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Double Sequence Spaces of Fuzzy Real Numbers of Paranormed Type Under An Orlicz Function

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Abstract: In this article we study different properties of convergent, null and bounded double sequence spaces of fuzzy real numbers defined by an Orlicz function. We study different properties like completeness, solidness, symmetricity etc.

Keywords: Fuzzy real number, solid space, symmetric space, completeness.

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I. INTRODUCTION

The notion of fuzzyness is widely applicable in many branches of Engineering and Technology. Throughout, a double sequence is denoted by $<X_{nk}>$, a double infinite array of elements X_{nk}, where each X_{nk} is a fuzzy real number.

The initial works on double sequences is found in Bromwich [2]. Later on it was studied by Hardy [4], Moricz [7], Basarir and Sonalcan [1], Sarma [11], Tripathy and Sarma [12] and many others. Hardy [4] introduced the notion of regular convergence for double sequences.

The concept of paranormed sequences was studied by Nakano [8] and Simmons [10] at the initial stage. Later on it was studied by many others.

An *Orlicz function M* is a mapping $M:[0, \infty) \to [0, \infty)$ such that it is *continuous*, *non-decreasing* and *convex* with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$, as $x \to \infty$.

Let D denote the set of all closed and bounded intervals $X = [a_1, a_2]$ on R, the real line. For $X, Y \in D$ we define

$$d(X, Y) = \max(|a_1 - b_1|, |a_2 - b_2|),$$

where $X = [a_1, a_2]$ and $Y = [b_1, b_2]$. It is known that (D, d) is a complete metric space.

A *fuzzy real number* X is a fuzzy set on R, *i.e.* a mapping $X : R \to I$ (=[0,1]) associating each real number t with its grade of membership $X(t)$.

The α - *level* set $[X]^\alpha$ of the fuzzy real number X, for $0 < \alpha \le 1$, defined as $[X]^\alpha = \{ t \in R : X(t) \ge \alpha \}$.

A *fuzzy real number X* is called *convex* if $X(t) \ge X(s) \wedge X(r) = \min(X(s), X(t))$, where $s < t < r$.

If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number X is called *normal*.

A fuzzy real number X is said to be *upper-semi continuous* if, for each $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$, for all $a \in I$ is open in the usual topology of R.

The set of all upper-semi continuous, normal, convex fuzzy real numbers is denoted by $R(I)$ and throughout the article, by a fuzzy real number we mean that the number belongs to $R(I)$.

The set R of all real numbers can be embedded in $R(I)$. For $r \in R$, $\bar{r} \in R(I)$ is defined by

$$\bar{r}(t) = \begin{cases} 1, & \text{for } t = r, \\ 0, & \text{for } t \neq r \end{cases}$$

A fuzzy real number X is called *non-negative* if $X(t) = 0$, for all $t < 0$. The set of all non-negative fuzzy real numbers is denoted by $R^*(I)$.

Let $\bar{d} : R(I) \times R(I) \to R$ be defined by

$$\bar{d}(X, Y) = \sup_{0 \le \alpha \le 1} d\left([X]^\alpha, [Y]^\alpha\right).$$

Then $\bar{d}$ defines a metric on $R(I)$.

The additive identity and multiplicative identity in $R(I)$ are denoted by $\bar{0}$ and $\bar{1}$ respectively.

II. DEFINITIONS AND PRELIMINARIES

A double sequence (X_{nk}) of fuzzy real numbers is said to be *convergent in Pringsheim's sense* to the fuzzy real number L if, for every $\varepsilon > 0$, there exists $n_0, k_0 \in N$ such that $\bar{d}(X_{nk}, L) < \varepsilon$, for all $n \geq n_0$, $k \geq k_0$.

A double sequence (X_{nk}) of fuzzy real numbers is said to be *regularly convergent* if it convergent in Pringsheim's sense and the following limits exist:

$$\lim_n \bar{d}(X_{nk}, L_k) = 0, \text{ for some } L_k \in R(I), \text{ for each } k \in N,$$

and $$\lim_k \bar{d}(X_{nk}, J_n) = 0, \text{ for some } J_n \in R(I), \text{ for each } n \in N.$$

A fuzzy real number sequence (X_k) is said to be *bounded* if $\sup_k |X_k| \leq \mu$, for some $\mu \in R^*(I)$.

Throughout the article ${}_2w_F$, $({}_2\ell_\infty)_F$, ${}_2c_F$, $({}_2c_0)_F$, ${}_2c_F^R$ and $({}_2c_0^R)_F$ denote the classes of *all*, *bounded*, *convergent in Pringsheim's sense*, *null in Pringsheim's sense*, *regularly convergent* and *regularly null* fuzzy real number sequences respectively.

A double sequence space E_F is said to be *solid* (or *normal*) if $<Y_{nk}> \in E_F$, whenever $|Y_{nk}| \leq |X_{nk}|$, for all $n, k \in N$, for some $<X_{nk}> \in E_F$.

Let $K = \{(n_i, k_i) : i \in N ;\ n_1 < n_2 < n_3 < \ldots \text{ and } k_1 < k_2 < k_3 < \ldots\} \subseteq N \times N$ and E_F be a double sequence space. A *K-step space* of E_F is a sequence space $\lambda_K^E = \{< X_{n_ik_i} > \in {}_2w_F : < X_{nk} > \in E_F\}$.

A *canonical pre-image* of a sequence $< X_{n_ik_i} > \in E_F$ is a sequence $<Y_{nk}>$ defined as follows:

$$Y_{nk} = \begin{cases} X_{nk}, & \text{if } (n,k) \in K, \\ 0, & \text{otherwise.} \end{cases}$$

A *canonical pre-image of a step space* λ_K^E is a set of canonical pre-images of all elements in λ_K^E.

A double sequence space E_F is said to be *monotone* if E_F contains the canonical pre-image of all its step spaces.

From the above definitions we have the following remark.

Remark. A sequence space E_F is solid $\Rightarrow$ E_F is monotone.

A double sequence space E_F is said to be *symmetric* if $(X_{\pi(n)\,\pi(k)}) \in E_F$, whenever $(X_{nk}) \in E_F$, where π is a permutation of N.

In this article we introduce the following sequence spaces of fuzzy real numbers:

Let $p = <p_{nk}>$ be a sequence of strictly positive real numbers.

$${}_2\ell_\infty(M, p) = \left\{ < X_{nk} > \in {}_2w_F : \lim_{n,k} \left\{ M\left(\frac{\bar{d}(X_{nk}, \bar{0})}{\rho} \right) \right\}^{p_{nk}} < \infty \right\}$$

$${}_2c_F(M, p) = \{< X_{nk} > \in {}_2w_F : \lim_{n,k} \left\{ M\left(\frac{\bar{d}(X_{nk}, L)}{\rho} \right) \right\}^{p_{nk}} = 0, \text{ for some } L \in R(I)\}$$

For $L = \bar{0}$ we get the class $({}_2c_F)_0(M, p)$.

Also a fuzzy sequence $<X_{nk}> \in {}_2c_F^R(M, p)$ if $<X_{nk}> \in {}_2c_F(M, p)$ and the following limits exist:

$$\lim_n \left\{ M\left(\frac{\bar{d}(X_{nk}, L_k)}{\rho} \right) \right\}^{p_{nk}} = 0, \text{ for some } L_k \in R(I)$$

$$\lim_k \left\{ M\left(\frac{\bar{d}(X_{nk}, J_n)}{\rho} \right) \right\}^{p_{nk}} = 0, \text{ for some } J_n \in R(I)$$

III. MAIN RESULTS

Theorem 3.1. *Let* $<p_{nk}>$ *be bounded. Then the classes of sequences* $({}_2\ell_\infty)_F(M, p)$, ${}_2c_F^R(M,p), ({}_2c_F^R)_0(M,p)$ *are complete metric spaces with respect to the metric defined by,*

$$f(X, Y) = \inf\left\{ r^{\frac{p_{nk}}{J}} > 0 : \sup_{n,k} M\left(\frac{\bar{d}(X_{nk}, Y_{nk})}{r}\right) \le 1 \right\}, \quad \text{where } J = \max(1, 2^{H-1})$$

Proof. We prove the result for ${}_2\ell_\infty(M,p)$. Let $< X_{nk}^i >$ be a Cauchy sequence in $({}_2\ell_\infty)_F(M, p)$.

Let ε > 0 be given. For a fixed $x_0 > 0$, choose $t > 0$ such that $M\left(\frac{tx_0}{2}\right) \ge 1.$ and $m_0 \in N$ be such that

$$f(X^i, X^j) < \frac{\varepsilon}{tx_0} \quad \text{for all } i, j \ge m_0.$$

$$\Rightarrow M\left(\frac{\bar{d}(X_{nk}^i, X_{nk}^j)}{r}\right) \le 1$$

$$\Rightarrow M\left(\frac{\bar{d}(X_{nk}^i, X_{nk}^j)}{f(X^i, X^j)}\right) \le 1 \le M\left(\frac{tx_0}{3}\right)$$

$$\Rightarrow \bar{d}(X_{nk}^i, X_{nk}^j) < \frac{tx_0}{3}.\frac{\varepsilon}{tx_0} = \frac{\varepsilon}{3}$$

$\Rightarrow < X_{nk}^j >_{j=1}^\infty$ is a Cauchy sequence of fuzzy real number for each $n, k \in N$.

Since $R(I)$ is complete there exists fuzzy numbers X_{nk} such that $\lim_{j\to\infty} X_{nk}^j = X_{nk}$ for each $n, k \in N$.

Taking $j \to \infty$ in (1) we have,

$$f(X_{nk}^i, X_{nk}) < \varepsilon$$

Using the triangular inequality

$$f(< X_{nk} >, \bar{0}) \le f(< X_{nk} >, < X_{nk}^j >) + f(< X_{nk}^j >, \bar{0})$$

we have $<X_{nk}> \in ({}_2\ell_\infty)_F(M, p)$. Hence $({}_2\ell_\infty)_F(M, p)$ is complete.

Theorem 3.2. *The space* $({}_2\ell_\infty)_F(M, p)$ *is symmetric but the spaces* ${}_2c_F(M,p)$, ${}_2c_F^R(M,p)$, $({}_2c_F)_0(M,p)$, $({}_2c_F^R)_0(M,p)$ *are not symmetric.*

Proof. Obviously the space $({}_2\ell_\infty)_F(M, p)$ is symmetric. For the other spaces consider the following example.

Example 3.1. Consider the sequence space ${}_2c_F(p)$. Let $M(x) = x$.

Let $p_{1k} = 2$ for all $k \in N$ and $p_{nk} = 3$, otherwise. Let the sequence $<X_{nk}>$ be defined by

$$X_{1k} = \bar{2} \quad \text{for all } k \in N.$$

and for $n > 1$,

$$X_{nk}(t) = \begin{cases} t+2 & \text{for } -2 \le t \le -1, \\ -t, & \text{for } -1 \le t \le 0, \\ 0, & \text{otherwise.} \end{cases}$$

Let $<Y_{nk}>$ be a rearrangement of $<X_{nk}>$ defined by

$$Y_{nn} = \bar{2}$$

and for $n \neq k$, $$Y_{nk}(t) = \begin{cases} t+2 & \text{for } -2 \le t \le -1, \\ -t, & \text{for } -1 \le t \le 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then $<X_{nk}> \in {}_2c_F(p)$ but $<Y_{nk}> \notin {}_2c_F(p)$. Hence ${}_2c_F(p)$ is not symmetric. Similarly the other spaces are also not symmetric.

Theorem 3.3. *The spaces* $({}_2\ell_\infty)_F(p)$, $({}_2c_0)_F(p)$ and $({}_2c_0^R)_F(p)$ *are solid.*

Proof. Consider the sequence space $({}_2\ell_\infty)_F(p)$. Let $<X_{nk}> \in ({}_2\ell_\infty)_F(p)$ and $<Y_{nk}>$ be such that $\bar{d}(Y_{nk}, \bar{0}) \le \bar{d}(X_{nk}, \bar{0})$.

The result follows from the inequality

$$\{\bar{d}(Y_{nk}, \bar{0})\}^{p_{nk}} \le \{\bar{d}(X_{nk}, \bar{0})\}^{p_{nk}}$$

Hence the space $({}_2\ell_\infty)_F(p)$ is solid. Similarly the other spaces are also solid.

Proposition 3.4. *The spaces* ${}_2c_F(p)$, $({}_2c^R)_F(p)$ *and* $m(p)$ *are not monotone and hence are not solid.*

Proof. The result follows from the following example:

Example 3.2. Consider the sequence space ${}_2c_F(p)$. Let $M(x) = x$.
Let $p_{nk} = 3$ for $n + k$ even and $p_{nk} = 2$, otherwise. Let $J = \{(n, k): n + k \text{ is even}\} \subseteq N \times N$. Let $<X_{nk}>$ be defined as:

For all $n, k \in N$, $$X_{nk}(t) = \begin{cases} t+3 & \text{for } -3 \le t \le -2, \\ nt(3n-1)^{-1} + 3n(3n-1)^{-1}, & \text{for } -2 \le t \le -1 + n^{-1}, \\ 0, & \text{otherwise.} \end{cases}$$

Then $<X_{nk}> \in {}_2c_F(p)$. Let $<Y_{nk}>$ be the canonical pre-image of $< X_{nk} >_J$ for the subsequence J of N. Then

$$Y_{nk} = \begin{cases} X_{nk} & \text{for } (n,k) \in J, \\ \bar{0} & \text{otherwise.} \end{cases}$$

Then $<Y_{nk}> \notin {}_2c_F(p)$. Thus ${}_2c_F(p)$ is not monotone. Similarly the other spaces are also not monotone. Hence the spaces ${}_2c_F(p)$, $({}_2c^R)_F(p)$ and $m(p)$ are not solid.

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