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Vague Reliability of a Network System Using Sugeno's Fuzzy Failure Rates

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Abstract: In General classical sets are used to analyze the system Reliability. But due to uncertainty in the present era the classical reliability is inadequate enough to describe the real situation of the system. Fuzzy logic tools attempt to review the reliability of the system with the help of membership function but in the present era when we have the favorable as well as the unfavorable conditions so the fuzzy logic is also not sufficient to study the reliability of the system. Present paper attempts to review the fuzzy/possibility tools when dealing with reliability of the network system. Various issues of reasoning-based approaches in this framework are reviewed, discussed and compared with the standard approaches of reliability. To analyze the vague reliability of the system, the failure rates are evaluated by sugeno's fuzzy failure rate estimation. A Vague inference engine is used to evaluate the failure rates in the form of trapezoidal vague numbers. A numerical example is also given to illustrate the method.

Keywords: - Fuzzy sets, Vague sets, Vague Numbers, Trapezoidal Vague Numbers (TrapVaN), Reliability, (α, β) - Cuts, Sugeno's Fuzzy Failure Rates, Vague inference engine AMS Subject Classification: - 62N86

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I. INTRODUCTION

Fuzzy set theory proposed by Zadeh [16] permits the replacement of the sharp boundaries in classical set theory by fuzzy boundaries. The concept of belongingness of an element in the context of classical sets changes to membership grade of the element to certain degree in fuzzy sets. The membership grade of an element *x* of the fuzzy set is given by a real number between zero and one. Due to fuzzy boundaries, this single value for the membership grade is the result of the combined effect of evidences in favour and against the inclusion of the element in the set [6]. The utility of the application of fuzzy sets depends on the capability of the user to construct appropriate membership functions, which are often very precise. In many contexts it is difficult to assign a particular real number as a membership grade and in such cases it may be useful to identify meaningful lower and upper bounds for the membership grade. Such a generalization of fuzzy sets is called vague sets.

Concept of vague sets was given by Gau and Buehrer [5] takes into account the favourable and unfavourable evidences separately providing a lower and an upper bound within which the membership grade may lie. In 1993 Chen [4] used the concept of fuzzy sets with the possibility theory. Again in 1995, Chen [2] presented the measures of similarity between vague sets. Recently, Chen [3] proposed fuzzy system reliability analysis based on vague set theory, where the reliabilities of the components of a system are represented by vague sets defined in the universe of discourse [0 1]. Chen's method has the advantages of modeling and analyzing the fuzzy system reliability in a more flexible and more intelligent manner. However, Chen's method can just apply to some special case of general vague set.

For the fast technology innovations, new product development is getting much complicated not only its system functions, but also on its system components. Therefore one of the important engineering tasks in design and development of a technical system is the reliability engineering. In [1] Kauffmann and Gupta pointed out that the discipline of reliability engineering encompasses a number of different activities, reliability modeling being the most important one. Cai [8] gave the concept of fuzzy methodology to system failure and describe the different aspects of fuzzy logic overview. Network systems are one of the most complicated system products in the real world. Network systems include many different system components in order to integrate sophisticated functions under system command and control. Network system reliability [13] problem is critical and important, because not only its expression into subsystems but also success of any subsystem may collapse.

The reliability of a system is the probability that the system will perform a specified function satisfactorily during some interval of time under specified operating conditions [3]. Traditionally, the reliability of a system behaviour is fully characterized in the context of probability measures, and the outcome of the top event is certain and precise as long as the assignment of basic events are descent from reliable information. Sharma & Pandey [11] gave an approach to evaluate the reliability of multistate fault tree model by applying the fuzzy logic. However in real life systems, the information may be inaccurate or might have linguistic representation. In such cases the estimation of precise values of probability becomes very difficult. In order to handle this situation, fuzzy approach [9 & 10] is used to evaluate the failure rate status. Sharma & Pandey [12] gave the concept of arithmetic operations on vague sets and vague set theoretic approach to fault tree analysis in which with help of membership and non-membership we can describe the overall characteristic of the system.

This paper is organized as follows: section 2 represents the basic concept of Fuzzy sets, vague sets, (α,β) −cuts for vague sets. Section 3 deals with the vague numbers, their types and arithmetic operations through (α,β) −cuts. In section 4 failure rate estimation is done through vague inference engine with the help of sugeno's fuzzy model and I have proposed an algorithm to evaluate the reliability of the network system using sugeno's fuzzy failure rate estimation and arithmetic operations based on (α,β) −cuts of vague numbers. In section 5 Vague Reliability of different networks is calculated through (α,β) −cuts. In section 6 a numerical example is given to illustrate the proposed algorithm the technique used to evaluate the failure rates. Conclusion is given in the last section.

II. BASIC NOTIONS AND DEFINITIONS OF VAGUE SETS (VS_S)

Fuzzy set theory was first introduced by Zadeh [16] in 1965. Let X be universe of discourse defined by $X = \{x_1, x_2 ... x_n\}$. The grade of membership of an element $x_i \in X$ in a fuzzy set is represented by real value between 0 and 1. It does indicate the evidence for $x_i \in X$, but does not indicate the evidence against $x_i \in X$. Gau W. L, Buehrer D. J. [5] in 1993 presented the concept of VS_s, and pointed out that this single value combines the evidence for $x_i \in X$ and the evidence against $x_i \in X$. VSs $\tilde{A}$ in X are characterized by a membership function $\mu_{\tilde{A}}(x)$ and a non membership function $\nu_{\tilde{A}}(x)$.

2.1 Definition of Vague Set: - Let E be a fixed set. A vague set $\tilde{A}$ of E is an object having the form $\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) >: x \in E\}$

Where the functions

$\mu_A: E \rightarrow [0\ 1]$ and $\nu_A: E \rightarrow [0\ 1]$ define respectively, the degree of membership and the degree of non-membership of the element $x \in E$ to the set A, which is a subset of E and for every $x \in E$, $0 \le \mu_A(x) + \nu_A(x) \le 1$.

When the universe of discourse E is discrete, a VaS_s $\tilde{A}$ can be written as

$$\tilde{A} = \sum_{i=1}^{n} [\mu_A(x),\ \ 1 - \nu_A(x)]/x, \forall\, x_i \in E$$

An $\mathbf{VS_s}$ $\tilde{A}$ with continuous universe of discourse E can be written as

$$\tilde{A} = \int_E [\mu_A(x), 1 - \nu_A(x)]/x, \forall x_i \in E$$

A vague set is represented pictorially as

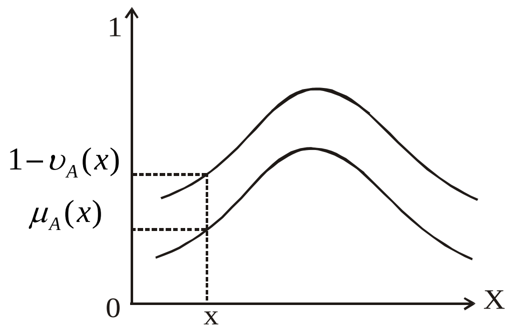

Fig. 1 Membership and non-membership functions of $\breve{A}$

2.2 (α,β) −cuts for VaS$_s$: - A set of (α,β) −cut generated by an VaS$_s$ $\widetilde{A}$, where $\alpha,\beta \in [0\ 1]$ are fixed numbers such that $\alpha+\beta \le 1$ is defined as

$$\tilde{A}_{\alpha,\beta} = \{(x,\mu_{\tilde{A}}(x),\nu_{\tilde{A}}(x)): x \in X, \mu_{\tilde{A}}(x) \ge \alpha, \nu_{\tilde{A}}(x) \le \beta, \alpha,\beta \in [0\ 1]\}$$

We define (α,β) −cut, denoted by $\widetilde{A}_{\alpha,\beta}$, as the crisp set of elements x which belongs to $\widetilde{A}$ at least to the degree α and which belongs to $\widetilde{A}$ at most to the degree β.

III. VAGUE NUMBERS (VaN)

An VaN $\tilde{A}$ is defined as follows:
(i) A vague subset of the real line.
(ii) Normal i.e. there is any $x_0 \in R$ such that $\mu_A(x_0); \alpha,\beta \in [0\ 1]$
(iii) Convex for the membership function $\mu_{\tilde{A}}(x)$ i.e.

$$\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda x_2] \ge \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)], \forall\, x_1, x_2 \in R, \lambda \in [0\ 1]$$

(iv) Concave for the non-membership function $\nu_{\tilde{A}}(x)$ i.e.

$$\nu_{\tilde{A}}[\lambda x_1 + (1-\lambda x_2] \le \max[\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2)], \forall\, x_1, x_2 \in R, \lambda \in [0\ 1]$$

Here we have two types of vague numbers:
(i) Triangular vague Numbers (ii) Trapezoidal vague Numbers
In the present paper we have introduced trapezoidal vague numbers by using (α,β) −cuts.

3.1 Trapezoidal vague Numbers (TrapVaN):-

Let $(a_1 \le a_2 \le a_3 \le a_4 \le a'_1 \le a_2 \le a_3 \le a'_4)$. A Trapezoidal vague number (TrVaN) $\tilde{A}$ in R, written as $(a_1, a_2, a_3, a_4, a'_1, a_2, a_3, a'_4)$ has the membership function and the non-membership function as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & for\ a_1 \le x \le a_2 \\ 1, & for\ a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & for\ a_3 \le x \le a_4 \\ 0, & otherwise \end{cases} \quad \text{and} \quad \nu_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a'_1}, & for\ a'_1 \le x \le a_2 \\ 0, & otherwise \\ \frac{x-a_3}{a'_4-a_2}, & for\ a_3 \le x \le a'_4 \\ 1, & otherwise \end{cases}$$

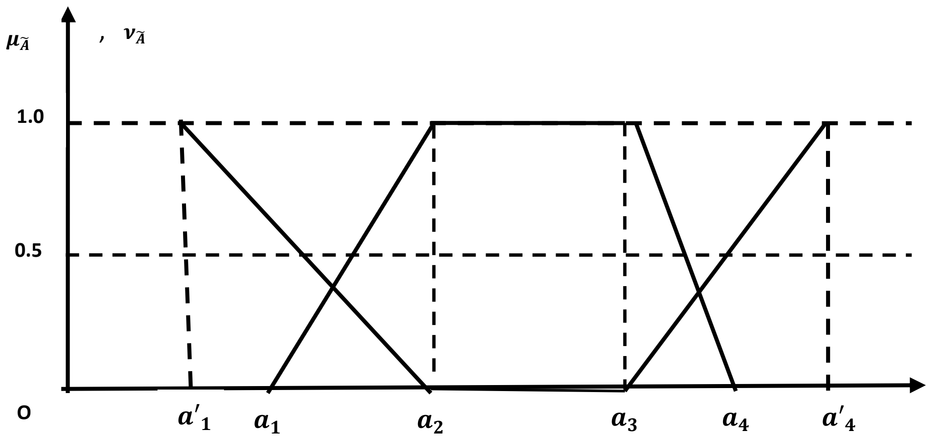

Fig. 2 Membership and non-membership functions of TrapVaN

3.2. Arithmetic operations on VaNs:-

The arithmetic operations denoted generally by *, of two VaNs is a mapping of an input subset of R x R (with elements $x = (x_1, x_2)$) onto an output subset of R (with elements denoted by y). Let $A_1 and\ A_2$ be two VaNs, and $(A_1 * A_2)$ the resultant of operations then:

$$A_1 * A_2(y) = \left\{ \begin{pmatrix} y, \\ \vee_y = x_1 * x_2[A_1(x_1) \wedge A_2(x_2)], \\ \wedge_y = x_1 * x_2[A_1(x_1) \vee A_2(x_2)] \end{pmatrix} T, \forall x_1, x_2, y \in R \right\}$$

With

$$\mu_{(A_1 * A_2)}(y) = \vee_y = x_1 * x_2[A_1(x_1) \wedge A_2(x_2)]$$

$$and \quad \nu_{(A_1 * A_2)}(y) = \wedge_y = x_1 * x_2[A_1(x_1) \vee A_2(x_2)]$$

The arithmetic operations on VaNs can be defined by using the (α,β) −cut method. Let $\alpha,\beta \in [0,1]$ be fixed numbers such that $\alpha+\beta \le 1$. A set of (α,β) −cut generated by an VaS$_s$ A is defined by:

$$\tilde{A}_{\alpha,\beta} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \geq \alpha, \nu_{\tilde{A}}(x) \leq \beta, \alpha, \beta \in [0\ 1]\}$$

The (α, β) −cut of Trapezoidal Vague Number is defined as usually by

$$A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)],\ [A'_1(\beta), A'_2(\beta)]\}$$

$$\alpha + \beta \leq 1, \alpha, \beta \in [0, 1]$$

Where

$$A_1(\alpha) = a_1 + \alpha(a_2 - a_1),\quad A_2(\alpha) = a_4 - \alpha(a_4 - a_3)$$

And $$A'_1(\beta) = a_2 - \beta(a_2 - a'_1),\quad A'_2(\beta) = a_3 + \beta(a'_4 - a_3)$$

With the following properties:

i) $A_1(\alpha)$, $A'_2(\beta)$ are continuous, monotonic increasing functions of α, respective β.

ii) $A_2(\alpha)$, $A'_1(\beta)$are continuous, monotonic decreasing functions of α, respective β.

IV. SUGENO'S FUZZY FAILURE RATE ESTIMATION

Sugeno fuzzy model was proposed by Takagi, Sugeno and Kang [17 & 18]. This method is similar to the Mamdani's method but in this method the first two parts fuzzify the inputs and apply the fuzzy operators. In Sugeno's method the output is linear or constant.

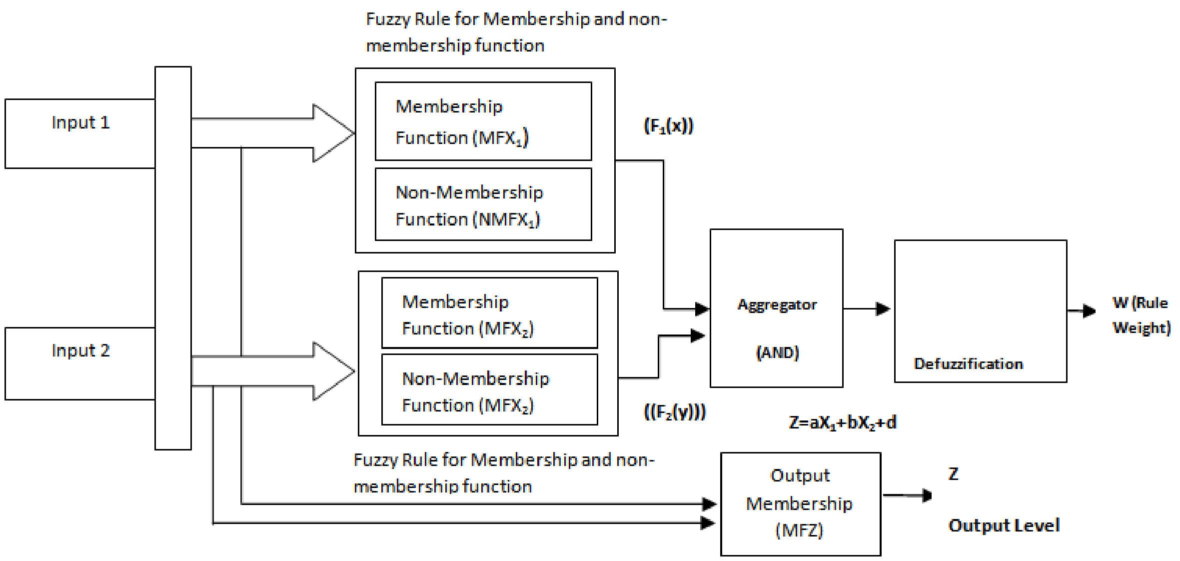

Fig-3(Vague Inference Engine)

4.1 Proposed Algorithm for Fuzzy failure rate estimation by sugeno's fuzzy model: - Failure / Repair rates [7] are important parameters in the estimation of reliability characteristics of any system. A small error in failure rate may lead to over / under estimation of system reliability. For systems having very sensitive applications, this risk must be avoided to the maximum possible extent. A standard method for determining a failure rate parameter is the maximum likelihood utilizing estimation from multiple data sets. Collection of failure data may involve following uncertainties:

1. Failure exactly occurs, but the failure time is not accurately observed or might be missed.
2. Failure doesn't occur or occurs partially. So, the reported failure time is based on censored observation.
3. Multiple failure data values need to be obtained under similar operating conditions. Operating Conditions, in all cases, cannot be uniquely explained and contain hazziness concerning the description.
4. It may involve human judgment, evaluation and decision at certain stages that may be vague.

Under the above-mentioned situations, it is appropriate to deal with the failure data by fuzzy techniques. We propose a method based on Sugeno's fuzzy model to estimate failure rate parameter by using the concepts of fuzzy numbers, fuzzy aggregation and defuzzification. Defuzzification is the process that creates a single assessment from the fuzzy conclusion set. The philosophy of the method is based on the two basic concepts of Sugeno's fuzzy model:-

(i) In first step we will find out the weight for the failure data set.

(ii) In second step we will find the out the output from the input given from the various data sets.

On the basis of these two steps we will get a weighted average of all the rules for output computed as:

For finding out the final output we will use

$$Final\ output = \frac{\sum_{m=1}^{N} W_m Z_m}{\sum_{m=1}^{N} W_m}$$

4.2 An Algorithm has been proposed to perform arithmetic operations among vague numbers through (α, β)-cuts, where the reliability of different components has been taken in the form of VaNs.

If $\tilde{A}$ is an VaN, then (α, β)-cut is given by

$$\tilde{A}_{\alpha,\beta} = \begin{cases}[A_1(\alpha), A_2(\alpha)]\ for\ degree\ of\ acceptance\ \ \alpha \in [0,1] \\ [A'_1(\beta), A'_2(\beta)]\ for\ degree\ of\ rejection\ \ \beta \in [0,1]\end{cases}\ with\ \alpha + \beta \le 1,$$

Here $\frac{dA_1(\alpha)}{d\alpha} > 0, \frac{dA_2(\alpha)}{d\alpha} < 0\ \ \forall\, \alpha \in (0,1), A_1(1) \le A_2(1)$

and $\frac{dA'_1(\beta)}{d\beta} < 0, \frac{dA'_2(\beta)}{d\beta} > 0\ \ \forall\, \beta \in (0,1), A'_1(0) \le A'_2(0)$

It is expressed as $\tilde{A}_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)], [A'_1(\beta), A'_2(\beta)], \alpha + \beta \le 1, \alpha, \beta \in [0,1]\}$

Step I: -First we will construct vague numbers for all the components of the network.

Step II: - In this step we will evaluate the (α, β)-cut for each vague number as in step I.

Step III: - If $\tilde{A} = (a_1, a_2, a_3, a_4; a'_1, a_2, a_3, a'_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; b'_1, b_2, b_3, b'_4)$ are two TrapVaN, then $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is also TrapVaN $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; a'_1 + b'_1, a_2 + b_2, a_3 + b_3, a'_4 + b'4$.

With the transformation z = x+y, we can find the membership function of acceptance (membership) VaS $\tilde{C} = \tilde{A} \oplus \tilde{B}$ by the $\alpha - cut$ method.

$\alpha -$cut for membership function of $\tilde{A}$ is $[a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]\ \forall\, \alpha \in [0,1]$

i.e. $x \in [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]$

$\alpha -$cut for membership function of $\tilde{B}$ is $[b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)]\ \forall\, \alpha \in [0,1]$

i.e. y$\in [b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)]$

So, z (= x+y) $\in \left[a_1 + b_1 + \alpha\big((a_2 - a_1) + (b_2 - b_1)\big), a_4 + b_4 - \alpha((a_4 - a_3) + (b_4 - b_3))\right]$

So, we have the membership (acceptance) function $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is

$$\mu_{\tilde{C}}(z) = \begin{cases}\frac{z - a_1 - b_1}{(a_2 - a_1) + (b_2 - b_1)}, & for\ a_1 + b_1 \le z \le a_2 + b_2 \\ 1, & for\ a_2 + b_2 \le z \le a_3 + b_3 \\ \frac{a_4 + b_4 - z}{(a_4 - a_3) + (b_4 + b_3)}, & for\ a_3 + b_3 \le z \le a_4 + b_4 \\ 0, & otherwise\end{cases}$$

This is the addition rule for membership function.

For non-membership function, β-cut of $\tilde{A}$ is $[a_2 - \beta(a_2 - a'_1), a_3 + \beta(a'_4 - a_3)]\forall\beta \in [0,1]$ i.e. $x \in [a_2 - \beta(a_2 - a'_1), a_3 + \beta(a'_4 - a_3)]$

β-cut of $\tilde{B}$ is $[b_2 - \beta(b_2 - b'_1), b_3 + \beta(b'_4 - b_3)]\forall\beta \in [0,1]$

i.e. $y \in [b_2 - \beta(b_2 - b'_1), b_3 + \beta(b'_4 - b_3)]$

So, z (= x+y) $\in \left[a_2 + b_2 - \beta\big((a_2 - a'_1) + (b_2 - b'_1)\big), (a_3 + b_3) - \beta((a'_4 - a_3) + (b'_4 - b_3))\right]$

So, we have the non-membership (rejection) function of $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is

$$\nu_{\tilde{C}}(z) = \begin{cases}\frac{a_2 + b_2 - z}{(a_2 - a'_1) + (b_2 - b'_1)}, & for\ a'_1 + b'_1 \le z \le a_2 + b_2 \\ 0, & for\ a_2 + b_2 \le z \le a_3 + b_3 \\ \frac{z - a_3 - b_3}{(a'_4 - a_3) + (b'_4 - b_3)}, & for\ a_3 + b_3 \le z \le a'_4 + b'_4 \\ 1, & otherwise\end{cases}$$

This is the rule for non-membership function.

Thus we have $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; a'_1 + b'_1, a_2 + b_2, a_3 + b_3, a'_4 + b'_4)$.

Step IV: - If $\tilde{A} = (a_1, a_2, a_3, a_4; a'_1, a_2, a_3, a'_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; b'_1, b_2, b_3, b'_4)$ are two TrapVaN, then $\tilde{P} = \tilde{A} \otimes \tilde{B}$ is approximated

TrapVaN $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4; a'_1 b'_1, a_2 b_2, a_3 b_3, a'_4 b'_4)$.

With the transformation z = xXy, we can find the membership function of acceptance (membership) VaS $\tilde{P} = \tilde{A} \otimes \tilde{B}$ by the $\alpha - cut$ method.

$\alpha -$cut for membership function of $\tilde{A}$ is $\mu_{\tilde{A}}(x) \ge \alpha \Rightarrow [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]\ \forall\, \alpha \in [0,1]$

i.e. $x \in [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]$

α −cut for membership function of $\tilde{B}$ is $\mu_{\tilde{B}}(x) \geq \alpha \Rightarrow [b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)] \ \forall \ \alpha \in [0,1]$
i.e. y∈ $[b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)]$
So, z (= xXy) ∈ $\left[\left(a_1 + \alpha(a_2 - a_1)\right)(b_1 + \alpha(b_2 - b_1)), (a_4 + \alpha(a_4 - a_3))(b_4 - \alpha(b_4 - b_3))\right]$
So, we have the membership (acceptance) function $\tilde{P} = \tilde{A} \otimes \tilde{B}$ is

$$\mu_{\tilde{P}}(z) = \begin{cases} \dfrac{-B_1 + \sqrt{B_1^2 - 4A_1(a_1b_1 - z)}}{2A_1}, & for\ a_1b_1 \leq z \leq a_2b_2 \\ 1, & for\ a_2b_2 \leq z \leq a_3b_3 \\ \dfrac{B_2 - \sqrt{B_2^2 - 4A_2(a_4b_4 - z)}}{2A_2}, & for\ a_3b_3 \leq z \leq a_4b_4 \\ 0, & otherwise \end{cases}$$

Where, $A_1 = (a_2 - a_1)(b_2 - b_1)$, $B_1 = b_1(a_2 - a_1) + a_1(b_2 - b_1)$, $A_2 = (a_4 - a_3)(b_4 - b_3)$, $B_2 = -(b_4(a_4 - a_3) + a_4(b_4 - b_3))$
For non-membership function, β-cut of $\tilde{A}$ is $\nu_{\tilde{A}}(x) \leq \beta \Rightarrow [a_2 - \beta(a_2 - a'_1), a_3 + \beta(a'_4 - a_3)] \forall \beta \in [0,1]$ i.e. $x \in [a_2 - \beta(a_2 - a'_1), a_3 + \beta(a'_4 - a_3)]$
β-cut of $\tilde{B}$ is $\nu_{\tilde{A}}(x) \leq \beta \Rightarrow [b_2 - \beta(b_2 - b'_1), b_3 + \beta(b'_4 - b_3)] \forall \beta \in [0,1]$
i.e. $y \in [b_2 - \beta(b_2 - b'_1), b_3 + \beta(b'_4 - b_3)]$
So, z (= xXy) ∈ $\left[\left(a_2 - \beta(a_2 - a'_1)\right)\left(b_2 - \beta(b_2 - b'_1)\right), \left(a_3 + \beta(a'_4 - a_3)\right), (b_3 + \beta(b'_4 - b_3))\right]$
So, we have the non-membership (rejection) function of $\tilde{z} = \tilde{A} \otimes \tilde{B}$ is

$$\nu_{\tilde{P}}(z) = \begin{cases} 1 - \dfrac{-B'_1 + \sqrt{B'^2_1 - 4A'_1(a'_1b'_1 - z)}}{2A'_1}, & for\ a'_1b'_1 \leq z \leq a_2b_2 \\ 0, & for\ a_2b_2 \leq z \leq a_3b_3 \\ 1 - \dfrac{B'_2 - \sqrt{B'^2_2 - 4A'_2(a'_4b'_4 - z)}}{2A'_2}, & for\ a_3b_3 \leq z \leq a'_4b'_4 \\ 1, & otherwise \end{cases}$$

Where, $A'_1 = (a_2 - a'_1)(b_2 - b'_1)$, $B'_1 = b'_1(a_2 - a'_1) + a'_1(b_2 - b'_1)$, $A'_2 = (a'_4 - a_3)(b'_4 - b_3)$ and $B'_2 = -(b'_4(a'_4 - a_3) + a'_4(b'_4 - b_3))$
This is the rule for non-membership function.
Thus we have $\tilde{P} = \tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4; a'_1b'_1, a_2b_2, a_3b_3, a'_4b'_4)$.
Step V: - Operations defined in step III and IV will be used to evaluate the reliability for the whole network by getting the defuzzified value. The defuzzified value will be gained as follows:

4.3 On the basis of the following steps we will generate another algorithm for the evaluation of the vague failure rate as follows:
(a) Failure rate is first estimated according to the existing procedure. The process must be done so many times that more than one number is available for estimating the failure rate where the membership function and non-membership function will be governed by the step III and step IV.
(b) Numbers obtained in (a) will be fuzzified according to their membership as well as their non-membership by using the fuzzufication process.
(c) Now by using the aggregation operations i.e. (OR=max) fuzzy union for membership as well as non-membership for the vague number and we will get a single vague number.
(d) Now we will use the defuzzification method for membership as well as non-membership to get a single crisp number.
(e) On the basis of (a) we will also provide the weights for the inputs and also fuzzify the membership and non-membership of the weights.
(f) Numbers obtained in (e) will be supplied to the fuzzy operations (AND=min) fuzzy intersection to the membership as well as non-membership for the weights and will get a single fuzzy number for membership as well as non-membership function for the weights.
(g) Now we will use the defuzzufication process for membership and non-membership to get a single output in the form of an interval.
On the basis of this algorithm we will get various weights and various outputs which will be based on the number of fuzzy rules which we have defined for our system.
For finding out the final output we will use

$$Final\ output = \frac{\sum_{m=1}^{N}(w_{m_1}+w_{m_2})(ax_m+bx_m+d)}{\sum_{m=1}^{N}(w_{m_1}+w_{m_2})}$$

on the basis of which we will get a single (crisp) value which will be used as failure rate.

V. RELIABILITY EVALUATION OF A SERIES, PARALLEL, PARALLEL-SERIES AND SERIES-PARALLEL NETWORKS: -

5.1 Series Networks:-This arrangement represents a system where subsystems form a series network. If any of subsystem fails, the series system experiences an overall system failure.

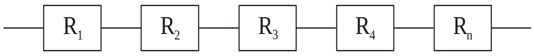

Fig. 4 Series Network

The vague reliability $\widetilde{R}_s = {}^{n}_{i}\otimes\tilde{R}_i$ can be evaluated by using the proposed algorithm

$$\widetilde{R_s} = \{(r_{11,}r_{12}r_{13},r_{14};r'_{11,}r'_{12}r'_{13},r'_{14}) \otimes (r_{21,}r_{22}r_{23},r_{24};r'_{21,}r'_{22}r'_{23},r'_{24}) \otimes \ldots\ldots\ldots\ldots\ldots\ldots. \otimes (r_{n1,}r_{n2}r_{n3},r_{n4};r'_{n1,}r'_{n2}r'_{n3},r'_{n4})\}$$

It can be approximated to a TrapVaN as

$$= (\textstyle\prod_{j=1}^{n} r_{j1}, \prod_{j=1}^{n} r_{j2}, \prod_{j=1}^{n} r_{j3}, \prod_{j=1}^{n} r_{j4}\ ; \prod_{j=1}^{n} r'_{j1}, \prod_{j=1}^{n} r'_{j2}, \prod_{j=1}^{n} r'_{j3}, \prod_{j=1}^{n} r'_{j4}$$

Where $\tilde{R}_j = (r_{j1},r_{j2},r_{j3},r_{j4};r'_{j1,}r'_{j2},r'_{j3},r'_{j4})$ is the vague reliability of the j[th] component for j=1, 2, 3, 4…n

5.2 Parallel Networks: - Consider a parallel network consisting of 'n' subsystems as shown in figure. The vague reliability $\widetilde{R}_p = 1 \ominus {}^{n}_{i}\otimes(1 \ominus \tilde{R}_i)$ of the parallel system shown in figure can be evaluated by using the proposed algorithm.

$$\widetilde{R_p} = 1 \ominus [(1 \ominus (r_{11,}r_{12}r_{13},r_{14};r'_{11,}r'_{12}r'_{13},r'_{14})) \otimes \ldots\ldots\ldots\ldots.\otimes (1 \ominus (r_{n1,}r_{n2}r_{n3},r_{n4};r'_{n1,}r'_{n2}r'_{n3},r'_{n4}))]$$

It can be approximated to a TrapVaN as

$$= [1-\prod_{j=1}^{n}(1-r_{j1}),1-\prod_{j=1}^{n}(1-r_{j2}),1-\prod_{j=1}^{n}(1-r_{j3}),1-\prod_{j=1}^{n}(1-r_{j4})\ ;1-\prod_{j=1}^{n}(1-r'_{j1}),1 -\prod_{j=1}^{n}(1-r'_{j2}),1-\prod_{j=1}^{n}(1-r'_{j3}),1-\prod_{j=1}^{n}(1-r'_{j4})]$$

Where $\tilde{R}_j = (r_{j1},r_{j2},r_{j3},r_{j4};r'_{j1,}r'_{j2},r'_{j3},r'_{j4})$ is the vague fuzzy reliability of the j[th] component for j=1, 2, 3, 4.n

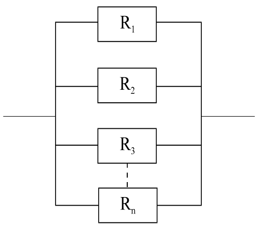

Fig. 5 Parallel networks

5.3 Parallel-series system: - Consider a parallel-series network consisting of 'm' connections connected in parallel and each connection contains 'n' subsystems as shown in the figure. The vague reliability is given by $\tilde{R}_{ps} = 1 \ominus {}^{m}_{i}\otimes(1 \ominus ({}^{n}_{i}\otimes\tilde{R}_{ki}))$ of the parallel-series network shown in figure. Reliability can be evaluated by the proposed algorithm, where $\tilde{R}_{kl}$ represents the reliability of the i[th] component at the k[th] network.

$$\widetilde{R_{ps}} = 1 \ominus {}^{m}_{i}\otimes[(1 \ominus \{(r_{11,}r_{12}r_{13},r_{14};r'_{11,}r'_{12}r'_{13},r'_{14}) \otimes (r_{21,}r_{22}r_{23},r_{24};r'_{21,}r'_{22}r'_{23},r'_{24}) \otimes \ldots\ldots\ldots\ldots\ldots\ldots. \otimes (r_{n1,}r_{n2}r_{n3},r_{n4};r'_{n1,}r'_{n2}r'_{n3},r'_{n4})\}]$$

$$= 1 \ominus {}^{m}_{i}\otimes[(1 \ominus (r_{11,}r_{12}r_{13},r_{14};r'_{11,}r'_{12}r'_{13},r'_{14})) \otimes \ldots\ldots\ldots\ldots.\otimes (1 \ominus (r_{n1,}r_{n2}r_{n3},r_{n4};r'_{n1,}r'_{n2}r'_{n3},r'_{n4}))]$$

$= 1 \ominus {}^{m}_{i}\otimes[(1 \ominus (r_{11,}r_{12}r_{13}, r_{14}; r'_{11,}r'_{12}r'_{13}, r'_{14})) \otimes \ldots\ldots\ldots\ldots \otimes (1 \ominus (r_{n1,}r_{n2}r_{n3}, r_{n4}; r'_{n1,}r'_{n2}r'_{n3}, r'_{n4}))]$

It can be approximated to a TrapVaN as

$$R_{ps} = 1 \ominus {}^{m}_{i}\otimes[1 \ominus \left(\prod_{j=1}^{n} r_{j1}, \prod_{j=1}^{n} r_{j2}, \prod_{j=1}^{n} r_{j3}, \prod_{j=1}^{n} r_{j4}; \prod_{j=1}^{n} r'_{j1}, \prod_{j=1}^{n} r'_{j2}, \prod_{j=1}^{n} r'_{j3}, \prod_{j=1}^{n} r'_{j4}\right)]$$

$= 1 \ominus {}^{m}_{i}\otimes[\left(\prod_{j=1}^{n}(1 \ominus r_{j1}), \prod_{j=1}^{n}(1 - r_{j2}), \prod_{j=1}^{n}(1 - r_{j3}), \prod_{j=1}^{n}(1 - r_{j4}); \prod_{j=1}^{n}(1 - r'_{j1}), \prod_{j=1}^{n}(1 -\right.$
r'j2),j=1n(1−r'j3),j=1n(1−r'j4)]

$$= 1 \ominus [(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r_{ji})\right), (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r_{ji})\right)), (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r_{ji})\right)), (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r_{ji})\right)); (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r'_{ji})\right)), (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r'_{ji})\right)), (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r'_{ji})\right)), (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r'_{ji})\right))]$$

$$= [\{1 \ominus (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r_{ji})\right)\}, \{1 - (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r_{ji})\right)\}, \{1 - (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r_{ji})\right))\}, \{1 - (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r_{ji})\right))\}, \{1 - (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r_{ji})\right))\}; \{1 - (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r'_{ji})\right))\}, \{1 - (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r'_{ji})\right))\}, \{1 - (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r'_{ji})\right))\}, \{1 - (\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1 \ominus r'_{ji})\right))\}]$$

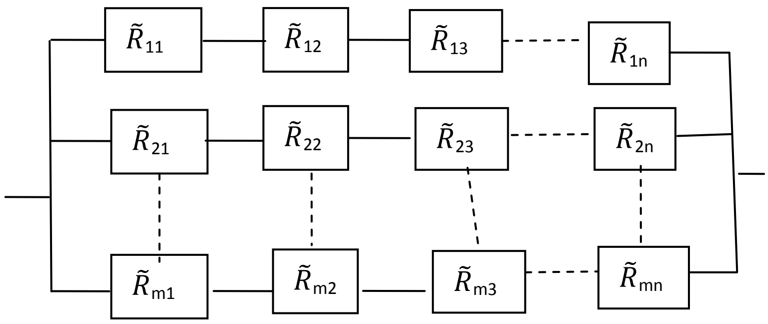

Fig. 6 Parallel-series networks

5.4 Series-parallel systems: - Consider a series-parallel network consisting of 'n' connections connected in parallel and each connection contains 'm' sub systems as shown in the figure. The vague reliability is given by $\tilde{R}_{sp} = {}_{k=i}^{n}\otimes(1 \ominus {}_{i=1}^{m}(1 - \tilde{R}_{ik}))$ of the series-parallel network shown in figure. Reliability can be evaluated by the proposed algorithm, where $\tilde{R}_{ik}$ represents the reliability of the k^th^ component at the i^th^ stage.

$$\widetilde{R_{sp}} = {}_{k=1}^{n}\otimes \{1 \ominus [(1 \ominus (r_{1k,}r_{2k}r_{3k}, r_{4k}; r'_{1k,}r'_{2k}, r'_{3k}, r'_{4k})) \otimes \ldots\ldots\ldots\ldots \otimes (1 \ominus (r_{mk}, r_{mk}, r_{mk}, r_{m4}; r'_{mk}, r'_{mk}, r'_{mk}, r'_{mk}))]$$

It can be approximated to a TrapVaN as

$= {}_{k=1}^{n}\otimes [1 - \prod_{j=1}^{n}(1 - r_{jk}), 1 - \prod_{j=1}^{n}(1 - r_{jk}), 1 - \prod_{j=1}^{n}(1 - r_{jk}), 1 - \prod_{j=1}^{n}(1 - r_{jk}); 1 - \prod_{j=1}^{n}(1 - r'_{jk}), 1 - \prod_{j=1}^{n}(1 - r'_{jk}), 1 - \prod_{j=1}^{n}(1 - r'_{jk}), 1 - \prod_{j=1}^{n}(1 - r'_{jk})]$

$= [\{\prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r_{jk})), \prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r_{jk})), \prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 - r_{jk})), \prod_{k=1}^{n}(1 - \prod_{j=1}^{n}(1 -$
rjk)};{k=1n(1−j=1n(1−r'jk)),k=1n(1−j=1n(1−r'jk)),k=1n(1−j=1n(1−r'jk)),k=1n(1−j=1n(1−r'jk))}]

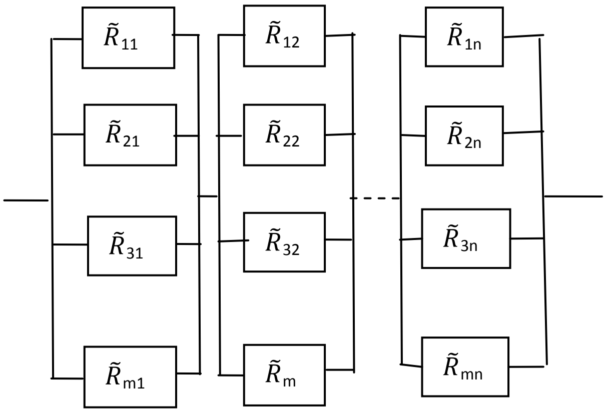

Fig. 7 Series-Parallel networks

VI. NUMERICAL COMPUTATIONS: -

A heavy current special machine demands continuous DC power supply during a particular period model [7 & 14] has been taken to illustrate our algorithm as shown in figure. The required power can be made available through convertor. In order to ensure uninterrupted power supply, two convertors are used, so that even if one fails, the other convertor provides the necessary current. The two convertors receive their power supplies from a sub-station which is connected to the main grid. We shall assume that the two convertors are basic components in addition to the grid and the sub-station. The machine becomes non-operative when there is no supply from the main grid, or when there if failure in the substation, or when both convertors fail to operate. The fault tree for the system is shown in figure 7.

Each event in this diagram is considered as the TrapVaN.

$\tilde{R}_1$ = represents the reliability of the grid failure (F_1)
$\tilde{R}_2$ = represents the reliability of the sub-station failure(F_2)
$\tilde{R}_3$ = represents the reliability of the switch of the DC supply to machine
$\tilde{R}_4$ = represents the reliability of the convertor I fails(F_4)
$\tilde{R}_5$ = represents the reliability of the convertor II fails(F_5)
$\tilde{R}_6$ = represents the reliability of power supply to the convertor
$\tilde{R}_7$ = represents the reliability of both convertors fail
$\tilde{R}_8$ = represents the reliability of DC power supply to the machine

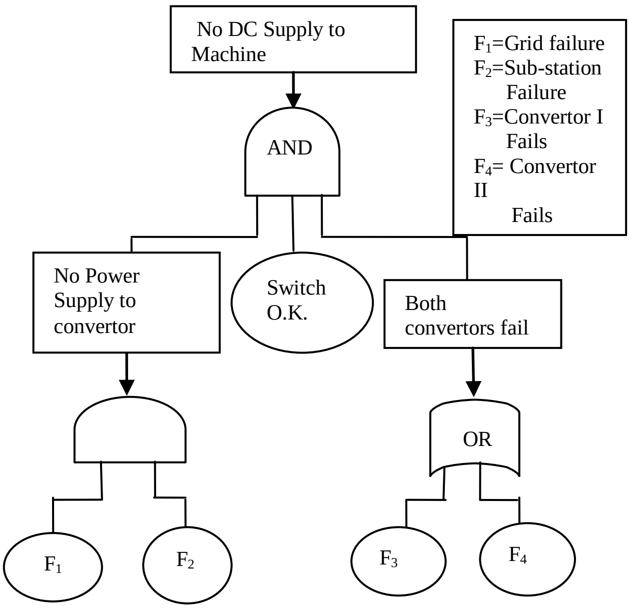

Fig. 8 Block Diagram for DC Power Supply

All the imprecise subsystems reliability $\tilde{R}_j$ are represented by TrapVaN $[\{(r_{j1,}r_{j2},r_{j3},r_{j4}),\ \mu_{ji}\};\{(r'_{j1},r'_{j2},r'_{j3},r'_{j4}),\ \nu_{ji}\}\big)\ for\ j=1\ldots..5.\ \&\ i=1,2,3,4\big]$. Let us evaluate the reliability of the desired system (No DC supply to machine)

The reliability value, for the occurrence of the event no power supply to the convertor, $\tilde{R}_6$:

$\tilde{R}_6=\tilde{R}_1\otimes\tilde{R}_2\cong$

$\{[(r_{11}r_{21},r_{12}r_{22},r_{13}r_{23},r_{14}r_{24}),\ \min(\mu_{ij},\mu_{ji})];[(r'_{11}r'_{21},r'_{12}r'_{22},r'_{13}r'_{23},r'_{14}r'_{24}),\min(\nu_{ij},\nu_{ji})]\}$

It is an Approximated TrapVaN.

Similarly, the reliability value, for the occurrence of event both convertors fail, $\tilde{R}_7$:

$\tilde{R}_7=1\ominus(1\ominus\tilde{R}_4)(1\ominus\tilde{R}_5)$

It is approximated to a TrapVaN as follows:

$$=\{[(1-\prod_{j=1}^{n}(1-r_{j1}),1-\prod_{j=1}^{n}(1-r_{j2}),1-\prod_{j=1}^{n}(1-r_{j3}),1-\prod_{j=1}^{n}(1-r_{j4});1-\prod_{j=1}^{n}(1-r''_{j1})),\min(\mu_{ij},\mu_{ji})],[(1-\prod_{j=1}^{n}(1-r'_{j2}),1-\prod_{j=1}^{n}(1-r'_{j3}),1-\prod_{j=1}^{n}(1-r'_{j4})),\min(\mu_{ij},\mu_{ji})]\}$$

By substituting the two above calculations and the given data value, we get the reliability value for the occurrence of the top event, no DC supply to machine, $\tilde{R}_8$:

$$\tilde{R}_8=\tilde{R}_1\otimes\tilde{R}_2\otimes\tilde{R}_3\otimes(1\ominus(1\ominus\tilde{R}_4)(1\ominus\tilde{R}_5))$$

It is approximated to a TrapVaN as follows

$$=\{[(r_{11}r_{21}r_{31}r_{41}(1-\prod_{j=1}^{5}(1-r_{j1})],r_{11}r_{22}r_{32}r_{42}[1-\prod_{j=1}^{5}(1-r_{j2})],r_{13}r_{23}r_{33}r_{43}[1-\prod_{j=1}^{5}(1-r_{j3})],r_{14}r_{24}r_{34}r_{44}[1-\prod_{j=1}^{5}(1-r_{j4})),\min(\mu_{ij},\mu_{ji})];[(r'_{11}r'_{21}r'_{31}r'_{41}[1-\prod_{j=1}^{5}(1-r'_{j1})],r'_{11}r'_{22}r'_{32}r'_{42}[1-\prod_{j=1}^{5}(1-r'_{j2})],r'_{13}r'_{23}r'_{33}r'_{43}[1-\prod_{j=1}^{5}(1-r'_{j3})],r'_{14}r'_{24}r'_{34}r'_{44}[1-\prod_{j=1}^{5}(1-r'_{j4})),\min(\mu_{ij},\mu_{ji})]\}$$

Let the vague reliability of events are

$$\tilde{R}_1=[(0.6931,0.7169,0.7989,0.8289);0.7],[(0.5898,0.6787,0.7804,0.8995);0.8]$$
$$\tilde{R}_2=[(0.6735,0.7865,0.8245,0.8925);0.7],[(0.6145,0.6736,0.7815,0.9125);0.8]$$
$$\tilde{R}_3=[(0.7818,0.8025,0.8992,0.9169);0.7],[(0.6735,0.8052,0.8289,0.9136);0.8]$$
$$\tilde{R}_4=[(0.7236,0.8034,0.8129,0.8912);0.7],[(0.6712,0.7028,0.8125,0.9024);0.8]$$
$$\tilde{R}_5=[(0.6134,0.6329,0.71848,0.8127);0.7],[(0.6022,0.7125,0.8043,0.9134);0.8]$$

So results for $\tilde{R}_6\ \ and\ \ \tilde{R}_7$ by using the calculations are as follows

$$\tilde{R}_6=[(0.4661,0.5639,0.6587,0.7387);0.7],[(0.3624,0.4572,0.6099,0.8208);0.8]$$
$$\tilde{R}_7=[(0.8931,0.8821,0.9382,0.9812);0.7],[(0.8467,0.9062,0.9572,0.9924);0.8]$$

By substituting the above two calculated values according to the data value, the reliability of the top event, no DC supply to the machine, $\tilde{R}_8$ is

$$\tilde{R}_8=[(0.4125,0.4942,0.5689,0.6748);0.7],[(0.3621,0.4642,0.5825,0.7845);0.8]$$

VII. CONCLUSION

In this research paper, I have proposed two algorithms one for a fuzzy failure rate estimation by using sugeno's fuzzy inference engine and another for the vague numbers by using the definition of (α,β)-cut method. Arithmetic operations of proposed Trapezoidal vague numbers (TarpVaN) are evaluated based on Vague (α,β)-cut method. Here, a method to analyze the vague reliability of network system which is based on vague set theory has been presented, where the components of the network system are trapezoidal vague numbers. (α, β)-cut of these TrapVaNs are evaluated. Arithmetic operations over these evaluated trapezoidal vague numbers through (α, β)-cut are used to analyze the vague reliability of the series, parallel, series-parallel and parallel-series network systems. The major advantage of using vague sets over the fuzzy sets is that vague sets are defined on the basis of membership and non-membership a function which separates the acceptance and rejection evidence for the membership of a connection in the network.

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