IOSR Journal of Engineering
May. 2012, Vol. 2(5) pp: 1111-1114

Some Results of half Canonical Cosine and Sine Transforms

S. B. Chavhan, V. C. Borkar
Department of mathematics Yeshwant Mahavidyalaya Nanded-432016

Abstract: In this paper, have study new results of half canonical cosine and sine transforms of generalized function.

Keyword: Canonical transform, half canonical cosine transform, half canonical sine transform, testing function space.

1. Introduction: Now a days, fractional integral transforms play an important role in signal processing, image reconstruction, pattern recognition, accostic signal processing [1],[2].A new generalized integral transform was obtained by Zayed[4]. Bhosale and Chaudhary [3], had extended fractional Fourier transform to the distribution of compact support. Definitions half canonical cosine and half canonical sine transform as

$$\{HCCTf(t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \qquad \text{for } b\neq 0$$

And

$$\{HCCTf(t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty (-i)\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} f(t) \quad \text{for } b\neq 0$$

Notation and terminology as per Zemanian[5],[6].This paper is organized as section 2 definition of testing function space .Section 3 definition of half canonical cosine transform and its inversion theorem. Section 4 definition of half canonical sine transform and its inversion theorem. Lastly conclusion is stated.

2. Definition of testing function space*E*:

An infinitely differentiable complex valued function ϕ on R^n belongs to $E(R^n)$, if for each compact set. $I\subset s_a$ where $s_a=\{t:\in R^n, \quad |t|\le a, a>0\}$ and for $k\in R^n$,

$$\gamma_{E,k}\phi(t)=\sup_{t\in I}\left|D^k\phi(t)\right|<\infty \qquad \text{k=0,1,2,3.....}$$

Note that space E is complete and a Frechet space,let E' denotes the dual space of E

3. Definition of half canonical cosine transform:

Half canonical cosine transform of $f(t)$ is given by

$$\{HCCTf(t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \qquad \text{for } b\neq 0$$

$$=\sqrt{d}\,.e^{\frac{i}{2}cds^2} f(d.s) \qquad \text{for b = 0}$$

Where, $$K_{HC}(t,s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}$$

Hence half canonical cosine transform of $f(t)$ is defined as

$$\{HCCT\ f(t)\}(s)=\langle f(t),K_{HC}f(t,s)\rangle$$

Since the range of integration for the half canonical cosine transform is just $[0,\infty]$ and not $(-\infty,\infty)$ using half canonical cosine transform is more convenient than using the canonical transform to deal with the even function.

3.1. Inverse of half canonical cosine transform:

Theorem 3.1: If $\{HCCT\, f(t)\}(s)$ is the half canonical cosine transform of $f(t)$ then

$$f(t)=e^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2}\sqrt{\frac{\pi i}{2b}}\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\{HCCT\, f(t)\}(s)\,ds$$

Proof: Using definition of half canonical cosine transform as

$$\{HCCTf(t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}$$

$$f(s)\sqrt{\frac{\pi ib}{2}}e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}=\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt$$

where, $f(s)=\{HCST\, f(t)\}(s)$

$$\therefore g(t)=e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t) \quad \text{and} \qquad C_1(s)=\int_0^\infty g(t)\cos\left(\frac{s}{b}t\right)dt$$

$$C_1(s)=\{HCCTg(t)\}\left(\frac{s}{b}\right)$$

where $\{HCCTg(t)\}\left(\frac{s}{b}\right)$ is half canonical cosine transform of $g(t)$. Half canonical cosine transform $g(t)$ with argument $\frac{s}{b}=\eta$ and $\frac{ds}{b}=d\eta$

$$C_1(s)=\{HCCTg(t)\}(\eta)$$

By invoking inversion formula we get

$$g(t)=\int_0^\infty \cos(\eta t)C_1(s)\,d\eta$$

$$g(t)=\int_0^\infty \cos\left(\frac{s}{b}t\right)C_1(s)\frac{ds}{b}$$

$$f(t)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}=\frac{1}{b}\int_0^\infty \cos\left(\frac{s}{b}t\right)\frac{\sqrt{\pi ib}}{\sqrt{2}}f(s)e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}ds$$

$$f(t)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}=\frac{\sqrt{\pi ib}}{\sqrt{2}}\times\frac{1}{b}\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}f(s)\,ds$$

$$f(t)=e^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2}\frac{\sqrt{\pi i}}{\sqrt{2b}}\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}\{HCCT\, f(t)\}(s)\,ds$$

4. Definition half canonical sine transform:

Half canonical sine transform of $f(t)$ is given by

$$\{HCCTf(t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty(-i)\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t) \qquad \text{for } b\neq 0$$

$$=\sqrt{d}\,.e^{\frac{i}{2}cds^2}f(d.s) \qquad \text{for b = 0}$$

where, $$K_{HS}(t,s)=\sqrt{\frac{2}{\pi ib}}\,e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}(-i)\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}$$

Hence half canonical sine transform of $f(t)$ is defined as

$$\{HCST\ f(t)\}(s)=\langle f(t),K_{HS}f(t,s)\rangle$$

Since the range of integration for the half canonical sine transform is just $[0,\infty]$ and not $(-\infty,\infty)$ using half canonical sine transform is more convenient than using the canonical transform to deal with the even function.

4.1 Inverse of half canonical sine transform:

Theorem4.1: If $\{HCST\ f(t)\}(s)$ is the half canonical sine transform of $f(t)$ then

$$f(t)=-i\sqrt{\frac{\pi i}{2b}}e^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2}\int_0^\infty \sin\left(\frac{s}{b}t\right)e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}\{HCST\ f(t)\}(s)\,ds$$

Proof: Using definition of half canonical sine transform as

$$\{HCST\ f(t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty(-i)\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt$$

$$F(s)=(-i)\sqrt{\frac{2}{\pi ib}}\,e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt$$

where, $f(s)=\{HCST\ f(t)\}(s)$

$$F(s)\sqrt{\frac{\pi ib}{2}}\,e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}=(-i)\int_0^\infty\sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)\,dt$$

$$C_1(s)=(-i)\int_0^\infty\sin\left(\frac{s}{b}t\right)g(t)$$

where, $C_1(s)=F(s)\sqrt{\frac{\pi ib}{2}}\,e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}$

$\therefore g(t)=e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)$ and $C_1(s)=\{HCSTg(t)\}\left(\frac{s}{b}\right)$

where $\{HCSTg(t)\}\left(\frac{s}{b}\right)$ is half canonical sine transform of $g(t)$. Half canonical sine transform $g(t)$ with argument $\frac{s}{b}=\eta$ and $\frac{ds}{b}=d\eta$

$$C_1(s)=\{HCSTg(t)\}(\eta)$$

By inversion formula we get

$$g(t)=(-i)\int_0^\infty\sin(\eta t)C_1(s)\,d\eta$$

$$g(t)=(-i)\int_0^\infty\sin\left(\frac{s}{b}t\right)C_1(s)\frac{ds}{b}$$

$$g(t)=(-i)\int_0^\infty\sin\left(\frac{s}{b}t\right)\sqrt{\frac{\pi ib}{2}}f(s)e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}\frac{ds}{b}$$

$$f(t)=(-i)\sqrt{\frac{\pi i}{2b}}e^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2}\int_0^\infty\sin\left(\frac{s}{b}t\right)e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}f(s)\,ds$$

$$f\left(t\right)=\left(-i\right)\sqrt{\frac{\pi i}{2b}}e^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2}\int_0^\infty \sin\left(\frac{s}{b}t\right)e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}\left\{HCST\ f\left(t\right)\right\}\left(s\right)ds$$

5. Conclusion:

In this paper half canonical cosine and half canonical sine transforms is generalized in the form the distributional sense, we have inversion theorem for this transforms are proved.

References:

[1] **Alieva Tatiana. and Bastianas Martin J.,:** "On Fractional Fourier transform moments," IEEE signal processing letters, Vol. 7, No.11, Nov. 2000.

[2] **Alieva Tatiana. and Bastiaans Martian J.,:** "Wigner Distribution and Fractional Fourier Transform for 2-Dimentional Symmetric Beams", JOSA A, Vol.17, No.12, P.2319 – 2323, Dec.2000.

[3] **Bhosale B.N. and Choudhary M.S.,: "**Fractional fourier transform of distributions of,compact support,"Bull.Cal.Math.Soc.,Vol.94,No.5,349-358,2002.

[4] **Zayed A. : "**A convolution and product theorem for the fractional Fourier transform", IEEE sign. proc. letters Vol.5, No.4, (1998), 101-103.

[5] **Zemanian A.H**., **:** "Distribution theory and transform analysis", McGraw Hill, New York, (1965).

[6] **Zemanian A.H., : "**Generalized Integral Transform", Inter Science Publisher's New York, (1968).