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Mixed Convection in a Vertical Channel with Sources, Sinks and Thermal Radiation

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Abstract: The study of mixed convection in a vertical channel with symmetric and asymmetric wall heating conditions including viscous dissipation in the presence of heat sources, sinks and thermal radiation is analyzed. The two boundaries are considered as isothermal-isothermal, isoflux-isothermal and isothermal-isoflux for the left and right walls of the channel and kept either at equal or at different temperature. The velocity and temperature fields are obtained by perturbation method for various parameters such as ratio of Grashoff number and Reynolds number (λ), the product of λ and Brinkman number for symmetric and asymmetric wall temperatures in the presence of heat sources, sinks and thermal radiation. The results are drawn graphically. The viscous dissipation enhances the flow reversal in the case of downward flow while it counters the flow in the case of upward flow.

Keywords: *Mixed Convection, Viscous Dissipation, Buoyancy force, Perturbation Method.*

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I. INTRODUCTION

Free and forced convection flows in vertical ducts has been investigated vastly. The majority of the recent studies have dealt with the circular tube geometry, but increasing attention is being focused on the parallel plate duct since this configuration is relevant to most applicable solar energy collection, in the cooling of modern electronic systems and in the convectional flat plate collector . The various areas of applications of free convection flow are found in heat transfer from transmission lines as well as from electronic devices, heat dissipation from the coil of a refrigerator to the surrounding air, heat transfer in nuclear fuel rods to the surrounding coolant.

Design information for mixed convection should reflect the interacting effects of free and forced convection. Heat transfer in mixed convection can be significantly different from its value in both pure free and pure forced convection. On the other hand, in buoyancy-opposed flow, the laminar mixed convection heat transfer can be lower than that for pure forced flow. Heat transfer in free and mixed convection in vertical channels occurs in many industrial processes and natural phenomena. Heat transfer by simultaneous radiation and convection has applications in numerous technological problems, including combustion, furnace design, the design of high temperature gas cooled nuclear reactors, nuclear reactor safety, fluidized – bed heat exchangers, fire spreads, solar ponds, solar collectors, natural convection in cavities and many others. On the other hand, it is worth mentioning that heat transfer by simultaneous radiation and convection is very important in the context of space technology and processes involving high temperatures.

Several papers on mixed convection in a parallel- plate vertical channel are available in the literature. However, most of these studies are based on the hypothesis that the effect of viscous dissipation is negligible. The fully developed region has been studied analytically. For instance, the boundary condition of uniform wall temperatures has been analyzed by Aung and Worku[1]. The cases of either uniform temperature or uniform heat flux at each boundary surface have been studied by Cheng, Kou and Huang [2] and by Hamadah and Wirtz [3]. The boundary conditions of linearly varying wall temperatures have been considered by Tao [4]. Some studies on the developing flow have been carried out by numerical methods. Aung and Worku [5, 6] have studied the developing flow with asymmetric wall temperatures [5] and with asymmetric wall heat fluxes [6]. The developing flow with asymmetric wall temperatures has been considered also by Ingham, Keen and Heggs [7] with particular reference to situations where reverse flow occurs. All the studies quoted above, as well as the references quoted in the review of the literature on this subject presented by Aung [8] , assume that viscous dissipation effects are negligible. On the other hand, Barletta [9,10] pointed out the relevant effects of laminar mixed convection with viscous dissipation in a vertical channel and flow reversal in a vertical duct with uniform heat fluxes. Recently Umavathi et. al. [11, 12] analyzed the effect of viscous dissipation with or without heat sources in a vertical channel. Vajravelu and Nayfeh [13] and Chamkha [14] have employed temperaturedependent heat generation or absorption effects. Umavathi et. al [15] studied the laminar mixed convection flow in a vertical porous stratum with symmetric and asymmetric wall heating. Umavathi, et. al.[16] discussed about the laminar magneto convection flow in a vertical channel in the presence of heat generation and heat absorption. T. Grosan. I. Pop [17] has analyzed the effects of thermal radiation on the steady fully developed mixed convection flow in a vertical channel such that the walls of the channels are subjected to uniform but different wall temperatures and has proved that the increase of both radiation and temperature parameters diminish the reversal flow and thus the radiation effect will stabilize the fluid motion. Effect of the radiation on natural convection in a vertical permeable channel has been investigated by Umavathi and Palaniappan [18] and it is proved that the radiation increases the rate of heat transfer thereby reducing the effect of natural convection. A novel voticity- velocity method was used by Wei-Mon Yan and Hung-Yi Li [19] to study the interaction of the thermal radiation with laminar mixed convection for a gray fluid in a vertical square duct and found that in the presence of radiation, the thermal development develops at a more rapid rate to that without radiation. The problem of steady, laminar, mixed convection boundary- layer flow over a vertical cone embedded in a porous medium saturated with a nanofluid, in the presence of the thermal radiation, has been analyzed by Ali. J. Chamkha et al [20] and numerical solutions with Rosseland diffusion approximation are presented. Ganesha Kumar and Gururaja Rao [21] made the parametric studies and correlations for the problem of combined conduction-mixed convection- radiation from a non- identically and discretely heated vertical plate. Local plate temperature distribution along the plate, local drag coefficient along the plate, peak plate temperature , mean drag coefficient, contributory roles of free convection, forced convection and radiation in fluid flow and heat transfer results are explored in detail. The effects of thermal radiation on laminar- forced and free convection along the wavy surface are studied by Cha'o–Kuang Chen et al [22] by transforming the wavy surface into a calculable planar coordinate system. It is seen that increasing values of Radiation- conduction parameter N_R induce development of the second harmonic of heat–transfer irreversibility, fluid friction, irreversibility and entropy generation.

Radioactive heat transfer of two fluid flow in a vertical porous channel has been analyzed by Umavathi and Manjula [23] by modeling the flow using Darcy-Lapwood –Brinkman equation. It is found that velocity and temperature can be controlled effectively by altering the values of Grashoff number , radiation parameter and heat generation or absorption coefficient. It is also found by Khaefinejad and Aghanajafi [24] that thermal radiation speeds the development of velocity and temperature fields, delay reverse flow occurrence and enhances total heat transfer but decreases buoyancy effects when investigated the effects of combined mixed convection and thermal radiation for laminar ascending flows. Radiation effects on natural convection MHD flow in a rotating vertical porous channel partially filled with a porous medium has been discussed by Dileep Singh Chauhan and Priyanka Rastogi[25].Thermal radiation having appreciable influence on the transient MHD Couette flow through a porous medium has been found by Baoku et al [26]. The steady flow of a viscous fluid through a porous medium bounded by a horizontal semi-infinite plate in the presence of thermal radiation in the fluid has been investigated by Raptis[27] The effect of radiation and chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface has been studied by Sudarshan Reddy et al [28]. Mukhopadhyay [29] made significant findings of effect of radiation on the boundary layer flow and heat transfer of a fluid with variable viscosity along a symmetric wedge indicating that due to variable fluid viscosity, flow separation is controlled. It is also found that the temperature decreases with increasing values of radiation parameter and Prandtl number. The combined effect of Hall current and radiation on an unsteady MHD free convective flow of a viscous incompressible electrically conducting fluid in a vertical channel with an oscillatory wall temperature has been studied by Sankar Kumar Guchhait et al [30]. All the results derived in this problem has similarities with the work published by Patil Mallikarjun B [31] when taken the radiation parameter F=0.

The study of heat generation or heat absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution; consequently, the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. The working fluid heat generation or absorption effects are important in certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic and /or endothermic chemical reactions and dissociating fluids in packed-bed reactors.

Radiation effect on flow and heat transfer is important in the context of space technology and processes involving high temperature. Many processes in new engineering areas occur at high temperatures, and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas.

The objective of this work is to analyze the effect of viscous dissipation for the laminar flow of Newtonian fluids in a vertical channel for symmetric and asymmetric wall heating conditions along with heat generation, heat absorption and radiation effects.

II. MATHEMATICAL FORMULATION

Consider steady, laminar, fully developed flow in a parallel-plate vertical channel. The physical configuration is described in Fig.1.Cartesian co-ordinate system is chosen with the transverse coordinate *Y* and the coordinate in the direction parallel to the walls is *X*. The origin of the axes is such that the channel walls are at the positions
Y =−*L*/2 and *Y* = *L*/2. The thermal conductivity, dynamic viscosity and thermal expansion coefficient are considered as constant. The Oberbeck-Boussinesq approximation is assumed to hold and for the evaluation of the gravitational body force, the density ρ is assumed to depend on temperature according to the equation of state $\rho = \rho_0 \left[1 - \beta(T - T_0)\right]$ (1

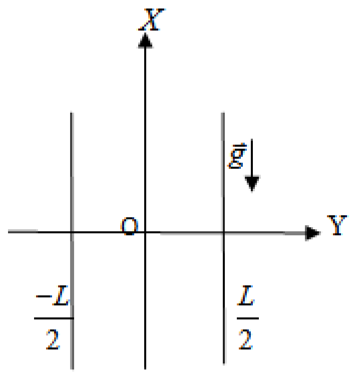

Fig. 1. Drawing of the parallel plate and rectangular coordinate axes

where *T* is the temperature, ρ_0 and β are the reference density and coefficient of thermal expansion at reference temperature T_0 . It is assumed that the component of velocity field q is the X-component U. The mass balance equation ,becomes $\partial U/\partial X = 0$ so that U depends on Y only. The transverse momentum equation gives

$$g\beta(T - T_0) - \frac{1}{\rho_0}\frac{dP}{dX} + \gamma\frac{d^2U}{dY^2} = 0 \tag{2}$$

$$\frac{dP}{dX} = A \tag{3}$$

where P = p +ρ_0gX is the difference between the pressure and the hydrostatic pressure. Therefore *P* depends only on *X* and the *X*-momentum balance equation is given by

$$\frac{dT}{dX} = 0$$

(4) Let the walls of the channel are constant for isothermal-isothermal case. In particular, the temperature of the boundary at *Y* =−*L*/2 is T_1 , while the temperature at *Y* = *L*/2 is *T*2 with *T*2 ≥ *T*1. These boundary conditions are compatible with Eq. (2) if and only if *dP/dX* is independent of *X*. Therefore, there exists a constant *A* such that $\frac{dP}{dX} = A$ On account of $\frac{dP}{dX} = A$, by evaluating the derivative of (4) with respect to X ,one obtains $\frac{dT}{dX} = 0$.
So that the temperature depends only on Y.
By taking into account the effect of viscous dissipation, heat generation and heat absorption with radiation the energy balance equation can be written as

$$\alpha\frac{d^2T}{dY^2} + \frac{\gamma}{C_p}\left(\frac{dU}{dY}\right)^2 \pm \frac{Q(T - T_0)}{\rho_0 C_P} - \frac{1}{\rho_0 C_P}\frac{dq_R}{dY} = 0 \tag{5}$$

$$\frac{d^4U}{dY^4} = \mp\frac{Q}{K}\frac{d^2U}{dY^2} + \frac{g\beta}{\alpha C_p}\left(\frac{dU}{dY}\right)^2 \pm \frac{QA}{\mu K} - \frac{g\beta}{K\gamma}\frac{dq_R}{dY} \tag{6}$$

The boundary conditions on U are both the no slip conditions $U = 0 \quad at \quad Y = \pm\frac{L}{2}$

(7)

And those induced by the boundary conditions on T and by equations (4) and *dP/dX= A*, namely

$$\left(\frac{d^2U}{dY^2}\right) = \frac{A}{\mu} - \frac{g\beta(T_1 - T_0)}{\nu} \quad at \quad Y = -\frac{L}{2} \quad \text{and} \quad \left(\frac{d^2U}{dY^2}\right) = \frac{A}{\mu} - \frac{g\beta(T_2 - T_0)}{\nu} \quad at \quad Y = \frac{L}{2}$$

(8) Equations (6) to Equations (8) can be written in a dimensionless form by employing the dimensionless form is

$$u = \frac{U}{U_0}, \theta = \frac{T - T_0}{\Delta T}, y = \frac{Y}{D}, Gr = \frac{g\beta\Delta TD^3}{\gamma^2}, R_e = \frac{U_0 D}{\gamma}, B_r = \frac{\mu U_0{}^2}{K\Delta T}, \lambda = \frac{Gr}{\mathrm{Re}},$$

$$\mathrm{R}_T = \frac{T_2 - T_1}{\Delta T}, P_r = \frac{\gamma}{\alpha}, \phi = \frac{QD^2}{K}, \frac{dq_r}{dY} = C(T - T_0)$$

(9)

where D=2L is the hydraulic diameter. The reference velocity U_0 and the reference temperature T_0 are given by

$$U_0 = -\frac{AD^2}{48\mu}; \qquad T_0 = \frac{T_1 + T_2}{2} \tag{10}$$

Moreover, the reference temperature difference ΔT is given by

$$\Delta T = T_2 - T_1 \text{ if } T_1 < T_2 \text{ and} \tag{11}$$

$$\Delta T = \frac{\gamma^2}{C_p D^2} \quad if \quad T_1 = T_2 \tag{12}$$

The heating parameter R_T becomes zero for symmetric heating $T_1 = T_2$ and unity for asymmetric heating $T_1 < T_2$. Using Equations (9), Equations (5)–(8) in non-dimensional form becomes

$$\frac{d^2\theta}{dy^2} = -Br\left(\frac{du}{dy}\right)^2 \mp \phi\theta + F^2\theta \quad \text{where } F^2 = \frac{CD^2}{K} \tag{13}$$

$$\frac{d^4u}{dy^4} - (F^2 \mp \phi)\frac{d^2u}{dy^2} = Br\lambda\left(\frac{du}{dy}\right)^2 \mp 48\phi + 48F^2 \tag{14}$$

$$u\left(-\frac{1}{4}\right) = u\left(-\frac{1}{4}\right) = 0$$

(15) $$\left(\frac{d^2u}{dy^2}\right)_{y=-\frac{1}{4}} = -48 + \frac{\lambda R_T}{2} \quad \text{and} \quad \left(\frac{d^2u}{dy^2}\right)_{y=\frac{1}{4}} = -48 - \frac{\lambda R_T}{2}$$

(16) Using Eqns. (9) and (10) in Eqn. (2) one obtains

$$\theta = -\frac{1}{\lambda}\left(48 + \frac{d^2u}{dy^2}\right)$$

(17)

The dimensionless temperature θ can be evaluated either by integrating Eqn (13) or by using Eqn. (17).

2.1 Special cases:

Case(1): No dissipations (*Br* = 0) If the viscous dissipation is negligible so that *Br* = 0,the dimensionless temperature θ and dimensionless velocity *u* are uncoupled. In this case the solutions of (14) using boundary

conditions (15) & (16) become $u = \frac{3}{2} - 24y^2 - \frac{2R_T\lambda}{(\phi - F^2)}\left[y - \frac{\sin(\sqrt{\phi - F^2}\,)y}{4\sin\left(\frac{\sqrt{\phi - F^2}}{4}\right)} \right]$ for the case of heat generation
(18)

and $u = \frac{3}{2} - 24y^2 + \frac{2R_T\lambda}{\phi + F^2}\left[y - \frac{\sinh(\sqrt{\phi + F^2}\,)y}{4\sinh\left(\frac{\sqrt{\phi + F^2}}{4}\right)} \right]$ for the case of heat absorption

(19)

Solving equation (13) using the boundary conditions $\theta\left(\pm\frac{1}{4}\right) = \pm\frac{R_T}{2}$ (20)

We get $\theta = \frac{R_T}{2}\frac{\sin(\sqrt{\phi - F^2}\,)y}{\sin\left(\frac{\sqrt{\phi - F^2}}{4}\right)}$ for the case of heat generation and (21)

$\theta = \frac{R_T}{2}\frac{\sinh(\sqrt{\phi + F^2}\,)y}{\sinh\left(\frac{\sqrt{\phi + F^2}}{4}\right)}$ for the case of heat absorption (22)

2.2 Case(2): No dissipations and no thermal radiation (*Br* = 0, $F = 0$) If there is no dissipations and no thermal radiation then the velocity is given by

$$u = \frac{3}{2} - 24y^2 - \frac{2R_T\lambda}{\phi}\left(y - \frac{\sin\sqrt{\phi}y}{4\sin\left(\frac{\sqrt{\phi}}{4}\right)} \right)$$ for the case of heat generation

and $u = \frac{3}{2} - 24y^2 + \frac{2R_T\lambda}{\phi}\left(y - \frac{\sinh\sqrt{\phi}y}{4\sinh\left(\frac{\sqrt{\phi}}{4}\right)} \right)$ for the case of heat absorption (23)

And the temperature using the equation (20) is $\theta = \frac{R_T}{2}\frac{\sin\sqrt{\phi}y}{\sin\left(\frac{\sqrt{\phi}}{4}\right)}$ for heat generation and

$\theta = \frac{R_T}{2}\frac{\sinh\sqrt{\phi}y}{\sinh\left(\frac{\sqrt{\phi}}{4}\right)}$ for the case of heat absorption (24)

2.3 Case(3): Purely free convection $(\lambda \to \pm\infty)$

In the case of asymmetric heating and when buoyancy forces are dominated, i.e. $(\lambda \to \pm\infty)$ when Equations (22) and (23) gives $\frac{u}{\lambda} = \frac{3}{2\lambda} - \frac{2R_T}{\phi - F^2} y + \frac{R_T}{2(\phi - F^2)} \frac{\sin(\sqrt{\phi - F^2})y}{\sin\left(\frac{\sqrt{\phi - F^2}}{4}\right)} - \frac{24y^2}{\lambda}$ (25)

as $\lambda \to \pm\infty, wkt \frac{1}{\lambda} \to 0$ & $R_T = 1$ for asymmetric heating, we get $\frac{u}{\lambda} = -\frac{2}{(\phi - F^2)}\left[y - \frac{1}{4}\frac{\sin(\sqrt{\phi - F^2})y}{\sin\left(\frac{\sqrt{\phi - F^2}}{4}\right)}\right]$ (26)

2.4 Case(4): Purely forced convection (λ = 0)

When buoyancy forces are negligible $\lambda = 0$ and viscous dissipation is relevant, so that a purely forced convection occurs. Under this condition Equations (18) and (19) become

$$u = \frac{3}{2} - 24y^2 \tag{27}$$

Using the above equation we get , $\frac{d^2\theta}{dy^2} = -Br(-48y)^2 \mp \phi\theta + F^2\theta$ (28)

Solving the above equation using the boundary conditions(20),we get

$\theta = C_1 \cos(\sqrt{\phi - F^2})y + C_2 \sin(\sqrt{\phi - F^2})y - \frac{(2304)Br}{\phi - F^2}\left(y^2 - \frac{2}{\phi - F^2}\right)$ for heat generation and (29)

$\theta = C_1 \cosh(\sqrt{\phi + F^2})y + C_2 \sinh(\sqrt{\phi + F^2})y + \frac{(2304)Br}{\phi + F^2}\left(y^2 + \frac{2}{\phi + F^2}\right)$ for heat absorption (30)

2.5 Case(5): Hagen-Poiseuille flow (λ = 0, □ = 0)

Solutions of Equations (13) and (14) for viscous fluid in the absence of buoyancy force and no source nor sink lead to the Hagen- Poiseuille velocity profile is

$$u = \frac{R_T\lambda}{F^2}\left(2y - \frac{1}{2}\frac{\sinh Fy}{\sinh\left(\frac{F}{4}\right)}\right) + 24\left(\frac{1}{16} - y^2\right) \tag{31}$$

$$\theta = \frac{R_T}{2}\frac{\sinh Fy}{\sinh\left(\frac{F}{4}\right)} \tag{32}$$

3. Analytical Solution by Perturbation Method

Perturbation series method is employed to solve equation (14) since the equation is nonlinear by defining the dimensionless parameter ε =Brλ= $\mathrm{Re}\,\mathrm{Pr}\frac{\beta g D}{C_p}$ as the perturbation parameter. The solution of equation (14) can be expressed by the perturbation expansion,

$$u(y) = u_0(y) + \varepsilon u_1(y) + \varepsilon^2 u_2(y) + ... = \sum_{n=0}^{\infty} \varepsilon^n u_n(y) \tag{33}$$

The first and higher order terms of ε give a correction to u_0 and θ_0 accounting for the viscous and Ohmic dissipation effects. Substituting equation (33) in equations(14) to (16) and equating the coefficients of like powers of ε on both sides, one obtains the boundary-value problem for n=0 as follows:

3.1 Isothermal-isothermal case: (T_1-T_2)

$\frac{d^4u_0}{dy^4} - (F^2 - \phi)\frac{d^2u_0}{dy^2} = -48\phi + 48F^2$ for the case of heat generation and (34)

$$\frac{d^4u_0}{dy^4} - (\phi + F^2)\frac{d^2u_0}{dy^2} = 48(\phi + F^2) \quad \text{for the case of heat absorption} \tag{35}$$

$$u_0\left(\frac{-1}{4}\right) = u_0\left(\frac{1}{4}\right) = 0, \tag{36}$$

$$\left.\frac{d^2u_0}{dy^2}\right|_{y=\frac{-1}{4}} = -48 + \frac{R_T\lambda}{2}, \quad \left.\frac{d^2u_0}{dy^2}\right|_{y=\frac{1}{4}} = -48 - \frac{R_T\lambda}{2} \tag{37}$$

Equations (34) and (35) are ordinary differential equations and exact solutions can be found easily.
These equations coincide with the solutions of equation(14) in the case Br = 0. The solutions of equations (34) and (35) subject to the boundary conditions (36) and (37) are,

$$u_0 = C_1 + C_2y + C_3\cos(\sqrt{\phi - F^2})y + C_4\sin(\sqrt{\phi - F^2})y - 24y^2 \quad \text{for the case of heat generation and} \tag{38}$$

$$u_0 = C_1 + C_2y + C_3\cosh(\sqrt{\phi + F^2})y + C_4\sinh(\sqrt{\phi + F^2})y - 24y^2 \quad \text{for the case of heat absorption} \tag{39}$$

Similarly the first order boundary value problem are

$$\frac{d^4u_1}{dy^4} - (F^2 - \phi)\frac{d^2u_1}{dy^2} = \left(\frac{du_0}{dy}\right)^2 \quad \text{for the case of heat generation and} \tag{40}$$

$$\frac{d^4u_1}{dy^4} - (F^2 + \phi)\frac{d^2u_1}{dy^2} = \left(\frac{du_0}{dy}\right)^2 \quad \text{for the case of heat absorption} \tag{41}$$

and the governing boundary conditions are $u_1\left(\frac{-1}{4}\right) = u_1\left(\frac{1}{4}\right) = 0,$

(42)

$$\left.\frac{d^2u_1}{dy^2}\right|_{y=\frac{-1}{4}} = \left.\frac{d^2u_1}{dy^2}\right|_{y=\frac{1}{4}} = 0$$

(43) Solving equations(40) and (41) using the above boundary conditions we get,

$$u_1 = C_5 + C_6y + C_7Cos(\sqrt{\phi - F^2})y + C_8Sin(\sqrt{\phi - F^2})y + l_1Cos2(\sqrt{\phi - F^2})y +$$
$$l_2Sin2(\sqrt{\phi - F^2})y + l_3y^2Cos(\sqrt{\phi - F^2})y + l_4y^2Sin(\sqrt{\phi - F^2})y +$$
$$l_5yCos(\sqrt{\phi - F^2})y + l_6ySin(\sqrt{\phi - F^2})y + l_7y^4 + l_8y^3 + l_9y^2$$

for the case of heat generation and (44)

$$u_1 = C_5 + C_6y + C_7Cosh(\sqrt{\phi + F^2})y + C_8Sinh(\sqrt{\phi + F^2})y + l_1Cosh2(\sqrt{\phi + F^2})y +$$
$$l_2Sinh2(\sqrt{\phi + F^2})y + l_3y^2Cosh(\sqrt{\phi + F^2})y + l_4y^2Sinh(\sqrt{\phi + F^2})y +$$
$$l_5yCosh(\sqrt{\phi + F^2})y + l_6ySinh(\sqrt{\phi + F^2})y + l_7y^4 + l_8y^3 + l_9y^2$$

for the case of heat absorption (45)

Hence the velocity upto the first order is $u = u_0 + \varepsilon u_1$ The dimensionless temperature field is obtained from equation (17) using velocity fields defined as in the equations (38), (39), (44) and (45) is given by

$$\theta = \frac{1}{\lambda}\Big\{(C_3 + \varepsilon C_7)(\phi - F^2)Cos(\sqrt{\phi - F^2})y + (C_4 + \varepsilon C_8)(\phi - F^2)Sin(\sqrt{\phi - F^2})y$$
$$-\varepsilon\Big[-4l_1(\phi - F^2)Cos2(\sqrt{\phi - F^2})y - 4l_2(\phi - F^2)Sin2(\sqrt{\phi - F^2})y$$
$$-l_3(\phi - F^2)y^2Cos(\sqrt{\phi - F^2})y - l_4(\phi - F^2)y^2Sin(\sqrt{\phi - F^2})y + l_{10}yCos(\sqrt{\phi - F^2})y$$
$$+l_{11}ySin(\sqrt{\phi - F^2})y + l_{12}Cos(\sqrt{\phi - F^2})y + l_{13}Sin(\sqrt{\phi - F^2})y + 12l_7y^2 + 6l_8y + 2l_9\Big]\Big\}$$

(46)
for the case of heat generation and

$$\theta = \frac{-1}{\lambda}\Big\{(C_3 + \varepsilon C_7)(\phi + F^2)Cosh(\sqrt{\phi + F^2})y + (C_4 + \varepsilon C_8)(\phi + F^2)Sinh(\sqrt{\phi + F^2})y$$
$$+\varepsilon\Big[4l_1(\phi + F^2)Cosh2(\sqrt{\phi + F^2})y + 4l_2(\phi + F^2)Sinh2(\sqrt{\phi + F^2})y$$
$$+l_3(\phi + F^2)y^2Cosh(\sqrt{\phi + F^2})y + l_4(\phi + F^2)y^2Sinh(\sqrt{\phi + F^2})y + l_{10}yCosh(\sqrt{\phi + F^2})y$$
$$+l_{11}ySinh(\sqrt{\phi + F^2})y + l_{12}Cosh(\sqrt{\phi + F^2})y + l_{13}Sinh(\sqrt{\phi + F^2})y + 12l_7y^2 + 6l_8y + 2l_9\Big]\Big\}$$

(47)
for the case of heat absorption.

3.2 Isoflux-isothermal case (q_1-T_2) walls:

For this case, the thermal boundary conditions for the channel walls can be written in the dimensional form as,

$$q_1 = -K\frac{dT}{dY}\, at\, Y = \frac{-L}{2} \quad and \quad T = T_2 \; at \; Y = \frac{L}{2} \tag{48}$$

using non-dimensional parameters from equation (9) with $\Delta T = \frac{q_1 D}{K}$ to give

$\left.\frac{d\theta}{dy}\right|_{y=\frac{-1}{4}} = -1, \quad \theta\left(\frac{1}{4}\right) = R_{qt}$ (49) where $R_{qt} = \frac{(T_2 - T_0)}{\Delta T}$ is the thermal ratio parameter for isoflux -isothermal case. Two more boundary conditions in terms of U are required to solve equation (6) in addition to the no-slip conditions at the channel walls. These are the conditions given in the equations (48) and (49) and are obtained from equation(2) as follows. Differentiating equation (2) wrt y along with dP/dX=A gives

$$\frac{d^3U}{dY^3} + \frac{g\beta}{\gamma}\frac{dT}{dY} = 0$$

(50)

using equation(9) to non-dimensionalise equation (50) we get $\frac{d^3u}{dy^3} + \lambda\frac{d\theta}{dy} = 0$ (51)

the above equation at the left wall y = $\frac{-1}{4}$ gives , $\frac{d^3u}{dy^3} = \lambda$ (52)

at right wall the boundary condition will remain same as that given for the isothermal-isothermal wall with R_T replaced by R_{qt} such that $\left.\frac{d^2u}{dy^2}\right|_{y=\frac{1}{4}} = -48 - \frac{\lambda R_T}{2}$ (53)

velocity field and temperature field can be obtained from equations (46) and (47) by using the boundary conditions (36), (37), (42) , (43) along with (52) and (53) as follows

$$u = C_1 + C_2 y + C_3 Cos(\sqrt{\phi - F^2})y + C_4 Sin(\sqrt{\phi - F^2})y - 24y^2 +$$
$$\varepsilon\Big[C_5 + C_6 y + C_7 Cos(\sqrt{\phi - F^2})y + C_8 Sin(\sqrt{\phi - F^2})y + l_1 Cos2(\sqrt{\phi - F^2})y$$
$$+ l_2 Sin2(\sqrt{\phi - F^2})y + l_3 y^2 Cos(\sqrt{\phi - F^2})y + l_4 y^2 Sin(\sqrt{\phi - F^2})y \qquad (54)$$
$$+ l_5 y Cos(\sqrt{\phi - F^2})y + l_6 y Sin(\sqrt{\phi - F^2})y + l_7 y^4 + l_8 y^3 + l_9 y^2\Big]$$

for heat generation and

$$u = C_1 + C_2 y + C_3 Cosh(\sqrt{\phi + F^2})y + C_4 Sinh(\sqrt{\phi + F^2})y - 24y^2 +$$
$$\varepsilon\Big[C_5 + C_6 y + C_7 Cosh(\sqrt{\phi + F^2})y + C_8 Sinh(\sqrt{\phi + F^2})y + l_1 Cosh2(\sqrt{\phi + F^2})y$$
$$+ l_2 Sinh2(\sqrt{\phi + F^2})y + l_3 y^2 Cosh(\sqrt{\phi + F^2})y + l_4 y^2 Sinh(\sqrt{\phi + F^2})y \qquad (55)$$
$$+ l_5 y Cosh(\sqrt{\phi + F^2})y + l_6 y Sinh(\sqrt{\phi + F^2})y + l_7 y^4 + l_8 y^3 + l_9 y^2\Big]$$

for heat absorption.

$$\theta = \frac{-1}{\lambda}\Big[-C_3(\phi - F^2)Cos(\sqrt{\phi - F^2})y - C_4(\phi - F^2)Sin(\sqrt{\phi - F^2})y + \varepsilon\{-C_7(\phi - F^2)Cos(\sqrt{\phi - F^2})y$$
$$-C_8(\phi - F^2)Sin(\sqrt{\phi - F^2})y - 4l_1(\phi - F^2)Cos2(\sqrt{\phi - F^2})y - 4l_2(\phi - F^2)Sin2(\sqrt{\phi - F^2})y +$$
$$2l_3 Cos(\sqrt{\phi - F^2})y + 2l_4 Sin(\sqrt{\phi - F^2})y - l_3 y^2(\phi - F^2)Cos(\sqrt{\phi - F^2})y - l_4 y^2(\phi - F^2)Sin(\sqrt{\phi - F^2})y$$
$$-l_5 y(\phi - F^2)Cos(\sqrt{\phi - F^2})y - l_6 y(\phi - F^2)Sin(\sqrt{\phi - F^2})y - 4l_3 y(\sqrt{\phi - F^2})Sin(\sqrt{\phi - F^2})y$$
$$+4l_4 y(\sqrt{\phi - F^2})Cos(\sqrt{\phi - F^2})y - 2l_5(\sqrt{\phi - F^2})Sin(\sqrt{\phi - F^2})y + 2l_6 y(\sqrt{\phi - F^2})Cos(\sqrt{\phi - F^2})y + 12l_7 y^2 + 6l_8 y + 2l_9\}\Big]$$

for heat generation and

(56)

$$\theta = \frac{-1}{\lambda}\Big[C_3(\phi + F^2)Cosh(\sqrt{\phi + F^2})y + C_4(\phi + F^2)Sinh(\sqrt{\phi + F^2})y + \varepsilon\{C_7(\phi + F^2)Cosh(\sqrt{\phi + F^2})y +$$
$$C_8(\phi + F^2)Sinh(\sqrt{\phi + F^2})y + 4l_1(\phi + F^2)Cosh2(\sqrt{\phi + F^2})y + 4l_2(\phi + F^2)Sinh2(\sqrt{\phi + F^2})y$$
$$+ 2l_3 Cosh(\sqrt{\phi + F^2})y + 2l_4 Sinh(\sqrt{\phi + F^2})y + l_3 y^2(\phi + F^2)Cosh(\sqrt{\phi + F^2})y + l_4 y^2(\phi + F^2)Sinh(\sqrt{\phi + F^2})y +$$
$$l_5 y(\phi + F^2)Cosh(\sqrt{\phi + F^2})y + l_6 y(\phi + F^2)Sinh(\sqrt{\phi + F^2})y + 4l_3 y(\sqrt{\phi + F^2})Sinh(\sqrt{\phi + F^2})y$$
$$+4l_4 y(\sqrt{\phi + F^2})Cosh(\sqrt{\phi + F^2})y + 2l_5(\sqrt{\phi + F^2})Sinh(\sqrt{\phi + F^2})y +$$
$$2l_6 y(\sqrt{\phi + F^2})Cosh(\sqrt{\phi + F^2})y + 12l_7 y^2 + 6l_8 y + 2l_9\}\Big]$$

for heat absorption. (57)

3.3 Isothermal-isoflux case (T_1-q_2) walls:

The thermal boundary condition for the channel walls can be written in the dimensional form as,

$$q_2 = -K\frac{dT}{dY} \ at \ Y = \frac{L}{2} \ \ and \ \ T = T_1 \ \ at \ \ Y = \frac{-L}{2}$$

(58)

The dimensionless form of the above equation is as follows:

$$\frac{d\theta}{dy} = -1 \ at \ y = \frac{1}{4} \ \ and \ \ \theta = R_{tq} \ at \ y = \frac{-1}{4} \ where \ R_{tq} = \frac{T_1 - T_0}{\Delta T} \qquad (59)$$

is the thermal ratio parameter for the isothermal -isoflux case. In analogous to the previous section of isoflux-isothermal walls, the dimensionless form of the thermal boundary condition at the right wall becomes,

$$\frac{d^3u}{dy^3} = \lambda \ \ at \ \ y = \frac{1}{4} \tag{60}$$

The other boundary condition at the left wall can be shown to be the same as that given in the isothermal-isothermal walls with R_T replaced by R_{tq} such that $\frac{d^2u}{dy^2} = -48 + \frac{\lambda R_{tq}}{2} \ \ at \ \ y = \frac{-1}{4}$.The integrating constant appeared in the equations for 'θ' & 'u' are evaluated using boundary conditions which are same as in the previous case and $\frac{d^3u}{dy^3} = \lambda \ \ at \ \ y = \frac{1}{4}$.Velocity field and temperature field can be obtained from equations by using the boundary conditions mentioned as above .it is as follows:

$$u = C_1 + C_2 y + C_3 Cos(\sqrt{\phi - F^2})y + C_4 Sin(\sqrt{\phi - F^2})y - 24y^2 + \varepsilon\Big[C_5 + C_6 y + C_7 Cos(\sqrt{\phi - F^2})y + C_8 Sin(\sqrt{\phi - F^2})y +$$
$$l_1 Cos2(\sqrt{\phi - F^2})y + l_2 Sin2(\sqrt{\phi - F^2})y + l_3 y^2 Cos(\sqrt{\phi - F^2})y + l_4 y^2 Sin(\sqrt{\phi - F^2})y + l_5 y Cos(\sqrt{\phi - F^2})y + l_6 y Sin(\sqrt{\phi - F^2})y$$
$$+ l_7 y^4 + l_8 y^3 + l_9 y^2\Big]$$

for heat generation and (61)

$$u = C_1 + C_2 y + C_3 Cosh(\sqrt{\phi + F^2})y + C_4 Sinh(\sqrt{\phi + F^2})y - 24y^2 + \varepsilon\Big[C_5 + C_6 y + C_7 Cosh(\sqrt{\phi + F^2})y + C_8 Sinh(\sqrt{\phi + F^2})y +$$
$$l_1 Cosh2(\sqrt{\phi + F^2})y + l_2 Sinh2(\sqrt{\phi + F^2})y + l_3 y^2 Cosh(\sqrt{\phi + F^2})y + l_4 y^2 Sinh(\sqrt{\phi + F^2})y + l_5 y Cosh(\sqrt{\phi + F^2})y +$$
$$l_6 y Sinh(\sqrt{\phi + F^2})y + l_7 y^4 + l_8 y^3 + l_9 y^2\Big]$$

for heat absorption (62)

$$\theta = \frac{-1}{\lambda}\Big[-C_3(\phi - F^2)Cos(\sqrt{\phi - F^2})y - C_4(\phi - F^2)Sin(\sqrt{\phi - F^2})y + \varepsilon\{-C_7(\phi - F^2)Cos(\sqrt{\phi - F^2})y -$$
$$C_8(\phi - F^2)Sin(\sqrt{\phi - F^2})y - 4l_1(\phi - F^2)Cos2(\sqrt{\phi - F^2})y - 4l_2(\phi - F^2)Sin2(\sqrt{\phi - F^2})y$$
$$+ 2l_3 Cos(\sqrt{\phi - F^2})y + 2l_4 Sin(\sqrt{\phi - F^2})y - l_3 y^2(\phi - F^2)Cos(\sqrt{\phi - F^2})y - l_4 y^2(\phi - F^2)Sin(\sqrt{\phi - F^2})y -$$
$$l_5 y(\phi - F^2)Cos(\sqrt{\phi - F^2})y - l_6 y(\phi - F^2)Sin(\sqrt{\phi - F^2})y - 4l_3 y(\sqrt{\phi - F^2})Sin(\sqrt{\phi - F^2})y$$
$$+ 4l_4 y(\sqrt{\phi - F^2})Cos(\sqrt{\phi - F^2})y - 2l_5(\sqrt{\phi - F^2})Sin(\sqrt{\phi - F^2})y + 2l_6 y(\sqrt{\phi - F^2})Cos(\sqrt{\phi - F^2})y + 12l_7 y^2 + 6l_8 y + 2l_9\}\Big]$$

for heat generation and (63)

$$\theta = \frac{-1}{\lambda}\Big[C_3(\phi + F^2)Cosh(\sqrt{\phi + F^2})y + C_4(\phi + F^2)Sinh(\sqrt{\phi + F^2})y + \varepsilon\{C_7(\phi + F^2)Cosh(\sqrt{\phi + F^2})y +$$
$$C_8(\phi + F^2)Sinh(\sqrt{\phi + F^2})y + 4l_1(\phi + F^2)Cosh2(\sqrt{\phi + F^2})y + 4l_2(\phi + F^2)Sinh2(\sqrt{\phi + F^2})y + 2l_3 Cosh(\sqrt{\phi + F^2})y +$$
$$2l_4 Sinh(\sqrt{\phi + F^2})y + l_3 y^2(\phi + F^2)Cosh(\sqrt{\phi + F^2})y + l_4 y^2(\phi + F^2)Sinh(\sqrt{\phi + F^2})y + l_5 y(\phi + F^2)Cosh(\sqrt{\phi + F^2})y$$
$$+ l_6 y(\phi + F^2)Sinh(\sqrt{\phi + F^2})y + 4l_3 y(\sqrt{\phi + F^2})Sinh(\sqrt{\phi + F^2})y + 4l_4 y(\sqrt{\phi + F^2})Cosh(\sqrt{\phi + F^2})y +$$
$$2l_5(\sqrt{\phi + F^2})Sinh(\sqrt{\phi + F^2})y + 2l_6 y(\sqrt{\phi + F^2})Cosh(\sqrt{\phi + F^2})y + 12l_7 y^2 + 6l_8 y + 2l_9\}\Big]$$

for heat absorption. (64)

IV. Results

The laminar and fully developed mixed convective flow in a vertical channel has been analyzed for viscous fluid in the presence of heat source, sink and radiation. The flow has been assumed to be parallel and each of the two boundaries has been considered as isothermal-isothermal, isoflux-isothermal and isothermal-isoflux. The governing equations have been written in a dimensionless form which is appropriate for the cases of different boundary conditions and same boundary conditions. The solutions of the dimensionless governing equations are determined by the perturbation series method by employing Br Gr /Re as the perturbation parameter. The dimensionless velocity and temperature have been evaluated both in the case of symmetric and asymmetric heating conditions.

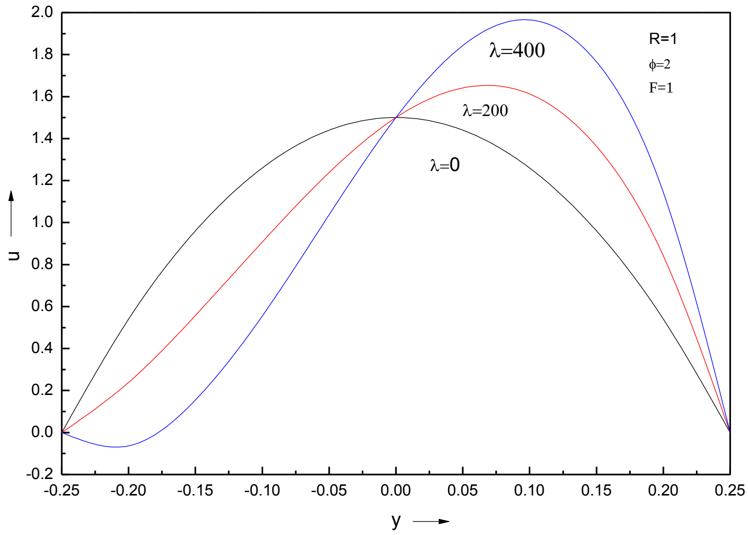

Fig.2. Plots of u versus y in the case of asymmetric heating for different values of λ

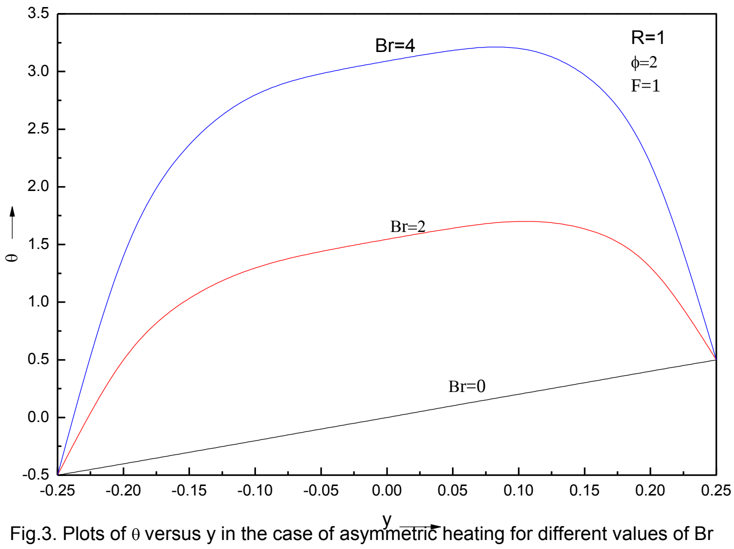

Fig.3. Plots of θ versus y in the case of asymmetric heating for different values of Br

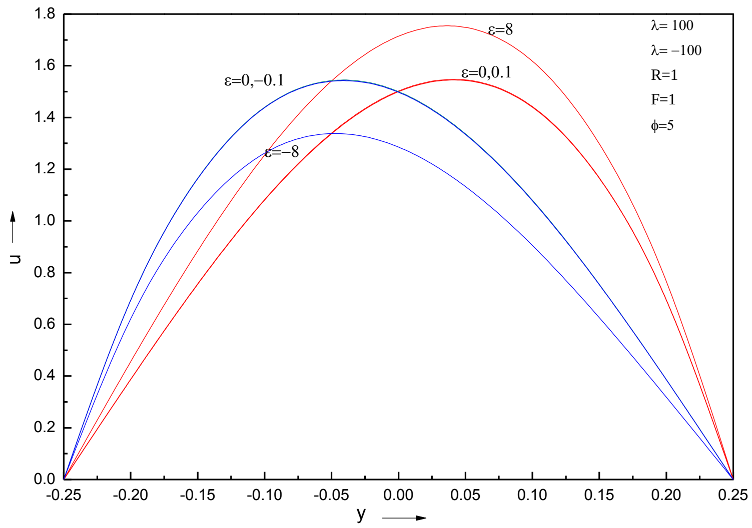

Fig.4. Plots of u versus y in the case of asymmetric heating for different values of λ and ε

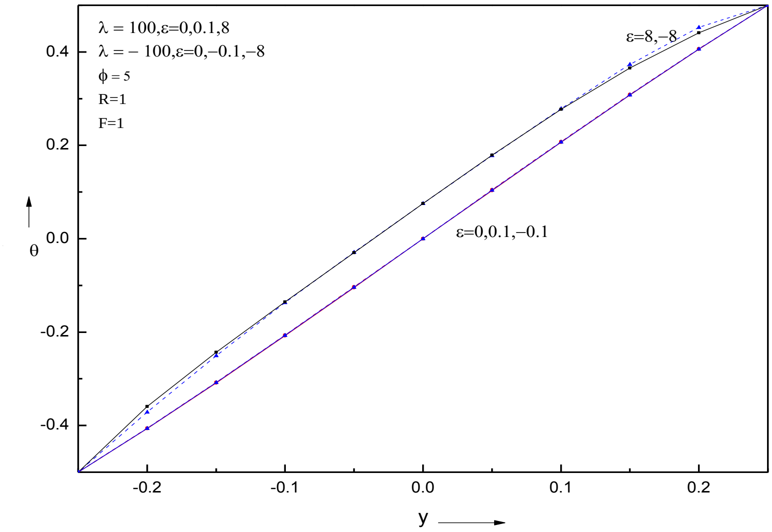

Fig5 : Plots of θ versus y in the case of asymmetric heating for different values of λ and ε

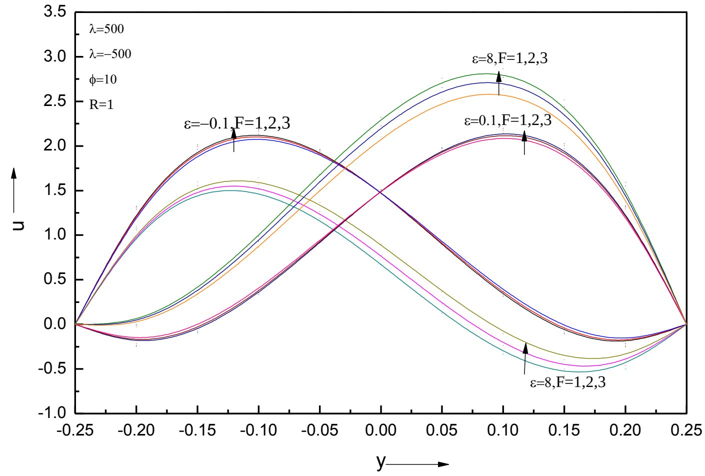

Fig.6. Plots of u versus y in the caseof asymmetric heating for different values of radiation parameter F

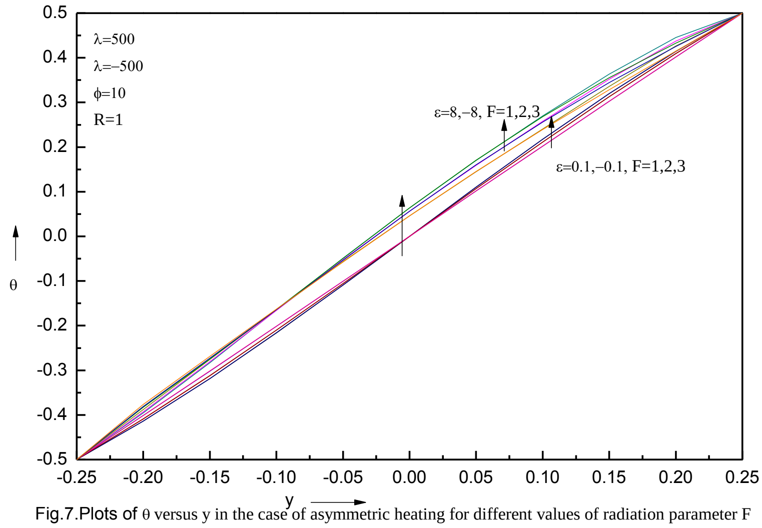

Fig.7.Plots of θ versus y in the case of asymmetric heating for different values of radiation parameter F

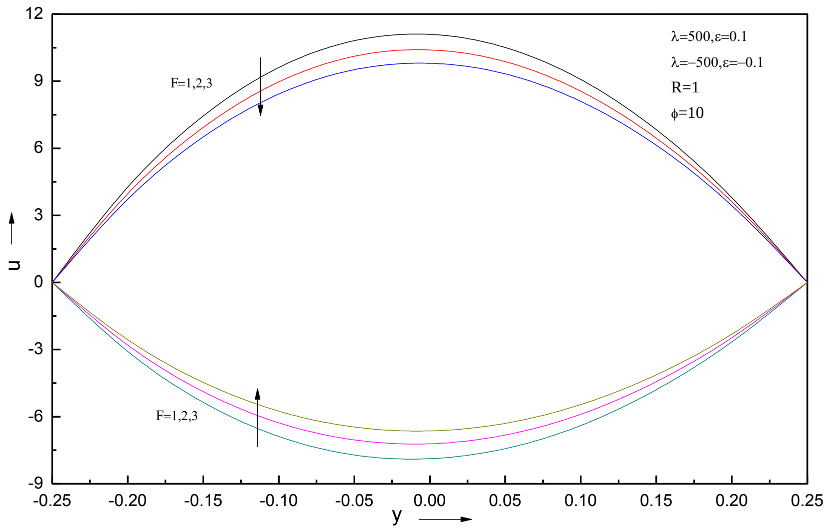

Fig.8. Plots of u versus y for different values of radiation parameter F for isoflux-isothermal case

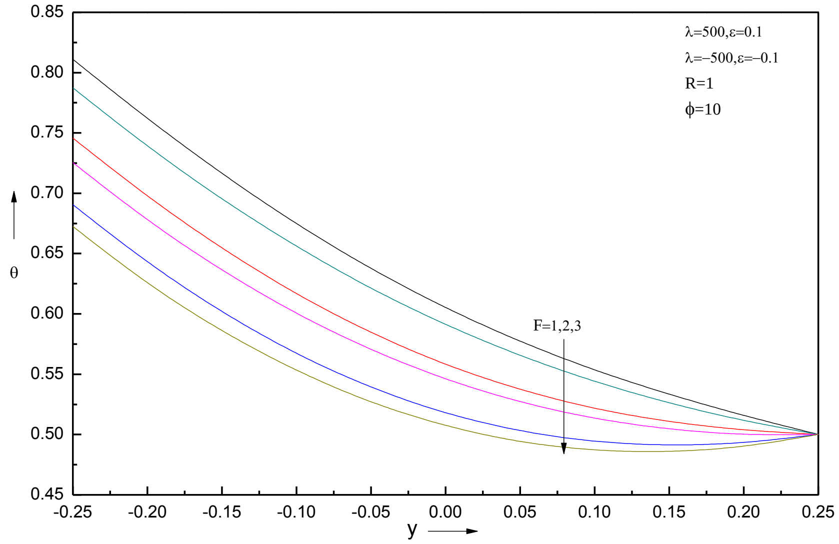

Fig. 9. Plots of θ versus y for different values of radiation parameter F for isoflux- isothermal case

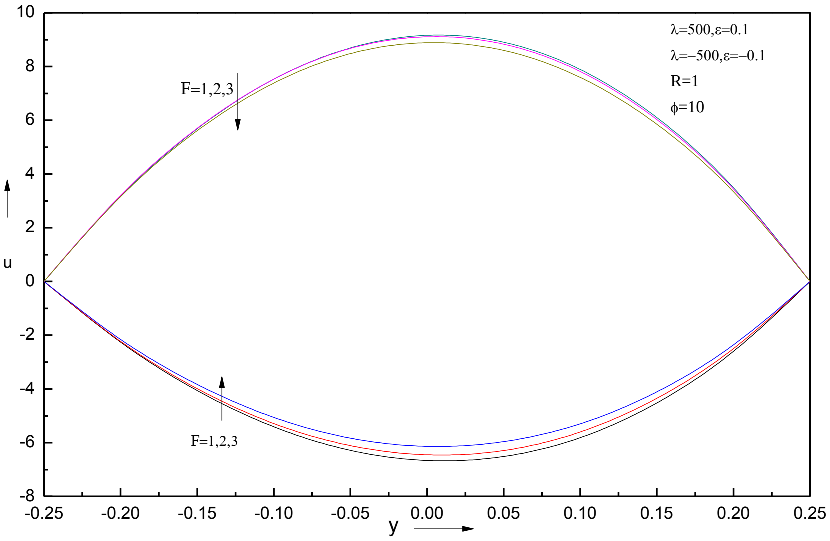

Fig.10.Plots of u versus y for different values of radiation parameter F for isothermal-isoflux case

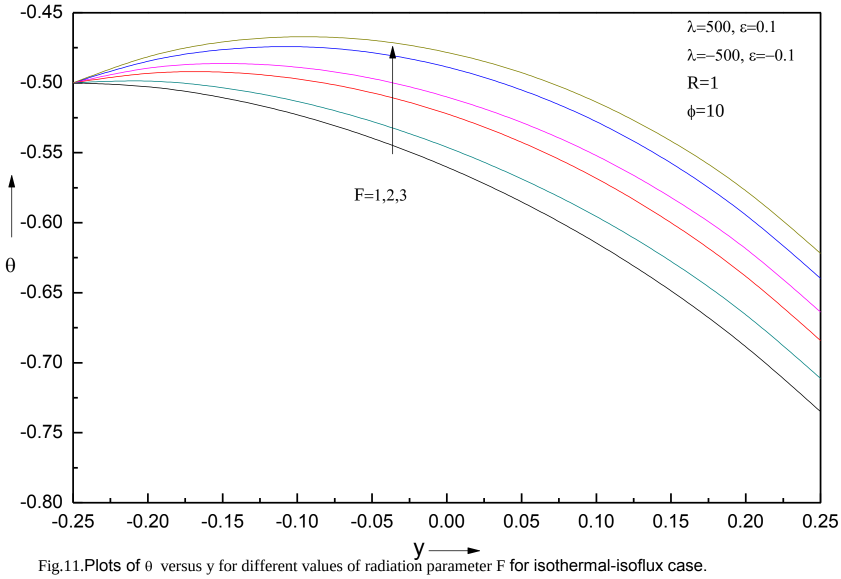

Fig.11.Plots of θ versus y for different values of radiation parameter F for isothermal-isoflux case.

V. Discussion

In fig.2, with R=1and F=1, it is noticed that the velocity profile is parabolic for λ=0 which corresponds to Hagen-Poiseulle flow. But with increase in λ=200 the velocity profile shifts to the right of the channel and with further increase in λ=400 we observe that the flow becomes reversal near the wall at y = -1/4 i.e. downward flow while it counter the flow in the case of upward flow. In other words, with increase in λ , buoyancy force will be greater than viscous forces, resulting in reversal of flow near the cooler wall.

In fig.3, the temperature profiles are drawn for Br=0 and F=1. It is found that the temperature is linear but with increase in Br values the temperature field increases but almost remain linear in the middle of the channel indicating that convection dominates in the boundary layer region. The figure shows that an increase in Br results in viscous dissipation increases which again results in temperature increase.

In fig.4, the effect of ε and ϕ on velocity profile for (T_1-T_2) >0 case is shown. For fixed values of ϕ and F , increase in ε from 0 to 8 it is seen that there is no reversal flow. However if ε is negative , there is no reversal flow towards the left wall but it is towards the right wall of the channel.

Fig.5 shows the effect of ε and ϕ on temperature profile for (T_1-T_2) >0 case. It is noticed that for fixed ϕ and F , increase in ε from 0 to 8 there is no reversal flow.

In fig.6, the effect of ε and F on velocity profile for (T_1-T_2) >0 case is depicted. It is infered that for fixed ϕ , increase in ε from 0.1 to 8 the reversal flow decreases with the increase in the heat generation coefficient. However if ε is negative and with increase in F there is reversal flow towards the right of the wall of the channel. Hence for the positive ε the reversal flow decreases while for negative ε, the flow becomes reversal for given values of ϕ. This means that as heat generation coefficient and radiation factor increases, the reversal flow decreases. For negative ε, the flow becomes reversal.

Fig.7, shows the effect of ε and F on temperature profiles. It is observed that for increasing values of F and ε, there is an increase in the temperature linearly. The same nature is observed for fixed value of ϕ and with increase in ε.

The effect of heat generation coefficient and mixed convection parameter λ on velocity and temperature fields for the isoflux-isothermal wall boundary conditions are shown in the figures 8 and 9. The velocity increases for downward flow and velocity decreases for upward flow with gradual increase in heat generation coefficient in fig.8 whereas the temperature profiles decreases with the increase in the heat generation coefficient as seen in fig.9.

Figures 10 and 11 displays the effect of λ and F on the flow field for isothermal-isoflux wall conditions for asymmetric heating. The velocity increases for upward flow and decreases for downward flow as the heat generation coefficient increases as seen in fig.10. The temperature decreases for increase in heat generation coefficient for both upward and downward flows as seen in fig.11.

VI. Conclusion

The problem of mixed convective flow in an infinite vertical channel with heat source, sink and thermal radiation in the presence of viscous dissipation is discussed. Three different combinations of thermal left-right wall conditions are presented. Analytical solution for the flow and temperature fields with reference to three different special cases are obtained. Graphical representations of all the results are presented for different parameters governing the flow and heat transfer. It is observed that the mixed convection parameter increases the velocity and temperature fields and also the additional radiation parameter positively effects in increasing the velocity and temperature fields.

References

[1]. Aung W, Worku G. Theory of fully developed combined convection including flow reversal. ASME Journal of Heat Transfer. 1986; 108: 485-488.

[2] Cheng CH, Kou HS, Huang W.H. Flow reversal and heat transfer of fully developed mixed convection in vertical channels, Journal Thermo Physics Heat transfer. 1990; 4: 375-383.

[3] Hamadah TT, Wirtz RA. Analysis of laminar fully developed mixed convection in a vertical channel with opposing buoyancy, ASME Journal Heat Transfer.1991;113: 507–510.

[4] Tao TN. On combined free and forced convection in channels, ASME Journal Heat Transfer. 1960; 82: 233–238.

[5] Aung W, Worku G. Mixed convection in ducts with asymmetric wall heat fluxes, ASME Journal of Heat Transfer.1987;109:947–951.

[6] Aung W, Worku G. Developing flow and flow reversal in a vertical channel with asymmetric wall temperature, ASME Journal of Heat Transfer. 1986;108: 299–304.

[7] Ingham DB, Keen DJ, Heggs PJ. Flows in vertical channels with symmetric wall temperatures and including situations where reverse flows occur, ASME Journal of Heat Transfer. 1986: 110: 910–917.

[8] Aung W.Mixed convection in internal flow, in Kakac S, Shah RK, Aung W (Eds) Handbook of single-phase convective heat transfer. 1987 New York , Wiley.

[9] Barletta A.Analysis of combined forced and free flow in a vertical channel with viscous dissipation and isothermal-isoflux boundary conditions, ASME Journal of Heat Transfer. 1999; 121: 349–356

[10] Barletta A.Fully developed mixed convection and flow reversal in a vertical rectangular duct with uniform wall heat fluxes, International Journal of Heat and Mass Transfer. 2002: 45: 641-654.

[11] Umavathi J. C, Prathap Kumar.J and Jaweriya Sulthana. Mixed convection flow in a vertical channel with boundary conditions of the third kind in the presence of heat source or sink. Appl. Math.Mech. Engg. 2012; 33(8): 1015-1034.

[12] Umavathi J. C, Chamkha A. J and Santhosh Veershetty. Fully developed mixed convection of a micro polar fluid in a vertical channel with boundary conditions of third kind. Int. J. Energy and Tech. 2012; 4(3) :1-9.

[13] Vajravelu K, Nayfeh. J.Hydro magnetic convection at a cone and a wedge. International communication in Heat and Mass Transfer. 1992; 19: 701-710.

[14] Chamkha A J. On laminar hydro magnetic mixed convection flow in a vertical channel with symmetric and asymmetric wall heating conditions, International communication in Heat and Mass Transfer. 2002;45: 2509–2525.

[15] Umavathi JC, Mallikarjun Patil B, Pop I . On laminar mixed convection flow in a vertical porous stratum with symmetric wall heating conditions. International Journal of Transport Phenomena. 2006; 8: .127–140

[16] Umavathi J. C.,Patil Mallikarjun B. and Narashima Murthy D. On laminar magnetoconvection flow in a vertical channel in the presence of heat generation and heat absorption, ASME Journal of Heat Transfer. 2013;135: 042503-1 to 042503-8.

[17] T Grosan . I.Pop. Effect of heat generated by an Exothermic reaction on the fully developed mixed convection flow in a vertical channel , Commun. Nonlinear Sci. Numer. Simul. 2010; 15: 471- 474.

[18] Umavathi J. C, Palaniappan.D. Effect of radiation on natural convection in a vertical permeable channel .ASME Journal of Heat Transfer. 1998; 57(2): 35-50.

[19] Wei-Mon Yan, Hung-Yi Li. Radiation effects on mixed convection heat transfer in a vertical square duct. International journal of Heat and Mass Transfer. 2001; 44 :1401-1410

[20] Chamka Ali J, Abbasbandy S., Rashad A M, Vajravelu K . Radiation effects on mixed convection about a cone embedded in a porous medium filled with a nanofluid . Meccanica. 2013; 48: 275-285

[21] Ganesh Kumar .G , Gururaja Rao C. Parametric studies and correlations for combined conduction.Meccanica.2012;47:929-937.

[22] Cha'o-Kuang Chen,Yue-Tzu Yang, Kuei-Hao Chang .Entropy generation of radiation effect on laminar-mixed convection along a wavy surface .Heat Mass Transfer. 2011; 47: 385-395

[23] Umavathi J. C, Manjula M. H , Radioactive Heat Transfer of Two Fluid Flow in a Vertical Porous Channel, Proceedings of International Symposium on Advances in Fluid Mechanics. 2001; 215-224.

[24] Khaefinejad B , Aghanajafi C. Laminar flow with Simultaneous effects of mixed convection and thermal radiation in a channel, Journal of Fusion Energy. 2009; 28: 83-90.

[25] Dileep Singh Chauhan and Priyanka Rastogi. Radiation effects on natural convection MHD flow in a rotating vertical porous channel partially filled with a porous medium. Applied Mathematical Sciences. 2010;4(13): 643-655.

[26] Baoku,I. G.;Israel-Cookey, C.; Olajuwon, B. I. Influence of thermal radiation on a transient MHD Couette flow through a porous medium, Journal of Applied Fluid Mechanics. 2012; 5(1): 81-87.

[27] Raptis .A. Radiation and flow through a porous medium. Journal of Porous Media. 2001;4(3).

[28] Sudarshan Reddy,G.; Ramana Reddy, G. V.; Jayarami Reddy ,K.. Radiation and chemical reaction effects on free convection MHD flow through a porous medium bounded by vertical surface. Advances in Applied Science Research.2012;3(3):1603- 1610.

[29] Mukhopadhyay, S . Effects of Radiation and Variable Fluid Viscosity on Flow and Heat Transfer along a Symmetric Wedge. Journal of Applied Fluid Mechanics. 2009;2(2): 29-34.

[30] Sankar Kumar Guchhait, Sanatan Das, Rabindra Nath Jana..The Combined Effects of Hall current and Radiation on MHD Free Convective Flow in a Vertical Channel with an Oscillatory Wall Temperature. Open Journal of Fluid Dynamics. 2013; 3(1) 9-22.

[31] Patil Mallikarjun B , Mixed convection in the presence of heat generation or heat absorption, *National conference on Recent Trends in Fluid Mechanics*, Tumkur University, Tumkur, 2012, 49-73