IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 08, Issue 11 (November. 2018), ||V (I) || PP 01-10

On three Parameter Weighted Quasi Akash Distribution: Properties and Applications

Anwar Hassan, Gulzar Ahmad Shalbaf and Bilal Ahmad Para
Department of Statistics, University of Kashmir, Srinagar (J&K), India
Corresponding Author: Anwar Hassan

Abstract: In this paper, we have introduced a weighted model of the Quasi Akash Distribution and the weight function considered here is $W(x) = x^c$, where the weight parameter is c. We have investigated different characteristics as well as the structural properties of the Weighted Quasi Akash Distribution (WQAD). We have also derived the Moment Generating Function, Characteristic Function, Reliability Function and Hazard Rate function of the introduced model. Finally model has been examined with real life data.

Keywords: Weighted Quasi Akash Distribution, Weighting Technique, Structural Properties and Maximum Likelihood Estimation.

Date of Submission: 08-11-2018 Date of acceptance: 22-11-2018

I. INTRODUCTION

A two parameter Quasi Akash distribution (QAD) having parameters θ and α introduced by Shanker (2016) is defined by its probability density function (pdf)

$$f(x;\theta,\alpha) = \frac{\theta^2}{\alpha\theta+2}(\alpha+\theta x^2)e^{-\theta x} \quad x>0,\theta>0,\alpha>0 \tag{1.1}$$

The corresponding cdf of (1.1) is given by

$$F(x;\theta,\alpha) = 1-\left[1+\frac{\theta x(\theta x+2)}{\alpha\theta+2}\right]e^{-\theta x}\ ,\ x>0,\theta>0,\alpha>0 \tag{1.2}$$

Shanker et al. (2016) have a detailed study on applications of QAD for modeling life time data.

II. WEIGHTED QUASI AKASH DISTRIBUTION (WQAD)

The concept of weighted distributions was given by Fisher (1934) to model the ascertainment bias. This concept was later on developed by Rao (1965) in a unified manner while modeling the statistical data when the standard distributions were not appropriate to record these observations with equal probabilities. Warren (1975) was the first to apply the weighted distributions in connection with sampling wood cells. Patil and Rao (1978) introduced the concept of size biased sampling and weighted distributions by identifying some of the situations where the underlying models retain their form. The statistical interpretation of weighted and size biased distributions was originally identified by Buckland and Cox (1964) in the context of renewal theory. As a result, weighted models were formulated in such situations to record the observations according to some weighted function. Different authors have reviewed and studied the various weighted probability models and illustrated their applications in different fields. Weighted distributions were applied in various research areas related to reliability, biomedicine, ecology and branching processes. For survival data analysis, Jing (2010) introduced the weighted inverse Weibull distribution and beta-inverse Weibull distribution as a new lifetime models. Ayesha, (2017) discussed the Size Biased Lindley Distribution as a new life time distribution and discussed its various statistical properties. Shankar (2017) discussed a Size-Biased Poisson-Shanker Distribution and its applications to handle various count data sets. Recently Shanker & Shukla (2018) discussed a generalized size-biased Poisson-Lindley distribution and Its Applications to model size distribution of freely-forming small group.

Assume X is a non negative random variable with probability density function (pdf) $f(x)$. Let $W(x)$ be the weight function which is a non negative function, then the probability density function of the weighted random variable X_w is given by:

$$f_W(x) = \frac{W(x) f(x)}{E(w(x))}, \quad x > 0,$$

where $w(x)$ be a non-negative weight function and $E(w(x)) = \int w(x) f(x) dx < \infty$.

In this paper, we have considered the weight function as $w(x) = x^c$ to obtain the weighted quasi Akash model. The probability density of weighted Quasi Akash distribution is given as:

$$f_w(x, c, \theta, \alpha) = \frac{x^c f(x, \theta, \alpha)}{E[x^c]},$$

$$f_w(x, c, \theta, \alpha) = \frac{x^c \theta^{(c+2)} (\alpha + \theta x^2) e^{-\theta x}}{c!(\alpha\theta + (c+1)(c+2))}, \qquad x > 0, \theta > 0, c > 0, \alpha > 0 \tag{2.1}$$

where $E(x^c) = \frac{c![\alpha\theta + (c+1)(c+2)]}{\theta^c (\alpha\theta + 2)}$.

The corresponding cdf of weighted Quasi Akash Distribution (WQAD) is obtained as

$$F_w(x; c, \theta, \alpha) = \int_0^x f_w(x; c, \theta, \alpha) dx$$

$$= \int_0^x \frac{x^c \theta^{(c+2)} (\alpha + \theta x^2) e^{-\theta x}}{c!(\alpha\theta + (c+1)(c+2))} dx$$, put $\theta x = t$, $\theta\, dx = dt$,

$as\ x \to 0, t \to 0\ \ and\ \ x \to x, t \to \theta x$, after simplification

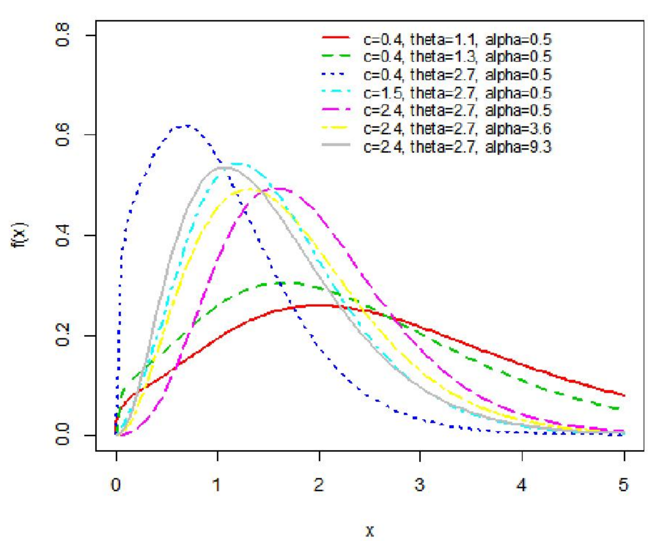
$$F_w(x; c, \theta, \alpha) = \frac{1}{c!(\alpha\theta + (c+1)(c+2))} (\alpha\theta\, \gamma(c+1, \theta x) + \gamma(c+3, \theta x)), \tag{2.2}$$

$$x > 0, \theta > 0, c > 0, \alpha > 0$$

Where c, θ and α are positive parameters and $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ is a lower incomplete gamma function.

Fig. 1 gives the description of some of the possible shapes of pdf plot of weighed Quasi Akash distribution for different values of the parameters c, θ and α. It illustrates that the density function of weighted Quasi Akash distribution is positively skewed. Fig. 2 shows the graphical overview of distribution function of WQAD, which is an increasing function.

Fig.1: pdf plot of Weighted Quasi Akash Distribution



III. SPECIAL CASES OF WQAD.

Case i: If we put $c=0$, then weighted Quasi Akash distribution (2.1) reduces to Quasi Akash distribution with probability density function as:

$$f(x;\theta,\alpha)=\frac{\theta^2}{\alpha\theta+2}(\alpha+\theta x^2)e^{-\theta x} \quad x>0,\theta>0,\alpha>0$$

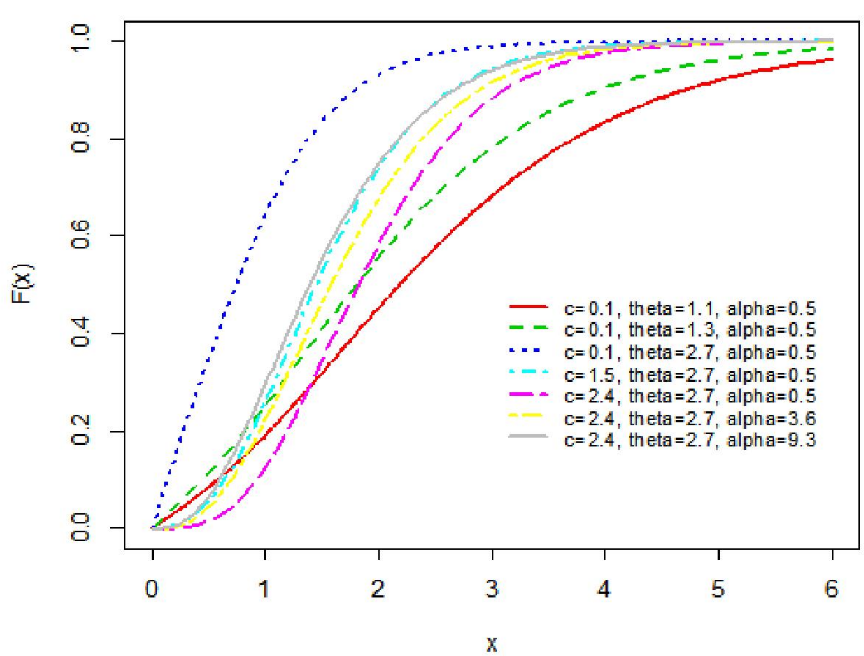
Case ii: if we put c=0 and $\alpha=\theta$, then weighted Quasi Akash distribution (2.1) reduces to Akash distribution with probability density function as:

$$f(x;\theta)=\frac{\theta^3}{\theta^2+2}(1+x^2)e^{-\theta x} \quad ;x>0,\theta>0.$$

Case iii: If we put c= $\alpha=0$, then weighted quasi Akash distribution reduces to gamma distribution. i.e;

$$f(x;\theta)=\frac{\theta^3}{2}x^2 e^{-\theta x} \quad ;x>0,\theta>0.$$

Fig.2: CDF plot of Weighted Quasi Akash Distribution



IV. Moments and Related Properties:

4.1 Moment Generating Function (MGF): The moment generating function of WQAD (2.1) is as follows

$$M_X(t)=E(e^{tx})=\int_0^\infty e^{tx} f_w(x)dx$$

$$=\int_0^\infty e^{tx}\frac{x^c\,\theta^{c+2}\,[\alpha+\theta x^2]}{(c)!\,[\alpha\theta+(c+1)(c+2)]}e^{-\theta x}dx$$

$$M_X(t)=\frac{1}{\alpha\theta+(c+1)(c+2)}\left[\frac{\alpha\theta}{\left(1-\frac{t}{\theta}\right)^{c+1}}+\frac{(c+1)(c+2)}{\left(1-\frac{t}{\theta}\right)^{c+3}}\right]$$

Thus, the moments about origin of WQAD obtained by differentiating with respect to t and equating to zero, we get

$$\mu_1' = \frac{(c+1)[\alpha\,\theta + (c+2)(c+3)]}{\theta\,[\alpha\,\theta + (c+1)(c+2)]}$$

$$\mu_2' = \frac{(c+1)(c+2)[\alpha\,\theta + (c+3)(c+4)]}{\theta^2\;[\alpha\,\theta + (c+1)(c+2)]}$$

Variance $= \mu_2' - (\mu_1')^2$

$$V(x) = \frac{(c+1)(c+2)[\alpha\,\theta + (c+3)(c+4)]}{\theta^2\;[\alpha\,\theta + (c+1)(c+2)]} - \left[\frac{(c+1)[\alpha\,\theta + (c+2)(c+3)]}{\theta[\alpha\theta + (c+1)(c+2)]}\right]^2$$

4.2 Harmonic mean: The harmonic mean for the proposed model is computed as

$$H.M = E\left[\frac{1}{X}\right] = \int_0^\infty \frac{1}{x} f_w(x; c, \alpha, \theta)\, dx$$

$$= \int_0^\infty \frac{1}{x}\;\frac{x^c\,\theta^{c+2}\;[\alpha + \theta\,x^2]}{(c\;)![\alpha\theta + (c+1)(c+2)]}\;e^{-\theta x}\;\;d\,x$$

$$E\left[\frac{1}{X}\right] = \frac{\theta[\alpha\theta + c(c+1)]}{c[\alpha\theta + (c+1)(c+2)]}, \qquad x>0, \theta>0, \alpha>0, c>0.$$

V. RELIABILITY, HAZARD AND REVERSE HAZARD FUNCTIONS:

In this, we have obtained the reliability, hazard rate, reverse hazard rate of the proposed weighted Quasi Akash distribution.

5.1 Reliability function $R(x)$ ***:***

The reliability function is defined as the probability that a system survives beyond a specified time .It can be computed as complement of the cumulative distribution function of the model. The reliability function or the survival function of weighted Quasi Akash distribution is computed as

$$R_w(x, c, \theta, \alpha) = 1 - F_w(x)$$

$$= 1 - \frac{[\alpha\theta\;\;\gamma(c+1, \theta\,x) + \gamma(c+3, \theta\,x)]}{(c)![\alpha\theta + (c+1)(c+2)]} \qquad (5.1.1)$$

$$x>0, \theta>0, c>0,$$

The graphical representation of the reliability function for the weighted Quasi Akash distribution is shown in fig. 3.

5.2 Hazard function:

The hazard function is also known as hazard rate ,defined as the instantaneous failure rate or force of mortality and is given as:

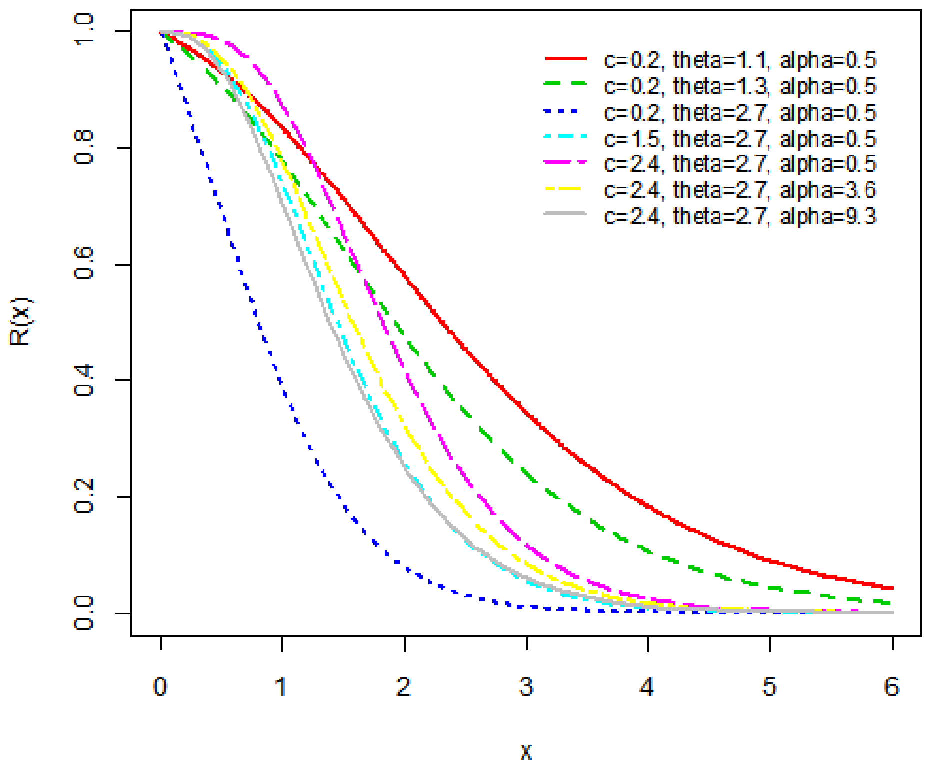
$$H.R = \frac{f_w(x, c, \theta, \alpha)}{R_w(x, c, \theta, \alpha)} = \frac{x^c\,\theta^{c+2}(\alpha + \theta\,x^2)\,e^{-\theta x}}{(c)![\alpha\,\theta + (c+1)(c+2)] - [\alpha\,\theta\;\;\gamma(c+), \theta\,x)\;+\;\;\gamma(c+3, \theta\,x)]}$$

5.3 Reverse Hazard Rate:

The reverse hazard rate of weighted Quasi Akash distribution is given as

$$R.H.R = h(x, c, \theta, \alpha) = \frac{x^c\,\theta^{\,c+2}\,(\alpha + \theta\,x^2)\,e^{-\theta x}}{\alpha\theta\;\;\gamma(c+1, \theta\,x)\;+\gamma(c+3, \theta\,x)}$$

Fig.3: Reliability function plot of Weighted Quasi Akash Distribution



VI. ORDER STATISTICS

Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the ordered statistics of the random sample $X_1, X_2, \ldots, X_n$ drawn from the continuous distribution with cumulative distribution function $F_X(x)$ and probability density function $f_X(x)$, then the probability density function of r^{th} order statistics $X_{(r)}$ is given by:

$$f_{X(r)}(X; c, \alpha, \theta) = \frac{(n)!}{(r-1)!(n-r)!} f(x)\,[F(x)]^{r-1}[1-F(x)]^{n-r} \quad , \qquad r = 1, 2, 3, \ldots, n$$

Using the equations (2.1) and (2.2), the probability density function of rth order statistics of weighted Quasi Akash distribution is given by:

$$f_{w(r)}(X; c, \alpha, \theta) = \frac{(n)!}{(r-1)!(n-r)!} \frac{x^c\ \theta^{\,c+2}(\alpha+\theta x^2)\ e^{-\theta x}}{(c)!\,[\alpha\theta+(c+1)(c+2)]} \left[\frac{\alpha\theta\quad \gamma(c+1,\,\theta x)+\gamma(c+3,\theta x)}{(c)!\,[\alpha\theta+(c+1)(c+2)]}\right]^{r-1} \left[1-\frac{\alpha\theta\ \ \gamma(c+1,\theta x)+\gamma(c+3,\theta x)}{(c)![\alpha\theta+(c+1)(c+2)]}\right]^{n-r}$$

Then the first order weighted Quasi Akash distribution is given by

$$f_{w(1)}(X; c, \alpha, \theta) = n\ \frac{x^c\ \theta^{\,c+2}(\alpha+\theta x^2)\ e^{-\theta x}}{(c)!\,[\alpha\theta+(c+1)(c+2)]} \left[1-\frac{\alpha\theta\ \ \gamma(c+1,\theta x)+\gamma(c+3,\theta x)}{(c)![\alpha\theta+(c+1)(c+2)]}\right]^{n-1}$$

And the pdf of nth order weighted Quasi Akash distribution is given by:

$$f_{w(n)}(X;c,\alpha,\theta)=n\ \frac{x^c\ \theta^{\ c+2}(\alpha+\theta x^2)\ \ e^{-\theta x}}{(c)!\,[\alpha\theta+(c+1)(c+2)]}\left[\frac{\alpha\theta\quad\gamma(c+1,\,\theta x)+\gamma(c+3,\theta x)}{(c)!\,[\alpha\theta+(c+1)(c+2)]}\right]^{n-1}$$

VII. RANDOM NUMBER GENERATION FROM WQLD

In this section, we discuss the random number generation from WQAD. We generally employ Inverse cdf Method for the generation of random numbers from a particular distribution. In this method the random numbers from a particular distribution are generated by solving the equation obtained on equating the cdf of a distribution to a number *u*. The number *u* is itself being generated from $U(0,1)$. Thus following the same steps for the purpose of random number generation from the WQAD, we will proceed as

$F_w(x\ ;c,\alpha,\theta)=u.$

Where $F_w(x\ ;c,\alpha,\theta)$ is the cdf of WQAD and u is the random number generated from uniform distribution i.e., $U(0,1)$.

$$\frac{1}{c!(\alpha\theta+(c+1)(c+2))}\alpha\theta\gamma(c+1,\theta x)+\gamma(c+3,\theta x)=u \qquad (7.1)$$

On solving the equation (7.1) for x, we will obtain the required random number from the WQAD. Main problem, which is being faced while using this method of generating the random numbers is to solve the equations which are usually complex and complicated. In order to overcome such hindrance, we use softwares like MATLAB, Mathematica, SAS or R for solving such a complex equation.

VIII. METHOD OF MAXIMUM LIKELIHOOD ESTIMATION OF WEIGHTED QUASI AKASH DISTRIBUTION

This is one of the most useful method for estimating the different parameters of the distribution. Let $X_1, X_2, X_3....X_n$ be the random sample of size n drawn from weighted Quasi Akash distribution, then the likelihood function of weighted Quasi Akash distribution is given as:

$$L(x\mid c,\theta,\alpha)=\prod_{i=1}^{n}f(x;c,\theta,\alpha)=\prod_{i=1}^{n}\frac{x^c\theta^{(c+2)}(\alpha+\theta x^2)e^{-\theta x}}{c!(\alpha\theta+(c+1)(c+2))}$$

The log likelihood function becomes:

$$\log L=c\log\sum_{i=1}^{n}x_i+n(c+2)\log\theta+\log\sum\theta+\log\sum(\alpha+\theta x_i^{\ 2})-\theta\sum_{i=1}^{n}x_i-n\log(c)!-n\log(c+1)(c+2). \qquad (8.1)$$

Differentiating the log-likelihood function with respect to *θ*, α and c.This is done by partially differentiate (8.1) with respect to *θ*, α and c and equating the result to zero, we obtain the following normal equations,

$$\frac{\partial\log L}{\partial\theta}=\frac{n(c+2)}{\theta}+\frac{\sum_{i=1}^{n}x_i^{\ 2}-\sum_{i=1}^{n}x_i}{\sum_{i=1}^{n}(\alpha+\theta x_i^{\ 2})}=0 \qquad (8.2)$$

$$\frac{\partial\log L}{\partial\alpha}=\frac{n}{\sum_{i=1}^{n}(\alpha+\theta x_i^{\ 2})}=0 \qquad (8.3)$$

$$\frac{\partial\log L}{\partial c}=\frac{1}{\log(c+1)}-\frac{1}{2(c+1)}-\frac{n(2c+3)}{(c+1)(c+2)}=0 \qquad (8.4)$$

By solving equations (8.1), (8.2) and (8.3), the maximum likelihood estimators of the parameters of the weighted Quasi Akash distribution are obtained using the numerical methods like Newton Raphson method.
We can compute the maximized unrestricted and restricted log likelihoods to construct the likelihood ratio (LR) statistics for testing the significance of weighted parameter of the proposed model. For example, we can use LR test to check whether the fitted weighted Quasi Akash distribution for a given data set is statistically "superior" to the fitted Quasi Akash distribution. In any case, hypothesis tests of the type $H_0 : \Theta = \Theta_0$ versus $H_1 : \Theta \neq \Theta_0$ can be performed using LR statistics. In this case, the LR statistic for testing H_0 versus H_1 is $\omega = 2(L(\hat{\Theta}) - L(\hat{\Theta}_0))$ where $\hat{\Theta}$ and $\hat{\Theta}_0$ are the MLEs under H_1 and H_0. The statistic ω is asymptotically ($as\ n \to \infty$) distributed as χ_k^2, with k degrees of freedom which is equal to the difference in dimensionality of $\hat{\Theta}$ and $\hat{\Theta}_0$. H_0 will be rejected if the LR-test p-value is <0.05 at 95% confidence level.

8.1 Simulation Study of ML estimators of WQAD

The performance of ML estimators of WQAD for different sample sizes (n=25,75,100,150,300,500) has been discussed in this section. Inverse cdf technique for data simulation for WQAD using R software has been employed. The process was repeated 500 times for calculation of bias, variance and MSE as given values in table 1. For two random parameter combinations of WQAD, we observe decreasing trend in average bias, variance and MSE as we increase the sample size. Hence, the performance of ML estimators is quite well, consistent in case of WQAD.

Table 1: Simulation Study of ML estimators for weighted Quasi Akash Distribution

Parameter	n	$c = 0.5, \alpha = 0.7, \theta = 0.8$			$c = 1.6, \alpha = 0.9, \theta = 1.9$		
		Bias	Variance	MSE	Bias	Variance	MSE
α	25	0.834899	0.211666	0.908722	0.994085	0.441057	1.429262
β		0.987854	0.121255	1.097111	1.02354	0.23541	1.283044
θ		0.099080	0.013196	0.023013	0.222888	0.020859	0.070538
α	75	0.683829	0.120782	0.588404	0.451215	0.226023	0.429618
β		0.865970	0.116740	0.866644	0.768745	0.511739	1.102708
θ		0.086580	0.012579	0.020075	0.211215	0.003211	0.047823
α	100	0.551480	0.096800	0.400930	0.33123	0.092468	0.202181
β		0.741917	0.084289	0.634729	0.623027	0.198717	0.586879
θ		0.053770	0.014081	0.016972	0.152203	0.003214	0.02638
α	150	0.449400	0.051392	0.253352	0.272492	0.009101	0.083353
β		0.681953	0.043215	0.508275	0.4114	0.111809	0.281058
θ		0.060080	0.005101	0.008711	0.154116	0.004113	0.027865
α	300	0.288480	0.007064	0.090285	0.11823	0.002912	0.01689
β		0.644068	0.011107	0.425930	0.201545	0.026483	0.067103
θ		0.066488	0.001229	0.005650	0.110114	0.004106	0.016231
α	500	0.185541	0.000280	0.034705	0.09878	0.000136	0.009894
β		0.522348	0.001699	0.274546	0.135097	0.001537	0.019788
θ		0.011522	0.000101	0.000234	0.090206	0.001208	0.009345

IX. MODEL COMPARISON BASED ON SIMULATED DATA FROM WQAD

In order to compare the Weighted Model with the base model on the basis of simulated data. We proceed by simulating a data from WQAD using inverse cdf technique discussed in the section 5.4. Two sets of random parameter combinations with sample sizes (n=10, 25, 100,300,500) have been taken into consideration for data generation. It is evident from the table 2 and table 3, that weighted parameter plays a highly significant role for large samples. Even though in small samples, the AIC, AICC, BIC and Negative Loglikelihood values are also minimum in case of Weighted model but the likelihood ratio test reveals that the role of weighted parameter exhibits a highly significant role in case of large samples only. LR statistic for testing H_0 versus H_1 is $\psi = 2(L(\hat{\Theta}) - L(\hat{\Theta}_0))$, where $\hat{\Theta}$ and $\hat{\Theta}_0$ are the MLEs under H_1 and H_0. The statistic ψ is asymptotically ($as\ n \to \infty$) distributed as χ_k^2, with k degrees of freedom which is equal to the difference in dimensionality of $\hat{\Theta}$ and $\hat{\Theta}_0$. H_0 will be rejected if the LR-test p-value is <0.01 (or LR Statistic value >6.635) at 99% confidence level

Table 2: Comparison of WQAD and QAD Based On Simulated Data.

$\hat{c} = 0.5, \hat{\alpha} = 1.5, \hat{\theta} = 1.0$				Parameter Estimates		Likelihood Ratio Statistic
Criterion	WQAD	QAD	Sample Size (n)	WQAD	QAD	
-logL	14.55305	14.57047	10	$\hat{c}$ =0.168 (0.931) $\hat{\alpha}$ =0.325 (0.668) $\hat{\theta}$ =1.46 (0.497)	$\hat{\alpha}$ =0.024 (0.321) $\hat{\theta}$ =1.398 (0.289)	0.0348
AIC	35.10609	33.14094				
AICC	39.10609	34.85522				
BIC	36.01385	33.74611				
-logL	48.08946	48.83002	25	$\hat{c}$ =0.991 (0.781) $\hat{\alpha}$ = 2.79 (1.012) $\hat{\theta}$ =1.07 (0.233)	$\hat{\alpha}$ =0.307 (0.427) $\hat{\theta}$ =0.896 (0.134)	1.4811
AIC	102.1789	101.66				
AICC	103.3218	102.2055				
BIC	105.8355	104.0978				
-logL	197.5296	200.0704	100	$\hat{c}$ =1.135 (0.321) $\hat{\alpha}$ =10.96 (2.354) $\hat{\theta}$ = 0.959 (0.13)	$\hat{\alpha}$ =0.222 (0.191) $\hat{\theta}$ =0.926 (0.071)	5.0815
AIC	401.0592	404.1408				
AICC	401.3092	404.2645				
BIC	408.8747	409.3511				
-logL	574.8448	580.1859	300	$\hat{c}$ =0.858 (0.246) $\hat{\alpha}$ =5.326 (1.966) $\hat{\theta}$ =1.001 (0.084)	$\hat{\alpha}$ =0.428 (0.185) $\hat{\theta}$ =0.934 (0.052)	10.682
AIC	1155.69	1164.372				
AICC	1155.771	1164.412				
BIC	1166.801	1171.779				
-logL	1951.565	1960.477	1000	$\hat{c}$ =0.781 (0.156) $\hat{\alpha}$ =2.469 (0.991) $\hat{\theta}$ =1.031 (0.034)	$\hat{\alpha}$ =0.288 (0.059) $\hat{\theta}$ =0.921 (0.021)	17.825
AIC	3909.129	3924.954				
AICC	3909.153	3924.966				
BIC	3923.852	3934.77				

Table 3: Model Comparison Based On Simulated Data from WQLD.

$\hat{c} = 1.2, \hat{\alpha} = 0.2, \hat{\theta} = 1.5$				Parameter Estimates		Likelihood Ratio Statistic
Criterion	WQLD	QLD	Sample Size (n)	WQLD	QED	
-logL	17.22829	17.82711	10	$\hat{c}$ =1.33 (0.80) $\hat{\alpha}$ =0.001 (0.89) $\hat{\theta}$ =1.41 (0.45)	$\hat{\alpha}$ =0.102 (0.561) $\hat{\theta}$ =0.96 (0.127)	1.19763
AIC	40.45658	39.65422				
AICC	44.45658	41.3685				
BIC	41.36433	40.25939				
-logL	42.26757	44.51678	25	$\hat{c}$ =2.05 (0.64) $\hat{\alpha}$ =0.001 (0.87) $\hat{\theta}$ =1.59 (0.33)	$\hat{\alpha}$ =0.112 (0.458) $\hat{\theta}$ =0.931 (0.08)	4.4984
AIC	90.53513	93.03356				
AICC	91.67799	93.57901				
BIC	94.19176	95.47131				

-logL	166.978	173.1069	100	$\hat{c}$ =2.04 (0.851) $\hat{\alpha}$ =0.798 (0.79) $\hat{\theta}$ =1.64 (0.251)	$\hat{\alpha}$ =0.101 (0.171) $\hat{\theta}$ =0.989 (0.069)	12.2578
AIC	339.956	350.2139				
AICC	340.206	350.3376				
BIC	347.7715	355.4242				
-logL	511.7328	522.3951	300	$\hat{c}$ =1.924 (0.641) $\hat{\alpha}$ =1.612 (0.68) $\hat{\theta}$ =1.59 (0.203)	$\hat{\alpha}$ =0.10 (0.269) $\hat{\theta}$ =1.019 (0.07)	21.3244
AIC	1029.466	1048.79				
AICC	1029.547	1048.831				
BIC	1040.577	1056.198				
-logL	1633.577	1679.299	1000	$\hat{c}$ =1.122 (0.25) $\hat{\alpha}$ =0.028 (0.09) $\hat{\theta}$ =1.51 (0.08)	$\hat{\alpha}$ =0.11 (0.1310) $\hat{\theta}$ =1.07 (0.02)	91.4451
AIC	3273.153	3362.598				
AICC	3273.177	3362.61				
BIC	3287.876	3372.414				

X. APPLICATIONS OF WEIGHTED QUASI AKASH DISTRIBUTIONS

Here we analyse the data set given in table 4, studied by Xu et al. (2003) and it represents the time to failure ($10^3 h$) of turbocharger of one type of engine.

Table 4: The time to failure of turbocharger data (*n* = 40) studied by Xu et al. (2003).

1.6	3.5	4.8	5.4	6.0	6.5	7	7.3	7.7	8
8.4	2	3.9	5	5.6	6.1	6.5	7.1	7.3	7.8
8.1	8.4	2.6	4.5	5.1	5.8	6.3	6.7	7.3	7.7
7.9	8.3	8.5	3	4.6	5.3	6	8.7	8.8	9

In order to compare the performance of weighted Quasi Akash Distribution vz Quasi Akash Distribution, we employ AIC (Akaike information criterion by Akaike (1976)), AICC (corrected Akaike information criterion) and BIC (Bayesian information criterion given by Schwarz (1987)) criterion. The better distribution corresponds to minimum of AIC, AICC and BIC values. The generic formulas for AIC, AICC and BIC are given as,

$$\text{AIC} = 2k-2\log L \qquad \text{AICC} = \text{AIC}+\frac{2k(k+1)}{n-k-1} \text{ and BIC} = k\log n-2\log L$$

where k is the number of parameters in the statistical model, n is the sample size and -log$L$ is the maximized value of the log-likelihood function under the considered model. R software was used to analyse the failure of turbocharger data (*n* = 40) studied by Xu et al. (2003). From table 5, it is clear that Weighted Quasi Akash Distribution (WQAD) has the lesser AIC, AICC, -logL and BIC values as compared to Quasi Akash Distribution. Hence we can conclude that the Weighted Quasi Akash distribution leads to a better fit than the Quasi Akash distribution. We also test the significance of weighted parameter c with the help of Kolmogorov Smirnov test. The p-value for WQAD is greater than 0.05, which signifies that data come from WQAD.

Table 5: ML estimates, -logL, AIC, AICC, BIC, Kolmogorov Smirnov statistic and KS p-values calculation for time to failure of turbocharger data (*n* = 40) studied by Xu et al. (2003).

Model	ML Estimates	-logL	AIC	AICC	BIC	KS-Distance	P-Value
Weighted Quasi Akash	$\hat{c}=5.93$ $\hat{\theta}=1.35$ $\hat{\alpha}=11.7$	86.84751	179.695	180.3617	184.7617	0.1179	0.6344
Quasi Akash	$\hat{\theta}=0.479$ $\hat{\alpha}=0.001$	94.50852	193.017	193.6837	196.3948	0.2166	0.0467

XI. CONCLUSION

In the present study we have studied a weighted Quasi Akash distribution as a new generalization Quasi Akash. The subject distribution is generated by using the weighting technique and taking the two parameter Quasi Akash distribution as the base distribution. Some mathematical properties along with reliability measures are discussed. The hazard rate function and reliability behavior of the weighted Quasi Akash distribution exhibits that subject distribution can be used as a lifetime model.

REFERENCES

[1]. Akaike, H. (1976). A new look at the statistical model identification. *IEEE Trans. Autom.Control* , 19 , 716–723.

[2]. Ayesha, A. (2017). Size Biased Lindley Distribution and Its Properties a Special Case of Weighted Distribution. *Applied Mathematics*, 08(06), 808-819.

[3]. Buckland, W. R., & Cox, D. R. (1964). Renewal Theory. *Biometrika*, *51*(1/2), 290. doi:10.2307/2334228

[4]. Fisher, R.A. (1934). The effects of methods of ascertainment upon the estimation of frequencies. *Ann. Eugenics*, 6, 13-25.

[5]. Jing (2010) Weighted Inverse Weibull and Beta-Inverse Weibull Distribution, Georgia Southern University Digital Commons@Georgia Southern.

[6]. Patil, G. P., & Rao, C. R. (1978). Weighted Distributions and Size-Biased Sampling with Applications to Wildlife Populations and Human Families. *Biometrics,* 34(2), 179. doi:10.2307/2530008

[7]. Rao, C. R. (1965). On discrete distributions arising out of method of ascertainment, in classical and Contagious Discrete, G.P. Patil .ed ;Pergamon Press and Statistical publishing Society, Calcutta, pp-320-332.

[8]. Schwarz, G. (1987). Estimating the dimension of a model. *Ann. Stat.,* 5, 461–464.

[9]. Shanker, R. (2016). A Quasi Akash Distribution. *ASR*, 30 (1), 135-159.

[10]. Shanker, R., & Shukla, K. K. A Generalized Size-Biased Poisson-Lindley Distribution and Its Applications to Model Size Distribution of Freely-Forming Small Group. Journal of Scientific Research, 10(2), 145, (2018)

[11]. Taillie, C., Patil, G.P. and Hennemuth, R. (1995). Modelling and analysis of recruitment distributions, *Ecological and Environmental Statistics*, 2(4), 315-329.

[12]. Warren, W. G. (1975). Statistical Distributions in Forestry and Forest Products Research. *A Modern Course on Statistical Distributions in Scientific Work*, 369-384. doi:10.1007/978-94-010-1845-6_27

[13]. Xu, K., Xie, M., Tang, L.C. and Ho, S.L. (2003). Application of neural networks in forecasting engine systems reliability, *Applied Soft Computing*, 2 (4), 255-268.

Anwar Hassan. " On three Parameter Weighted Quasi Akash Distribution: Properties and Applications." IOSR Journal of Engineering (IOSRJEN), vol. 08, no. 11, 2018, pp. 01-10.