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Propagation of Plane Waves in a Rotating Magneto-Thermoplastic Fiber-Reinforced Medium with Voids under G-N Theory

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Abstract: The present study is concerned with the possibility of plane wave propagation in a rotating magneto-thermoelastic and fire-reinforced material with voids. Following the concepts of Cowin and Nunziato [1] in handling void materials and of Green and Nagdhi [3, 4] in dealing with thermoelastic behavior of solids, governing equations of motion for the problem have been written in tensor notation. In the present discussion Cowin's linear theory applicable to elastic materials with voids and Green and Nagdhi's generalized thermoelastic models II and III have been used. The possible velocity of plane wave propagation has been obtained as the solution of an algebraic equation involving a determinant whose elements contain the material parameters, the direction of applied magnetic field, rotation and the direction of plane wave propagation. A particular case derived from our general discussion has been investigated in detail and the numerical results based on parameter values have been presented graphically.

Key Words: Fibre-reinforced media, material with void, plane wave, magnetic permeability and electric conductivity, Green-Nagdhi models II and III

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I. INTRODUCTION

The purpose of generating more strength to a solid without adding too much weight to it or increasing its load bearing capacity is fulfilled through the process of fibre-reinforcing. Actually the process was followed in ancient times where people used horse hair in mortar and straws in mud to generate more strength in these materials during building of their houses. The process of fiber-reinforcing has been developing continuously with advanced technology and the products are in use in various fields. Carbon fibre is ideal as a strengthening member in pipes for deep water installations. Most concrete construction includes steel reinforcing, at least nominally. Fibre-reinforced materials are used for structures vulnerable to more or less violent vibrations during an earthquake and for similar disturbances. Study of wave propagation in fibre-reinforced medium can justify the effectiveness of fibre-reinforcing in civil engineering and geophysics.

Governing equations in classical theory of elasticity are based on one intuitive assumption that a solid is a continuum. There is no denying the fact that although this assumption is valid for a wide class of solids, there remains a lot where this assumption seems to be inadequate. Geological materials like rocks and soils, and manufactured materials like ceramics and pressed powder and many others belong to this class where material voids play quite a significant role. To study the effects of loadings on such materials Cowin and Nunziato [1] developed a new theory in which they have introduced a new parameter $\emptyset$ depending on the change in local volume fraction of the solid, in the stress-strain relations. Some basic theorems related to materials with voids are discussed by Cowin [6], Goodman and Cowin [7], Cowin and Nunziato [8], Puri [9], Chandrasekharaiah [13] and Issan [11]. They developed a linear theory applicable to elastic materials with voids for the mathematical study of the mechanical behavior of such materials. The modified linear theory when applied to the propagation of longitudinal waves in a porous medium shows some distinct characters of its own. The propagation of longitudinal waves in an elastic medium is seen to be significantly affected due to the presence of voids in the medium while the transverse wave propagation remains unaffected. Increasing uses of these materials suggest that the study of solid mechanics problems needs to be extended to fibre-reinforced media as well as medium with voids.

It is known that mechanical loadings are not the only cause for deformation in elastic solids; thermal ladings also can play a vital role in producing deformations in structures and machines subject to generation and flow of heat. Thermoelasticity takes care of the deformations and stresses produced due to thermal loadings along with the deformation and stresses produced due to mechanical loadings. Clearly the governing equations of motion in thermoelasticity consist of the coupled equations involving mechanical and thermal stresses and the equation governing heat flow in the solid. The parabolic type of heat conduction equation as it was adopted initially in the classical theory encountered a serious drawback in the sense that the speed of the heat propagation in the solid could be infinite, which is absolutely unrealistic. In order to get the parabolic heat conduction equation replaced by the hyperbolic type heat conduction equation scientists developed a number of models. Green and Nagdhi have formulated three models (I, II, III) of thermoelasticity for isotropic homogeneous materials [2, 3, 4]. Model I of Green and Nagdhi theory after linearization reduces to the classical theory of thermoelasticity. Energy dissipation has not been taken into consideration in model II while model III takes care of it. Both the models of type II and III imply a finite speed of propagation for heat waves. Investigation of various problems characterizing the two theories has been discussed by Chandrasekharaiah [13, 14]. Further modifications in the constitutive equations of thermoelasticity were done by Green and Nagdhi [2, 3] to accommodate a wider class of heat flow problems.

Another interesting field of recent study is the field of magneto-thermoelasticity in which interacting effects of applied magnetic field on elastic and thermal deformations of a solid are studied. Such studies have applications in several areas, particularly in nuclear devices, biomedical engineering and geomagnetic investigations. Some of the works related to the interaction of the electromagnetic field, the thermal field and the electric field may be available in literature viz, Abd-Alla and Al-Dawy [15, 16], Ezzat and Othman [17], Ezzat [18], Ezzat et al. [19], Wang et al. [20], Othman and Song [21], Othman [22], Othman et al. [23], Othman and Said [24] etc. A number of discussions relating wave propagation in rotating isotropic or transversely isotropic media was reported in literature, some of which are the works of RoyChoudhuri [25], Gupta and Gupta [26], Singh [30] etc. RoyChoudhuri and Banerjee [31] studied the propagation of time-harmonic coupled electromagnetoelastic dilatational thermal shear waves using the thermoelasticity theory of type II [3]. Thermoelastic plane waves in a rotating transversely isotropic medium has been studied by Ahmad and Khan [32]. A number of discussion relating to fibre-reinforced materials was made by England and Rogers [33], Belfield et al. [34], Othman [36], Othman [37], Markham [38], Zorammuana [39] etc.

The present discussion aims at investigating the propagation of plane waves in a rotating thermoelastic fiber-reinforced medium with and without energy dissipation under Green-Nagdhi model. A magnetic field of uniform magnitude is supposed to be acting on the medium but there is no body force. Fiber-reinforcing of general type has been considered and the governing equations of motion are framed taking into account of the thermoelastic characteristics of the material, rotational effects and the applied magnetic field. Equations have been presented using tensor notations. Possibilities of plane wave propagation in the medium have been studied in this discussion. Effects of rotation, applied magnetic field, and temperature of the material on plane wave propagation have been examined severally and jointly. Some numerical results have been presented in the form of graphs based on the particular values of the parameters involved to assess the effects of various parameters on the plane wave propagation in the medium.

II. NOMENCLATURE

τ_{ij} , ε_{ij} = the Cartesian components of the stress and strain tensor
u_i = displacement vector
λ, μ_T = elastic constants
φ= the volume fraction field
$\alpha^*,\ \beta^*,\ \mu_L - \mu_T$ = reinforcement parameters
ν= is a temperature parameter
ρ = the material density
Ω = angular velocity vector
H , h, H_0 = magnetic field, induced magnetic field and applied magnetic field
J = current density
E = induced electric field
ϵ_0 , μ_0 = electrical conductivity and magnetic permeability
T^*= reference temperature
K = coefficient of thermal conductivity
K^* = additional material constant
c_e = specific heat of the solid at constant strain
α_T= coefficient of linear thermal expansion
$\xi, \alpha, \beta, \omega_d, \bar{k}$ = void constants
ω = wave number

III. FORMULATION OF THE PROBLEM AND SOLUTION

Following Belfield et al. [34] the stress-strain relations for linearly fibre- reinforced elastic medium may be expressed in tensor form as

$$\tau_{ij}=\lambda\epsilon_{kk}\ \delta_{ij} + 2\mu_T\ \epsilon_{ij} + \alpha^*\left(a_k a_m \epsilon_{km}\delta_{ij} + a_i a_j \epsilon_{kk}\right) + 2(\mu_L-\mu_T)$$
$$\left(a_i a_k \epsilon_{kj} + a_j a_k \epsilon_{ki}\right) + \beta^*\left(a_k a_m a_i a_j \epsilon_{km}\right) + \beta\varphi\delta_{ij} - \nu T\delta_{ij} \quad (1)$$

where τ_{ij} are the Cartesian components of the stress tensor;

$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ are the strain components, related to the displacement vector u_i ; λ, μ_T are elastic constants; α^*, β^*, $(\mu_L-\mu_T)$ are reinforcement parameters; ν is a temperature parameter; φ is the volume fraction field and $\boldsymbol{a} = (a_1, a_2, a_3)$ such that $a_1^2 + a_2^2 + a_3^2 = 1.$

For a rotating elastic medium the equation of motion, in absence of body force, can be written as

$$\tau_{ij,j} = \rho\left[\ddot{u}_i + \{\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u}) + 2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}}\}_i\right] \quad (2)$$

In (2), ρ denotes the material density, $\boldsymbol{\Omega}$ is the angular velocity vector, overhead dot represents differentiation with respect to time and the suffix i after second bracket represent the i th component of the vector inside.

If, in addition, the solid is under the action of magnetic field $\boldsymbol{H}$, then the governing field equations involving the displacement $\boldsymbol{u} = u_i(\boldsymbol{x};\ t)$ and the temperature $T(x;\ t)$, for a fiber-reinforced void material with rotation in absence of body force may be written as [Maity et al. [28, 29]]

$$(\lambda+\mu_T) + u_{k,ki} + \mu_T u_{i,kk} + \alpha^*\left(a_k a_m u_{k,mi} + a_i a_j u_{k,kj}\right) + (\mu_L-\mu_T)[a_i a_k u_{k,jj} + a_i a_k u_{j,kj}$$
$$+ a_j a_k\left(u_{k,ij} + u_{i,kj}\right)] + \beta\varphi_{,i} - \nu T_{,i} + \beta^* a_i a_j a_k a_m u_{k,mj} + (J\times B)_i$$
$$= \rho\left[\ddot{u}_i + \{\Omega\times(\Omega\times u) + 2\Omega\times\dot{u}\}_i\right] \quad (3)$$

$$\alpha\varphi_{,kk} - \omega_d\dot{\varphi} - \xi\varphi - \beta u_{k,k} + mT = \rho\bar{k}\ddot{\varphi} \quad (4)$$

and $K\dot{T}_{,kk} + K^*T_{,kk} - mT^*\varphi_{,t} - \nu T^*\dot{u}_{k,k} = \rho c_e\ \ddot{T}$ (5)

where $K = 0$ represents the heat conduction without energy dissipation. α, ξ, β, ω_d, $\bar{k}$ are void constants. The term $\mathbf{J}\times\mathbf{B}$ in (3) arises from the presence of the applied magnetic field. Due to the application of the initially applied magnetic field $\mathbf{H_0}$, an induced magnetic field $\mathbf{h}$, an induced electric field $\mathbf{E}$ and a current density $\mathbf{J}$ are developed. For a slowly moving homogeneous electrically conducting medium, the simplified system of linear equations of electrodynamics are

$$\boldsymbol{\nabla}\times\boldsymbol{h} = \boldsymbol{J} + \epsilon_0\dot{\boldsymbol{E}},\ \boldsymbol{\nabla}\times\boldsymbol{E} = -\mu_0\dot{\boldsymbol{h}},\ \boldsymbol{\nabla}\cdot\boldsymbol{h} = 0,\ \boldsymbol{E} = -\dot{\boldsymbol{u}}\times\boldsymbol{B} \quad (6)$$

Where ϵ_0 is the electrical conductivity and μ_0 is the magnetic permeability so that $\mathbf{B} = \mu_0\mathbf{H}$ is the magnetic field in the medium due to total magnetic field $\mathbf{H} = \mathbf{H_0} + \mathbf{h}$, arising from applied field $\boldsymbol{H_0}$ and induced field $\boldsymbol{h}$.

If we assume that $\mathbf{H_0} = (\mathrm{H_{01}}, \mathrm{H_{02}}, \mathrm{H_{03}})$ and $\boldsymbol{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$, then utilizing relations (6) and neglecting products of $\mathbf{h}$ with $\mathbf{u}$ and its derivatives, the governing equations of motion (3), (4) and (5) for a void medium in thermoelasticity with and without energy dissipation under the action of applied magnetic field and rotation may be written in tensor notation as

$$(\lambda+\mu_T) + u_{k,ki} + \mu_T u_{i,kk} + \alpha^*\left(a_k a_m u_{k,mi} + a_i a_j u_{k,kj}\right)$$
$$+ (\mu_L-\mu_T)\left[a_i a_k u_{k,jj} + a_i a_k u_{j,kj} + a_j a_k\left(u_{k,ij} + u_{i,kj}\right)\right] + \beta^* a_i a_j a_k a_m u_{k,mj} + \mu_0 H_0^2 u_{j,ji}$$
$$- \mu_0 H_{0k} H_{0i} u_{j,jk} - \mu_0 H_{0m} H_{0k}\left(u_{k,im} + u_{i,km}\right) - \mu_0^2\epsilon_0 H_0^2(\ddot{u}_i - \ddot{u}_k) + \beta\varphi_{,i} - \nu T_{,i}$$
$$= \rho\left[\ddot{u}_i + \Omega_k\Omega_i u_k - \Omega^2 u_i + 2\epsilon_{ijk}\Omega_j\dot{u}_k\right] \quad (7)$$

$$\alpha\varphi_{,kk} - \omega_d\dot{\varphi} - \xi\varphi - \beta u_{k,k} + mT = \rho\bar{k}\ddot{\varphi} \quad (8)$$

$$K\dot{T}_{,kk} + K^*T_{,kk} - mT^*\varphi_{,t} - \nu T^*\dot{u}_{k,k} = \rho c_e\ \ddot{T} \quad (9)$$

Where T^* is the reference temperature, $\nu = (3\lambda+2\mu)\alpha_t$, K is the coefficient of thermal conductivity, K^* is the additional material constant, ρ is the mass density, c_e is the specific heat of the solid at constant strain, α_t is the coefficient of linear thermal expansion, λ and μ are Lames constants. In (7), ϵ_{ijk} represents the Levi-civita tensor which has a non-zero value only if i, j, k are all distinct and has a value 1 if i, j, k are in cyclic order, whereas, it has a value -1 when they are acyclic.

IV. PLANE WAVE PROPAGATION

In order to examine the possibility of a plane wave propagation in the medium under consideration we shall assume a solution of governing equations (7), (8) and (9) in the form

$$(u_i, \varphi, T) = (Ap_i, B, C)\exp\{\tau\,(q\,n_s x_s - \omega t)\},\ i = 1, 2, 3;\ \tau = \text{-}1 \quad (10)$$

The speed of the wave is $c_n = \frac{\omega}{Re(q)}$ (11)

The direction of plane wave propagation is represented by the unit vector $\boldsymbol{n} = (n_1, n_2, n_3)$, while the direction of particle displacement is denoted by the unit vector $\boldsymbol{p} = (p_1, p_2, p_3)$. A, B and C appearing in (10) are constants.

Substituting (10) into (7), (8), (9) and using $\tau^2 = -1$, $n_k n_k = 1$ we get

$$(n_{ik}p_k + \theta p_i)A - (\beta\tau n_i q)B + (\nu\tau n_i q)C = 0 \tag{12}$$
$$(\beta\tau q n_k p_k)A - \left[\left(\rho\bar{k}\omega^2 - \xi\right) + \alpha q^2 + \tau\omega\omega_d\right]B + mC = 0 \tag{13}$$
$$(\nu T^*\omega^2\tau q p_k n_k)A + (mT^*\tau\omega)B + [(k\tau\omega - k^*)q^2 + \rho c_e\omega^2]C = 0 \tag{14}$$

where $\eta_{ik} = q^2[F_{ik} + M_{ik}] + R_{ik}$

$$F_{ik} = (\lambda + \mu_T)n_i n_k + \alpha^*\left(a_k a_m n_i n_m + a_i a_j n_j n_j\right) + (\mu_L - \mu_T)\left(a_i a_k + a_i a_l n_k n_l + a_j a_k n_i n_j\right) + \beta^* a_i a_j a_k a_m n_m n_j$$

$$M_{ik} = \mu_0 H_0^2 n_i n_k - \mu_0 H_{0i} H_{0j} n_j n_k - \mu_0 H_{0m} H_{0k} n_i n_m + \frac{\mu_0^2\epsilon_0\omega^2 H_{0k} H_{0i}}{q^2} \tag{15}$$

$$R_{ik} = \rho\Omega_k\Omega_i - 2\rho\epsilon_{ijk}\,\omega\tau\Omega_j$$

$$\theta = \left[\mu_T + (\mu_L - \mu_T)a_j a_k n_k n_j\right] - \mu_0^2\epsilon_0\omega^2 H_0^2 + q^2\mu_0 H_{0m} H_{0k} n_m n_k - \rho(\Omega^2 + \omega^2)$$

Eliminating A, B and C from (12), (13), (14) we get

$$\begin{vmatrix} (\eta_{ik} + \theta\delta_{ik})p_k & -\beta\tau n_i q & \nu\tau n_i q \\ \beta\tau q n_k p_k & -\left[\left(\rho\bar{k}\omega^2 - \xi\right) + \alpha q^2 + \tau\omega\omega_d\right] & m \\ \nu T^*\omega^2\tau q p_k n_k & mT^*\tau\omega & [(k\tau\omega - k^*)q^2 + \rho c_e\omega^2] \end{vmatrix} = 0 \tag{16}$$

The determinantal equation (16) yields an algebraic equation in q^2 with complex coefficients which will determine the wave speed c_n in (11). It is clear that the velocity of the plane wave propagation depends on the elastic behavior of the fibre-reinforced material, direction n_i of propagation of the wave, applied magnetic field, temperature coefficients, rotation of the medium and void character of the material. As a particular derivation from our general results above we consider propagation of a longitudinal plane wave for which the directions of particle displacement p_i and the direction of wave propagation n_i are the same. Accordingly, $p_i = n_i$ and $p_i\, n_i = 1$. In this case we consider a fibre reinforced elastic body with fibre-reinforcing direction $\boldsymbol{a} = (0, 0, 1)$ is rotating with uniform angular velocity $\boldsymbol{\Omega} = \Omega(0, 0, 1)$ (Fig.1). Let us suppose that a uniform magnetic field $\mathbf{H_0} = \mathrm{H}_0(0, 1, 0)$ is applied to the body. We investigate propagation of a plane wave in the medium in a direction specified by the unit vector $\boldsymbol{n} = (0, 0, 1)$.
In the present case equation (16) transforms into

$$\begin{vmatrix} q^2 X + Y & -\beta\tau q & \nu\tau q \\ \beta\tau q & -\left[\left(\rho\bar{k}\omega^2 - \xi\right) + \alpha q^2 + \tau\omega\omega_d\right] & m \\ \nu T^*\omega^2\tau q & mT^*\tau\omega & [(k\tau\omega - k^*)q^2 + \rho c_e\omega^2] \end{vmatrix} = 0 \tag{17}$$

where $X = (\lambda + \mu_T) + 2\alpha^* + 3(\mu_L - \mu_T) + \beta^* + \mu_0 H_0^2$
and $Y = 3\mu_L - 3\rho(\Omega^2 + \omega^2) - 2\epsilon_0\mu_0^2 H_0^2\omega^2 + \rho\Omega^2$

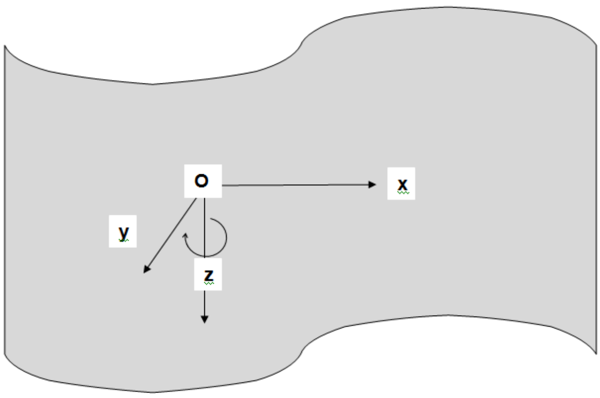

Fig. 1 Geometry of the problem

The determinantal equation (17) can be decomposed into two algebraic equations of velocity, v

$$a_0 v^6 + a_1 v^4 + a_2 v^2 + a_3 = 0 \tag{18}$$
and
$$b_0 v^6 + b_1 v^4 + b_2 v^2 + b_3 = 0 \tag{19}$$
where $a_0 = Y\rho c_e\left(\rho\bar{k}\omega^2 - \xi\right)$

$$a_1 = Y\left[\alpha\rho c_e\omega^2 - k^*\left(\rho\bar{k}\omega^2 - \xi\right) - k\omega^2\omega_d\right] + X\rho c_e\omega^2\left(\rho\bar{k}\omega^2 - \xi\right) - m\beta\nu T^*\omega^2 + \beta^2\rho c_e\omega^2 + \nu^2T^*\omega^2\left(\rho\bar{k}\omega^2 - \xi\right)$$

$a_2 = X\left[\alpha\rho c_e\omega^2 - k^*\left(\rho\bar{k}\omega^2 - \xi\right) - k\omega^2\omega_d\right] - Y\alpha k^*\omega^2 - \beta^2k^*\omega^2 + \alpha\nu^2T^*\omega^4$
$a_3 = -X\alpha k^*\omega^4$
$b_0 = Y(\rho c_e\omega^2\omega_d + m^2T^*)$
$b_1 = \left[X(\rho c_e\omega^2\omega_d + m^2T^*) + Y\left\{k\left(\rho\bar{k}\omega^2 - \xi\right) - k^*\omega_d\right\} + m\beta\nu T^* + \nu^2T^*\omega^2\omega_d\right]\omega^2$
$b_2 = [\alpha kY + X\{k(\rho\bar{k}\omega^2 - \xi) - k^*\omega_d\} + \beta^2k]\omega^4$
$b3 = X\alpha k\omega^6$

Using Cardan's method in equations (18) and (19), as Singh and Tomar [35], we obtain respectively

$$Z_1^3 + 3H_1Z_1 + G_1 = 0 \tag{20}$$

and $$Z_2^3 + 3H_2Z_2 + G_2 = 0 \tag{21}$$

where

$$Z_1 = a_0v^2 + \frac{a_1}{3}, \qquad H_1 = \frac{a_0a_2}{3} - \frac{a_1^2}{9} \ \ and \ \ G_1 = \frac{2a_1^3}{27} - \frac{a_0a_1a_2}{3} + a_0^2a_3$$

$$Z_2 = b_0v^2 + \frac{b_1}{3}, \qquad H_1 = \frac{b_0b_2}{3} - \frac{b_1^2}{9} \ \ and \ \ G_1 = \frac{2b_1^3}{27} - \frac{b_0b_1b_2}{3} + b_0^2b_3$$

The roots of the equation (20) and (21) are given by

$$Z_{11} = S_1, \qquad Z_{12} = \frac{1}{2}\left(-S_1 + \tau\sqrt{3}T_1\right), \qquad Z_{13} = \frac{1}{2}\left(-S_1 - \tau\sqrt{3}T_1\right);$$

and $Z_{21} = S_2, Z_{22} = \frac{1}{2}\left(-S_2 + \tau\sqrt{3}T_2\right),\ Z_{23} = \frac{1}{2}\left(-S_2 - \tau\sqrt{3}T_2\right)$ respectively

where $S_1 = U_1 + W_1, T_1 = U_1 - W_1, U_1^3 = \frac{1}{2}\left[-G_1 + \sqrt{G_1^2 + 4H_1^3}\right], W_1 = -\frac{H_1}{U_1}$;

and $S_2 = U_2 + W_2, T_2 = U_2 - W_2, U_2^3 = \frac{1}{2}\left[-G_2 + \sqrt{G_2^2 + 4H_2^3}\right], W_2 = -\frac{H_2}{U_2}$;

Hence, the three roots of equations (18) and (19) are respectively given by

$$V_1^2 = \frac{1}{a_0}\left(S_1 - \frac{a_1}{3}\right), \qquad V_{11}^2 = \frac{1}{a_0}\left(-\frac{1}{2}S_1 + \frac{\sqrt{3}}{2}\tau T_1 - \frac{a_1}{3}\right),$$

$$V_{12}^2 = \frac{1}{a_0}\left(-\frac{1}{2}S_1 - \frac{\sqrt{3}}{2}\tau T_1 \ \frac{a_1}{3}\right) \tag{22}$$

and $$V_2^2 = \frac{1}{b_0}\left(S_2 - \frac{b_1}{3}\right),\ V_{21}^2 = \frac{1}{b_0}\left(-\frac{1}{2}S_2 + \frac{\sqrt{3}}{2}\tau T_2 - \frac{b_1}{3}\right),$$

$$V_{22}^2 = \frac{1}{b_0}\left(-\frac{1}{2}S_2 - \frac{\sqrt{3}}{2}\tau T_2 - \frac{b_1}{3}\right) \tag{23}$$

For real roots of U_1 and U_2 the above equations (22) and (23) represent the possible real velocities V_1 and V_2 as

$$V_1^2 = \frac{1}{a_0}\left(Re(S_1) - \frac{a_1}{3}\right) \tag{24}$$

$$V_2^2 = \frac{1}{b_0}\left(Re(S_2) - \frac{b_1}{3}\right) \tag{25}$$

V. NUMERICAL RESULTS AND DISCUSSIONS

The present study focuses on the effects of fibre reinforcing, rotation, magnetic field and void pores of the medium on the propagation of plane wave in a solid. For numerical discussion we have considered three sets of values of relevant parameters from the works of Othman et al.[22], Markham[38], Zorammuana [39] as given below;

$\lambda = 9.4 \times 10^9 N.m^{-2}, \quad \mu_T = 1.89 \times 10^9 N.m^{-2},\ \mu_L = 2.45 \times 10^9 N.m^{-2}, \quad \rho = 1.7 \times 10^3 Kg.m^{-3},$
$\lambda = 5.65 \times 10^9 N.m^{-2},\ \mu_T = 2.46 \times 10^9 N.m^{-2},\ \mu_L = 5.66 \times 10^9 N.m^{-2}, \quad \rho = 2.26 \times 10^3 Kg.m^{-3}$
$\lambda = 7.59 \times 10^9 N.m^{-2},\ \mu_T = 3.5 \times 10^9 N.m^{-2},\ \mu_L = 7.07 \times 10^9 N.m^{-2}, \quad \rho = 1.6 \times 10^3 Kg.m^{-3}$
$\alpha = 3.668 \times 10^{-4} N, \beta = 1.13849 \times 10^{11} N.m^{-2}, \xi = 1.475 \times 10^{12} N.m^{-2}, \bar{k} = 1.753 \times 10^{-15} N.m^{-2},$
$\alpha^* = -1.28 \times 10^9 N.m^{-2}, \beta^* = 0.32 \times 10^9 N.m^{-2}$

Using the above parameter values the velocities of plane wave propagation are found from equations (24) and (25) and the nature of plane wave propagation following GN-II and GN-III models have been examined.
Numerical results based on the considered parameter values show that the wave velocities V_1 and V_2 have almost similar behaviour in respect of GN-II and GN-III models, as is evident from the graphs presented in Figs. 2 and 3. It is clear that when ω is fixed V_1 and V_2 decrease with the increase of Γ (Figs. 2(a), 2(c) and 3(a), 3(c)).

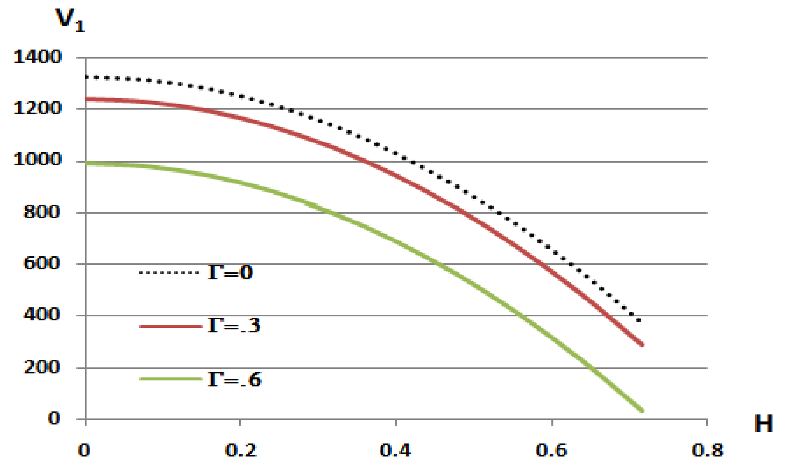

Fig. 2(a). Effect of rotation on wave velocity for G-N model II (ω=0.2)

On the other hand when Γ is fixed, V_1 increases and V_2 decreases with the increase of ω (Figs. 2(b), 2(d) and 3(b), 3(d)).

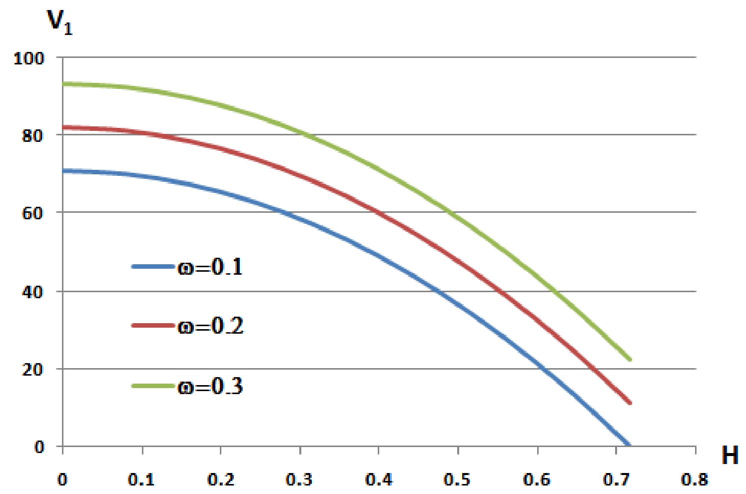

Fig. 2(b). Effect of wave number on wave velocity for G-N model II (Γ=0.4)

Figs. 2(e), 2(f) and 3(e), 3(f) show that when both ω and Γ are fixed, increase of reinforcing parameter ($\mu_L - \mu_T$ decreases $V1$ and $V2$. Figs. 4(a), 4(b) show that for fixed ω and Γ, $V1$ and $V2$ values in GN-III model exceed those in GN-II model. Fig. 5 indicates the effects of reinforcing parameter on wave velocities.

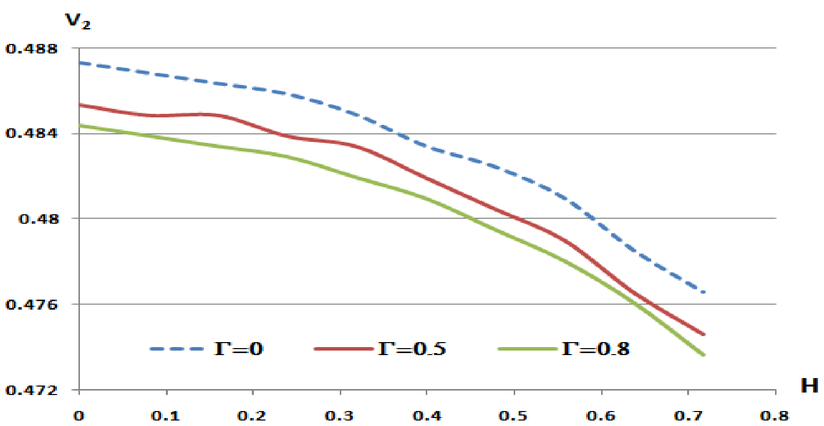

Fig. 2(c). Effect of rotation on wave velocity for G-N model II (ω=0.2)

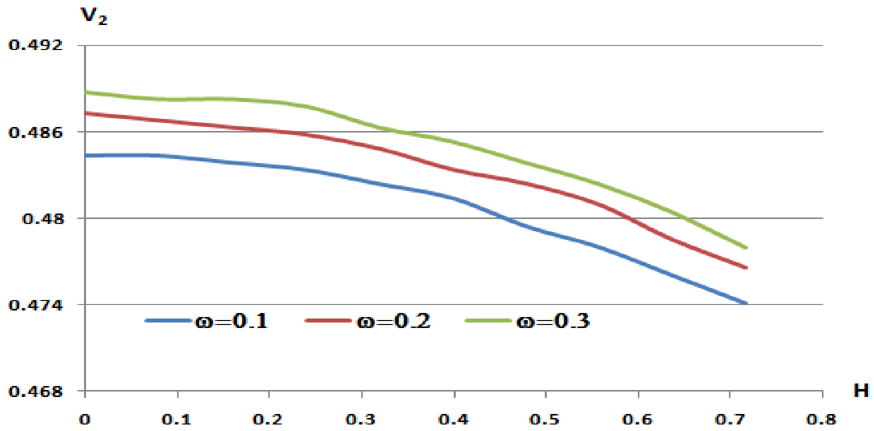

Fig. 2(d). Effect of wave number on wave velocity for G-N model II (Γ=0.4)

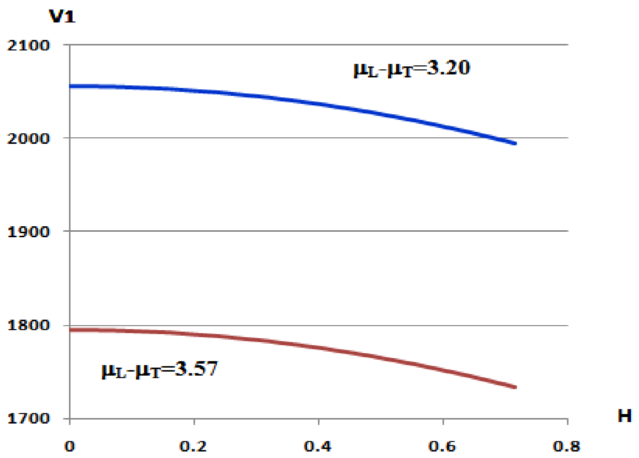

Fig. 2(e). Effect of reinforcing parameter on wave velocity for G-N model II (ω=0.2 and Γ=0.4)

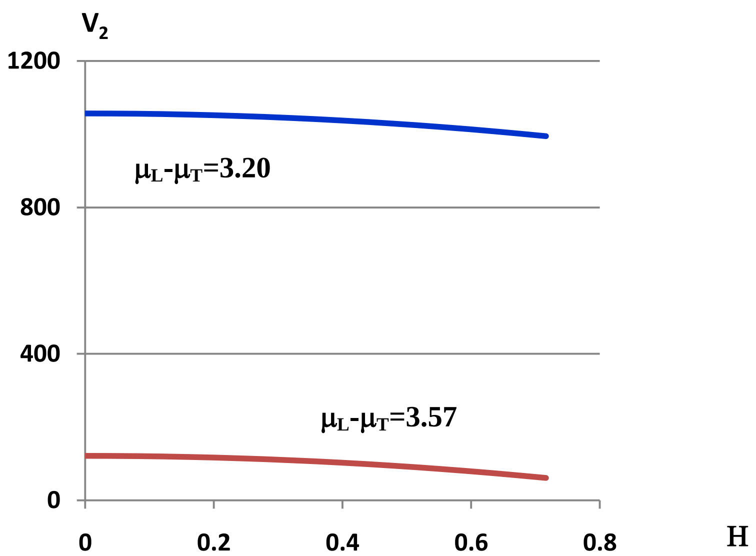

Fig. 2(f). Effect of reinforcing parameter on wave velocity for G-N model II (ω=0.2 and Γ=0.4)

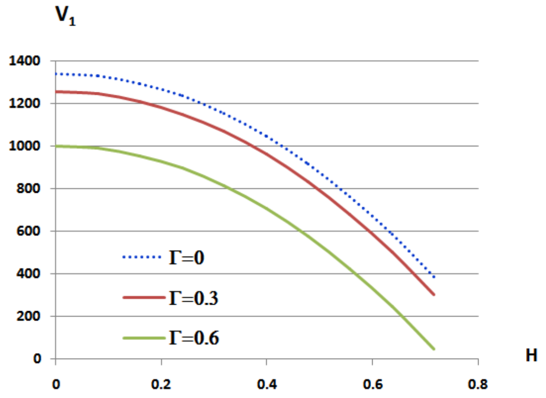

Fig. 3(a). Effect of rotation on wave velocity for G-N model III (ω=0.2)

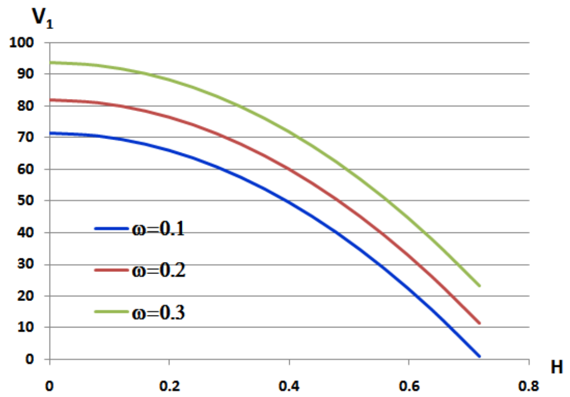

Fig. 3(b). Effect of wave number on wave velocity for G-N model III (Γ=0.4)

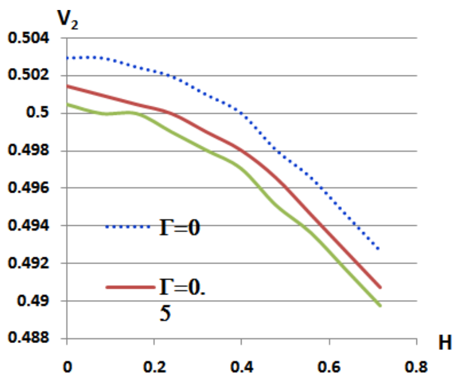

Fig. 3(c). Effect of rotation on wave velocity for G-N model III (ω=0.2)

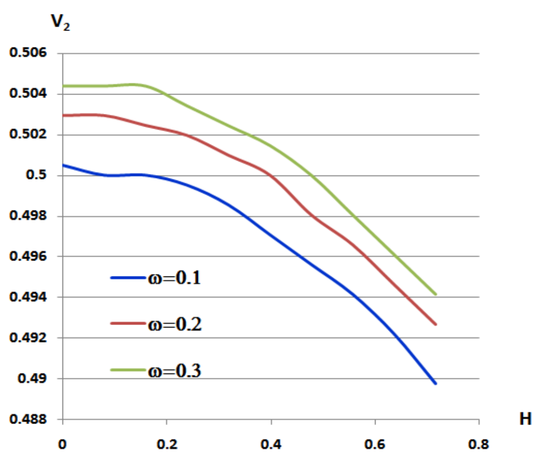

Fig. 3(d). Effect of wave number on wave velocity for G-N model III (Γ=0.4)

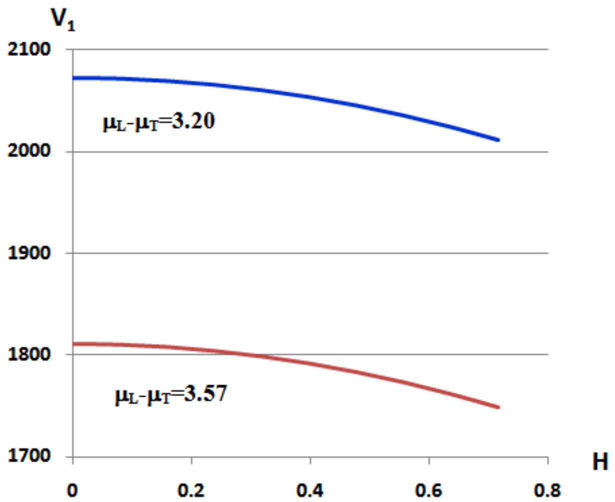

Fig. 3(e). Effect of reinforcing parameter on wave velocity for G-N model III (ω=0.2 and Γ=0.4)

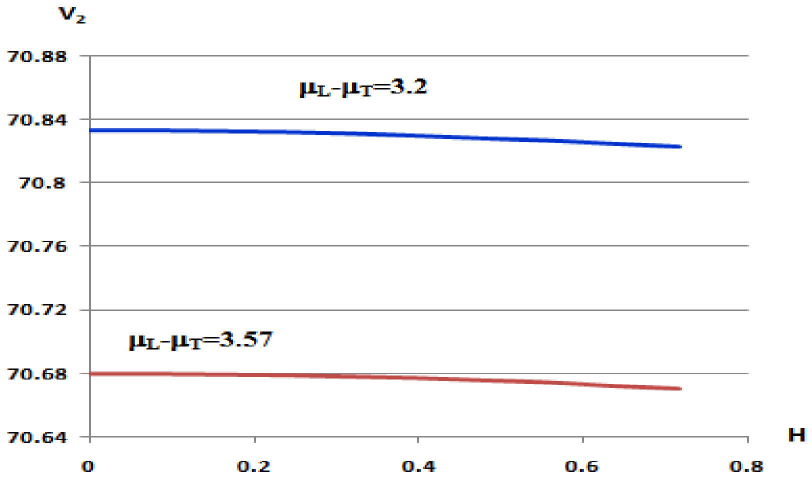

Fig. 3(f). Effect of reinforcing parameter on wave velocity for G-N model III (ω=0.2 and Γ=0.4)

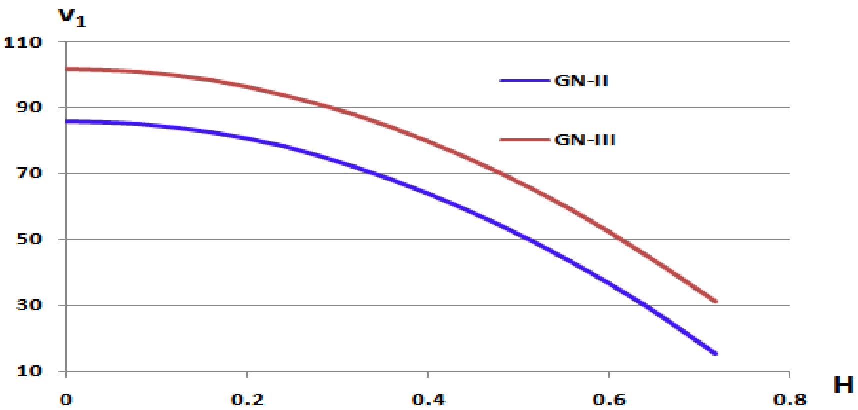

Fig. 4(a) Comparison of the velocities of wave propagation for GN-II and GN-III models (ω=0.2, Γ=0.4)

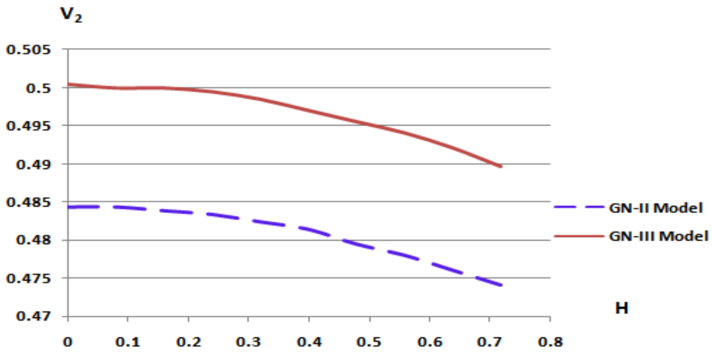

Fig. 4 (b) Comparison of the velocities of wave propagation for GN-II and GN-III models (ω=0.2, Γ=0.4)

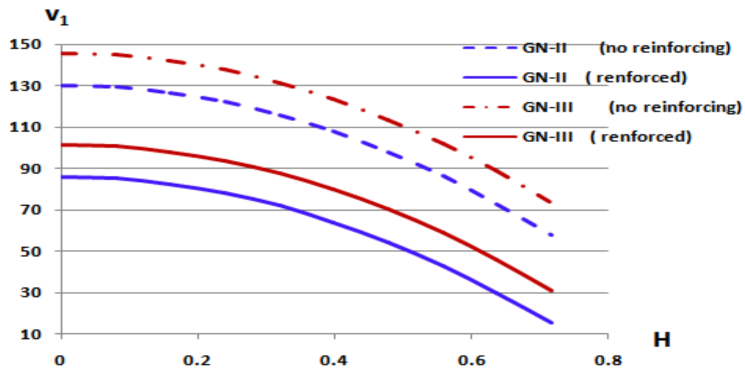

Fig. 5 Effect of fibre-renforcing on velocity of propagation for GN-II and GN-III models (ω=0.2, Γ=0.4)

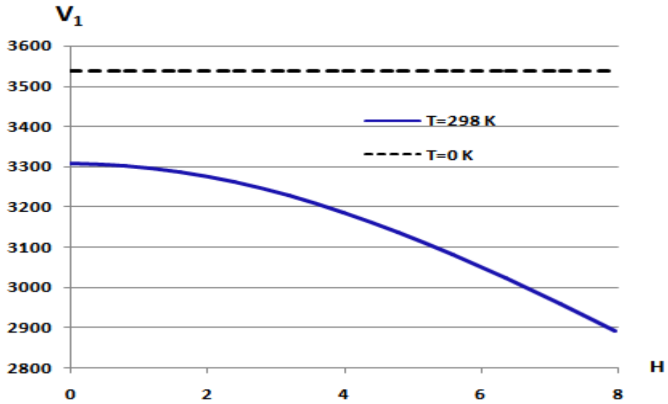

Fig. 6 Effect of temperature on wave velocity V_1 in absence of voids in the medium

It is seen that for fixed ω and Γ, reinforcing decreases the wave velocity V_1 for both the models GN-II and GN-III. Fig. 6 gives an idea for the effect of temperature on wave velocities. It is found that for medium having no voids, the effect of temperature is to decrease V_1 with the increase of magnetic field.

VI. CONCLUSION

In the present paper the numerical calculations carried out for wave's propagation thermally conducting fiber-reinforced void media under the action of uniform magnetic field using Green-Naghdi theory of both type II and III leads to following conclusions.

1.There are significant differences in the field quantities under Green-Naghdi theory of both type II and III.
2.The magnetic effect has a significant effect on the distribution of the field quantities.
3.The method that was used in this article is applicable to a wide range of problems in hydrodynamics.
4.The effects of reinforcement, void, temperature are more influence Green-Naghdi theory of III than Green-Naghdi theory of II.
5.The all physical quantities are very depending on all types of velocities.

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