249253632a^{15}b + 108006678a^{16}b}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-569088a^{17}b + 8398a^{18}b + 22153940827063831976400b^2 - 23139738814018920063390ab^2}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{9933711099522196655760a^2b^2 - 5738820296938637651392a^4b^2 + 2322425070729799904856a^5b^2}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-448268772128253649664a^6b^2 + 89755345390166601392a^7b^2 - 8730337031956713888a^8b^2}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{1055369472607482604a^9b^2 - 56752913346162176a^{10}b^2 + 4458667436531568a^{11}b^2 - 133022238102208a^{12}b^2}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{6976042787224a^{13}b^2 - 106790311680a^{14}b^2 + 3716282960a^{15}b^2 - 22372272a^{16}b^2 + 493506a^{17}b^2}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-17925525907364074526550b^3 + 21516156363100184047872ab^3 - 9933711099522196655760a^2b^3}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{1600403274473963897496a^4b^3 - 607219120610904465664a^5b^3 + 190415570648151178512a^6b^3}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-25391094996880797184a^7b^3 + 4176061103280434940a^8b^3 - 280686885125320960a^9b^3}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{28815447847634448a^{10}b^3 - 1028399437485056a^{11}b^3 + 69991902775704a^{12}b^3 - 1250170208000a^{13}b^3}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{57384877680a^{14}b^3 - 397729280a^{15}b^3 + 12118314a^{16}b^3 + 7567822489717658616192b^4}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-11827389742977955137336ab^4 + 5738820296938637651392a^2b^4 - 1600403274473963897496a^3b^4}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{116886064094067881704a^5b^4 - 29718620638310828224a^6b^4 + 7483938508477753608a^7b^4}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-678604057567962240a^8b^4 + 93576506319484760a^9b^4 - 4136807371336640a^{10}b^4 + 365316837457208a^{11}b^4}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-7769044890880a^{12}b^4 + 461716368120a^{13}b^4 - 3728712000a^{14}b^4 + 150391384a^{15}b^4}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{\prod_{\Omega=1}^{18}\{a-2\Omega\}}
 \end{aligned}$$

$$\begin{aligned}
 & - 2458271220624423404328b^5 + 3325546317323479116032ab^5 - 2322425070729799904856a^2b^5 \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{607219120610904465664a^3b - 116886064094067881704a^4b^5 + 4258621100102048680a^6b^5} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-720221439853014528a^7b^5 + 149549567547044808a^8b^5 - 8809810923513600a^9b^5 + 1035181358595384a^{10}b^5} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-27078647022336a^{11}b^5 + 2078656264776a^{12}b^5 - 19917058560a^{13}b^5 + 1040310648a^{14}b^5} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{479271213946622028480b^6 - 832659328882303559224ab^6 + 448268772128253649664a^2b^6} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-190415570648151178512a^3b^6 + 29718620638310828224a^4b^6 - 4258621100102048680a^5b^6} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{80696899873601440a^7b^6 - 8721445194990528a^8b^6 + 1523988869787384a^9b^6 - 52617684863744a^{10}b^6} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{5338008453232a^{11}b^6 - 62572758976a^{12}b^6 + 4211628008a^{13}b^6 - 84654065201507029112b^7} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{114754994980386645248ab^7 - 89755345390166601392a^2b^7 + 25391094996880797184a^3b^7} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-7483938508477753608a^4b^7 + 720221439853014528a^5b^7 - 80696899873601440a^6b^7 + 791050371234552a^8b^7} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-49480910519040a^9b^7 + 7395423461712a^{10}b^7 - 113577444864a^{11}b^7 + 10056336264a^{12}b^7} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{9092335240513010304b^8 - 16598022165655290516ab^8 + 8730337031956713888a^2b^8} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-4176061103280434940a^3b^8 + 678604057567962240a^4b^8 - 149549567547044808a^5b^8} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{8721445194990528a^6b^8 - 791050371234552a^7b^8 + 3728705795100a^9b^8 - 102742754400a^{10}b^8} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{13309856820a^{11}b^8 - 982146241369204236b^9 + 1276900600798194944ab^9 - 1055369472607482604a^2b^9} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{280686885125320960a^3b^9 - 93576506319484760a^4b^9 + 8809810923513600a^5b^9 - 1523988869787384a^6b^9} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{49480910519040a^7b^9 - 3728705795100a^8b^9 + 6564120420a^{10}b^9 + 62317491573823200b^{10}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-116930668507644420ab^{10} + 56752913346162176a^2b^{10} - 28815447847634448a^3b^{10}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{4136807371336640a^4b^{10} - 1035181358595384a^5b^{10} + 52617684863744a^6b^{10} - 7395423461712a^7b^{10}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{102742754400a^8b^{10} - 6564120420a^9b^{10} - 4336214993731764b^{11} + 5224908392674048ab^{11}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-4458667436531568a^2b^{11} + 1028399437485056a^3b^{11} - 365316837457208a^4b^{11} + 27078647022336a^5b^{11}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-5338008453232a^6b^{11} + 113577444864a^7b^{11} - 13309856820a^8b^{11} + 165231363762816b^{12}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-314648876615320ab^{12} + 133022238102208a^2b^{12} - 69991902775704a^3b^{12} + 7769044890880a^4b^{12}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-2078656264776a^5b^{12} + 62572758976a^6b^{12} - 10056336264a^7b^{12} - 7512309428744b^{13}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{8027640199936ab^{13} - 6976042787224a^2b^{13} + 1250170208000a^3b^{13} - 461716368120a^4b^{13} + 19917058560a^5b^{13}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{-4211628008a^6b^{13} + 166874823360b^{14} - 319151847768ab^{14} + 106790311680a^2b^{14} - 57384877680a^3b^{14}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{3728712000a^4b^{14} - 1040310648a^5b^{14} - 4857736536b^{15} + 4249253632ab^{15} - 3716282960a^2b^{15}} \\
 & + \frac{\prod_{\Omega=1}^{18}\{a-2\Omega\}}{}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{397729280a^3b^{15} - 150391384a^4b^{15} + 56930688b^{16} - 108006678ab^{16} + 22372272a^2b^{16} - 12118314a^3b^{16}}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \\
 & + \frac{-986442b^{17} + 569088ab^{17} - 493506a^2b^{17} + 4560b^{18} - 8398ab^{18} - 38b^{19}}{\prod_{\Omega=1}^{18}\{a-2\Omega\}} \} ] \quad (7)
 \end{aligned}$$

**C DERIVATION OF THE MAIN SUMMATION FORMULA:**

Putting  $c = \frac{a+b-37}{2}$  and  $z = \frac{1}{2}$  in equation (2), we get

$$(a-b) {}_2F_1\left(a, b; \frac{a+b-37}{2}; \frac{1}{2}\right) = (a-b-37) {}_2F_1\left(a, b-1; \frac{a+b-37}{2}; \frac{1}{2}\right) + (a-b+37) {}_2F_1\left(a-1, b; \frac{a+b-37}{2}; \frac{1}{2}\right)$$

Now using the same parallel method of Ref [4], the summation formula is obtained.

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