

New Generating Functions for Measures of Inaccuracy

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Abstract: - The paper intended to obtain new generating functions for measures of inaccuracy and relative information measures. The derivative of it, at 1 and 0 includes several well-known results. The particular and limiting cases of the same are also to be examined.

Keywords: -, Directed Divergence, Generalized Entropy, Generating Functions, Measure Of Entropy And Measures Of Inaccuracy Measures Of Information.

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I. INTRODUCTION

Information theory is the significant method of unspecific and indefinite situation where exact interpretation is next to impossible. From the very beginning, the information theory has made a great advancement with its implementation in various possible fields, for example- artificial intelligence, computer science, transportation models city population operation research control engineering. In 1996 Golomb [1] determined an information generating functions of the probability distribution. He verified that the first moment of self-information is Shannon's [2] measure of entropy. Golomb [1] achieved an easy mathematical expression, measures of generating functions for discrete and continuous distribution, geometric distribution, zeta distribution, exponential distribution, Pareto distribution and normal distribution. Subsequently, Golomb [1] and Guiasu and Reischer [3] gave generating function for Shannon's [2] and Kullback- Leibler's [4] for corresponding measure of information; and also defined generating function for relative information or cross entropy or directed divergence of P from Q. Golomb [1] provided generating functions for Shannon measure of entropy and Kullback Leibler's [4] measures of information theory. Galomb [1] proposed the generating function with the derivatives, evaluated at a certain place, create the moments of self-information of distribution.

In 1898, Guiasu and C. Reischer [3] contributed substantially with the relative information generating function. Its derivatives give famous statistical measures such as the Kullback-Leibler [4] divergence between two probability distribution. In this literature, Guiasu and Reisher [3] contain Golomb's [1] generating function as a particular case. It holds simple pattern for the binomial and poison distribution, neither of which fits into Golomb's [1] model. Apart from its consolidation, the relative information generating function advocates new information including the standard deviation of variation of information. Kapur [5] came up with the generating function for other measures of entropy and relative information. He also examined these measures of information based on two, three, four or m probability distribution.

In 1990, D.S. Hooda and U. Singh [9] proposed information improvement generating function with its derivative at point 1. It gives Theils [10] measures of information improvement; and also explained comprehensive functions in the domain of Economics. The expression for discrete distribution is enhanced and verified in such a way that the information improvement generating functions recommended a new information pointer as the standard deviation of the variation of information.

In 1995 D.S.Hooda and U.S. Bhaker [11] introduced a weighted entropy generating functions. In 1995 D.S. Hooda et al. [13] contributed substantially in relative information generating functions with its applications. In 2007 D.S. Hooda, Kumar et al. [12] put forward some new findings on relative information generating functions. In 2009, K.C. Jain and Amit Shrivastava [7] indicated some new weighted generating functions of discrete probability distribution with applications. Its derivatives give some renowned measures of information and analyzed some specific cases of advocated functions. D.S. Hooda and D.K. Sharma [8] proposed generalized 'useful' information generating functions and two new relative 'useful' information generating functions are developed. The information generating recommended functions improve the efficiency to estimate various measures illustrated by application of these proposed information generating functions for systematic and constant geometric and exponential probability distribution. Amit Shrivastava and M. Shikha

[14] offered a new weighted information generating functions for discrete probability distribution with the derivative at point 1, provides some renowned measures of information. In 2011, K.C. Deshmukh et al. [14] advocated weighted generating functions for measures of fuzzy improvement and the knowledge of measure of entropy can be used to upgrade the literature of information energy. P. Jha, Dewangan, R.Verma [6] advocated some generating functions for measures of probabilistic entropy and directed divergence. It has some resemblance and contradiction in the results of four type of new generalized measures. Further, the knowledge of information energy helps us obtain many new measures of information energy. In the literature, we attempted to achieve one, two, three and four parametric measures of information energy. Here, we have obtained generating functions with the various measures of inaccuracy and verified that the derivative at 1 and 0 of proposed generating functions incorporates several well-known results. The particular and limiting cases of obtained generating functions are also discussed. In the section two, some preliminaries are examined along with the discussion on the basic concepts, definitions and properties of generating functions for measures of inaccuracy. In the section three, we found generating functions for several measures of inaccuracy corresponding to Shannon's [2] measures of entropy, Kapur's [5] , [17] measures of entropy, Renyi's [18] measures of entropy, Kerridge's [19] measure of entropy, Bose-Einstein's [5], Havrda Charvat's [20] , Kapur's [5], [17],Fermi-Dirac [5] measure of entropy expanded new two parametric generating functions; and also discussed particular and limiting cases of obtained generating functions. In the section four, the conclusive comments of the literature are discussed. The reference and citation for the paper are mentioned in the section five.

II. PRELIMINARIES

For any probability $P = \{p_1, p_2, \dots, p_n\}$ and $Q = \{q_1, q_2, \dots, q_n\}$ are two probability distributions.

Afterwards, Golomb [1] defined an information generating function of probability distribution function P as

$$f(t) = -\sum_{i=1}^n p_i t^i \tag{2.1}$$

as the generating function with property,

$$f'(1) = S(P) \tag{2.2}$$

where S(P) is Shannon's [2] measure of entropy for probability distribution P, from (2.1) we get

$$f^r(1) = (-1)^{r-1} \sum_{i=1}^n p_i (-\ln p_i)^r, \quad r = 1, 2, \dots \tag{2.3}$$

Golomb [1] recommended that the first moment of self-information is Shannon [2] that is $-\ln p_i$ is the self-information of i^{th} outcomes, from (2.3)

$$(-1)^{r-1} f^r(1) \tag{2.4}$$

The expected value of the r^{th} power of the self-information given by equation (2.4)

In 1994, Kapur [5] developed the generating functions for relative information or measures of inaccuracy of probability distribution $P = \{p_1, p_2, \dots, p_n\}$ from another probability distribution be $Q = \{q_1, q_2, \dots, q_n\}$ as

$$h(t) = -\sum_{i=1}^n p_i q_i^t \tag{2.5}$$

with the property,

$$h'(0) = -\sum_{i=1}^n p_i \ln q_i \tag{2.6}$$

which is Kerridge's [8] measures of inaccuracy. Thus $h(t)$ can be regarded as a generating function for Kerridge's [8] measures of inaccuracy. The measure of inaccuracy (2.6) is based on Shannon's [2] measure of entropy. Kapur [5] has generalized the generating function $h(t)$ for Kerridge's [8] measures of inaccuracy with the help of parameter as

$$h_\alpha(t) = \frac{1}{1-\alpha} \left[\left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right)^t - 1 \right], \quad \alpha \neq 1, \alpha > 0 \tag{2.7}$$

$$h'_\alpha(0) = \frac{1}{1-\alpha} \ln \left[\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right], \quad \alpha \neq 1, \alpha > 0 \tag{2.8}$$

It is Renyi's [7] measure of inaccuracy therefore, $h_\alpha(t)$ can be regarded as a generating function for Renyi's measure of inaccuracy.

III. NEW GENERATING FUNCTIONS FOR MEASURES OF INFORMATION BASED ON ONE PROBABILITY DISTRIBUTION

Measures Of Inaccuracy

(I) Generating Function for Kapur's [5] Measures of Inaccuracy

Let,

$$h_{\alpha,\beta}(t) = \frac{1}{\beta-\alpha} \left[\left\{ \left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right) \left(\frac{\sum_{i=1}^n p_i^\beta q_i^{1-\beta}}{\sum_{i=1}^n p_i^\beta} \right) \right\}^t - 1 \right], \text{ Where } \beta > 1, \alpha < 1 \text{ or } \beta < 1, \alpha > 1 \quad (3.1)$$

with property,

$$h'_{\alpha,\beta}(0) = \frac{1}{\beta-\alpha} \ln \left[\left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right) \left(\frac{\sum_{i=1}^n p_i^\beta q_i^{1-\beta}}{\sum_{i=1}^n p_i^\beta} \right) \right] \quad (3.2)$$

It is measure of inaccuracy corresponding to Kapur [5] measure of entropy

$$\frac{1}{\beta-\alpha} \ln \left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\beta} \right) \quad \beta \neq \alpha, \beta > 1, \alpha < 1 \text{ or } \beta < 1, \alpha > 1 \quad (3.3)$$

So $h_{\alpha,\beta}(t)$ is generating function for Kapur's [5] measures of inaccuracy. It is obvious that

$$h_{\alpha,1}(t) = \frac{1}{1-\alpha} \left[\left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right)^t - 1 \right] \quad (3.4)$$

$$h_{\alpha,1}(t) = h_\alpha(t)$$

$$\lim_{\alpha \rightarrow 1} h_{\alpha,1}(t) = h_\alpha(t) \quad (3.5)$$

(II) Let,

$$h_{\alpha,\beta}^c(t) = \frac{1}{\beta-\alpha} \left[\left\{ \left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right)^t \left(\frac{\sum_{i=1}^n p_i^\beta q_i^{1-\beta}}{\sum_{i=1}^n p_i^\beta} \right) \right\}^{ct} - 1 \right] \quad \beta \neq \alpha, c > 0, 0 < \alpha < 1, \beta > 1 \text{ or } c > 0, 0 < \beta < 1, \alpha > 1 \quad (3.6)$$

$$[h_{\alpha,\beta}^1(t)]' = \frac{1}{\beta-\alpha} \left\{ \left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right) \left(\frac{\sum_{i=1}^n p_i^\beta q_i^{1-\beta}}{\sum_{i=1}^n p_i^\beta} \right) \right\}^t \ln \left[\left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right) \left(\frac{\sum_{i=1}^n p_i^\beta q_i^{1-\beta}}{\sum_{i=1}^n p_i^\beta} \right) \right] \quad (3.7)$$

$$[h_{\alpha,\beta}^c(0)]' = \frac{1}{\beta-\alpha} \left[\ln \left\{ \left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right) \left(\frac{\sum_{i=1}^n p_i^\beta q_i^{1-\beta}}{\sum_{i=1}^n p_i^\beta} \right) \right\}^c \right] \quad (3.8)$$

the mathematics expression (3.8) is the measure of inaccuracy corresponding to Kapur's [5] measure of entropy

$$\frac{1}{\beta-\alpha} \ln \frac{\sum_{i=1}^n p_i^\alpha}{\left(\sum_{i=1}^n p_i^\beta \right)^c}$$

So, $h_{\alpha,\beta}^c(t)$ is generating function for Kapur's [5] measure of inaccuracy. It is obvious that

$$[h_{\alpha,\beta}^1(t)]' = h_{\alpha,\beta}(t) \quad (3.9)$$

$$\lim_{\alpha \rightarrow 1} h_\alpha(t) = h(t) \quad (3.10)$$

Thus, $h_{\alpha,\beta}^k(t)$ includes all the generating functions as aforementioned.

(III) Let,

$$h_{\alpha,\beta}(t) = \frac{1}{1-\alpha} \left[\left\{ \left(\frac{\sum_{i=1}^n p_i^{\alpha+\beta-1}}{\sum_{i=1}^n p_i^{\alpha+\beta-1} q_i^{2-\alpha-\beta}} \right) \left(\frac{\sum_{i=1}^n p_i^\beta q_i^{1-\beta}}{\sum_{i=1}^n p_i^\beta} \right) \right\}^t - \beta \right], \alpha \neq \beta, \alpha > 0, \beta > 0, \alpha+\beta-1 > 0, \alpha \neq 1 \quad (3.11)$$

$$h'_{\alpha,\beta}(0) = \frac{1}{1-\alpha} \ln \left\{ \left(\frac{\sum_{i=1}^n p_i^{\alpha+\beta-1}}{\sum_{i=1}^n p_i^{\alpha+\beta-1} q_i^{2-\alpha-\beta}} \right) \left(\frac{\sum_{i=1}^n p_i^\beta q_i^{1-\beta}}{\sum_{i=1}^n p_i^\beta} \right) \right\} \alpha > 0, \beta > 0, \alpha+\beta-1 > 0, \alpha \neq 1, \alpha \neq \beta$$

which is a measure of Inaccuracy of Kapur's [5] measure of entropy. So, $h_{\alpha,\beta}(t)$ is the generating function for Kapur's [5] measure of inaccuracy,

$$h'_{\alpha,1}(0) = \frac{1}{1-\alpha} \ln \left\{ \left(\frac{\sum_{i=1}^n p_i^\alpha}{\sum_{i=1}^n p_i^\alpha q_i^{1-\alpha}} \right) \right\}$$

It is apparent that when $\beta = 1$

$h_{\alpha,\beta}(t)$ is reduced to $h_\alpha(t)$ discussed in section two

and if $\alpha = 0$ in addition to $\beta = 1$

$$h_{\alpha,\beta}(t) = n^{t-1}$$

so

$$h^*_{0,1}(1) = \ln n$$

which is measure of inaccuracy corresponding to Hartley's [10] measure of entropy.

Thus, $h_{\alpha,\beta}(t)$ is generating function for Hartley's [10] measure of inaccuracy.

(IV) New Measure Of Inaccuracy Corresponding To Fermi Dirac Measure Of Entropy

$$\text{Let, } h(t) = -\sum_{i=1}^n p_i q_i^t - \sum_{i=1}^n (1-p_i)(1-q_i)^t \tag{3.12}$$

$$h'(0) = -\sum_{i=1}^n p_i \ln q_i - \sum_{i=1}^n (1-p_i) \ln(1-q_i) \tag{3.13}$$

it is measure of inaccuracy that belongs Fermi Dirac [5] [C.f Kapur 1994, P 102] measure of entropy.

$$-\sum_{i=1}^n p_i \ln p_i - \sum_{i=1}^n (1-p_i) \ln(1-p_i) \tag{3.14}$$

So $h(t)$ is the generating function for Fermi-Dirac's [5] Kapur [1994, P102] measure of inaccuracy.

(V) Let,

$$h_{a,b}(t) = -\sum_{i=1}^n (p_i - a_i)(q_i - a_i)^t - \sum_{i=1}^n (b_i - p_i)(b_i - q_i)^t + \sum_{i=1}^n (b_i - a_i)^t, \quad a_i \leq p_i \leq b_i \tag{3.15}$$

$$h_{a,b}'(0) = -\sum_{i=1}^n (p_i - a_i) \ln(q_i - a_i) - \sum_{i=1}^n (b_i - p_i) \ln(b_i - q_i) + \sum_{i=1}^n (b_i - a_i) \ln(b_i - a_i), \tag{3.16}$$

It is measure of inaccuracy corresponding to Kapur's [5] measure of entropy.

$$-\sum_{i=1}^n (p_i - a_i) \ln(p_i - a_i) - \sum_{i=1}^n (b_i - p_i) \ln(b_i - p_i) + \sum_{i=1}^n (b_i - a_i) \ln(b_i - a_i), \quad a_i \leq p_i \leq b_i \tag{3.17}$$

so, $h_{a,b}(t)$ is generating function for Kapur [5] measures of inaccuracy.

$a_i = 0, b_i = 1$ which is natural constraint in (3.15), (3.16), (3.17),

if $a_i = 0, b_i = 1$ in (3.15), it reduced to (3.12)

If $a_i = 0, b_i = 1$ in (3.16), it reduced to (3.13)

(VI) Let

$$h(t) = -\sum_{i=1}^n (p_i - a_i)(q_i - a_i)^t - \sum_{i=1}^n (b_i - p_i)(b_i - q_i)^t + \frac{1}{a} \sum_{i=1}^n \{1 + a(p_i - a_i)\} \{1 + a(q_i - a_i)\}^t + \frac{1}{a} \sum_{i=1}^n \{1 + a(b_i - p_i)\} \{1 + a(b_i - q_i)\}^t - \sum_{i=1}^n \left\{ \left(\frac{b_i - a_i}{a} \right) (1 + a)^{t+1} \right\} \tag{3.18}$$

$a_i \leq p_i \leq b_i$

$$h'(0) = -\sum_{i=1}^n (p_i - a_i) \ln(q_i - a_i) - \sum_{i=1}^n (b_i - p_i) \ln(b_i - q_i) + \frac{1}{a} \sum_{i=1}^n \{1 + a(p_i - a_i)\} \ln\{1 + a(q_i - a_i)\} + \frac{1}{a} \sum_{i=1}^n \{1 + a(b_i - p_i)\} \ln\{1 + a(b_i - q_i)\} - \sum_{i=1}^n \left\{ \left(\frac{b_i - a_i}{a} \right) \frac{1}{a} (1 + a) \ln(1 + a) \right\} \tag{3.19}$$

$a_i \leq p_i \leq b_i$

which is measure of inaccuracy Kapur's [5] measure of entropy

$$-\sum_{i=1}^n (p_i - a_i) \ln(p_i - a_i) + \frac{1}{a} \left[\sum_{i=1}^n (1 + a(p_i - a_i)) \ln\{1 + a(p_i - a_i)\} \right] - \sum_{i=1}^n (b_i - p_i) \ln(b_i - p_i) + \frac{1}{a} \sum_{i=1}^n \{1 + a(b_i - p_i)\} \ln\{1 + a(b_i - p_i)\} - \sum_{i=1}^n \left\{ \left(\frac{b_i - a_i}{a} \right) (1 + a) \ln(1 + a) \right\}$$

(VII) Let,

$$h_{\frac{a}{b}}(t) = -\sum_{i=1}^n p_i q_i^t + \frac{a}{b} \sum_{i=1}^n \left(1 + \frac{b}{a} p_i \right) \left(1 + \frac{b}{a} q_i \right)^t - \frac{a}{b} \sum_{i=1}^n p_i \left(1 + \frac{b}{a} \right)^{t+1}, \quad b > 0, 0 < a \leq 1 \tag{3.20}$$

$$h'_{\frac{a}{b}}(0) = -\sum_{i=1}^n p_i \ln q_i + \frac{a}{b} \sum_{i=1}^n \left(1 + \frac{b}{a} p_i \right) \ln \left(1 + \frac{b}{a} q_i \right) - \frac{a}{b} \left(1 + \frac{b}{a} \right) \ln \left(1 + \frac{b}{a} \right), \quad b > 0, 0 < a \leq 1 \tag{3.21}$$

It is a measure of inaccuracy corresponding to two parametric measure of entropy.

$$-\sum_{i=1}^n p_i \ln p_i + \frac{a}{b} \sum_{i=1}^n \left(1 + \frac{b}{a} p_i \right) \ln \left(1 + \frac{b}{a} p_i \right) - \frac{a}{b} \left(1 + \frac{b}{a} \right) \ln \left(1 + \frac{b}{a} \right) p_i, \quad b > 0, 0 < a \leq 1$$

And (3.20) is the generating function corresponding to two parametric measure of inaccuracy

$$-\sum_{i=1}^n p_i q_i \ln q_i + \frac{a}{b} \sum_{i=1}^n \left(1 + \frac{b}{a} p_i \right) \ln \left(1 + \frac{b}{a} q_i \right) - \frac{a}{b} \left(1 + \frac{b}{a} \right) \ln \left(1 + \frac{b}{a} \right), \quad b > 0, 0 < a \leq 1$$

Again (3.20) becomes

$$h_{\frac{a}{1}}(t) = -\sum_{i=1}^n p_i q_i^t + \frac{1}{a} \sum (1 + ap_i)(1 + aq_i)^t - \frac{1}{a} \sum_{i=1}^n p_i (1 + a)^{t+1}, \quad a > 0 \quad (3.22)$$

$$h'_{\frac{a}{1}}(0) = -\sum_{i=1}^n p_i \ln q_i + \frac{1}{a} \sum (1 + ap_i) \ln(1 + aq_i) - \frac{1}{a} (1 + a) \ln(1 + a), \quad a > 0 \quad (3.23)$$

it is the measure of inaccuracy of Kapur's [5] measures of entropy

$$-\sum_{i=1}^n p_i \ln p_i + \frac{1}{a} \sum_{i=1}^n (1 + ap_i) \ln(1 + ap_i) - \frac{1}{a} (1 + a) \ln(1 + a) p_i, \quad a > 0$$

so, $h_{\frac{a}{1}}(t)$ is generating function related to Kapur's [5] measure of inaccuracy

by taking $a=b$ in (3.20) we attain,

$$h(t) = -\sum_{i=1}^n p_i q_i^t + \sum (1 + p_i)(1 + q_i)^t - 2^{t+1} \quad (3.24)$$

Then

$$h'(t) = -\sum_{i=1}^n p_i q_i^t \ln q_i + \sum_{i=1}^n (1 + p_i)(1 + q_i)^t \ln(1 + q_i) - 2^{t+1} \ln 2$$

$$h'(0) = -\sum_{i=1}^n p_i \ln q_i + \sum_{i=1}^n (1 + p_i) \ln(1 + q_i) - 2 \ln 2 \quad (3.25)$$

which is measure of inaccuracy for Bose Einstein's [5] (Kapur [5]) [C.F. 1994 P 102] measure of entropy.

$$-\sum_{i=1}^n p_i \ln p_i + \sum_{i=1}^n (1 + p_i) \ln(1 + p_i) - 2 \ln 2 \quad (3.26)$$

So $h(t)$ is generating function for Bose Einstein's [5] (Kapur [5]) [C.F. 1994 P 102] measure of inaccuracy.

[VIII] Let,

$$h_{\frac{a}{b}}(t) = -\sum_{i=1}^n p_i q_i^t + \frac{a}{b} \sum_{i=1}^n (1 + \frac{b}{a} p_i)^{t+1} - \frac{a}{b} \sum_{i=1}^n (1 + \frac{b}{a})^{t+1} p_i - \frac{a}{b} \sum_{i=1}^n (q_i + \frac{b}{a} p_i)^{t+1} + \frac{a}{b} \sum_{i=1}^n (q_i)^t (q_i + \frac{b}{a} p_i) (1 + \frac{b}{a}), \quad b > 0, 0 < a \leq 1 \quad (3.27)$$

$$h'_{\frac{a}{b}}(0) = -\sum_{i=1}^n p_i \ln q_i + \frac{a}{b} \sum_{i=1}^n \ln(1 + \frac{b}{a} p_i) (1 + \frac{b}{a} p_i) - \frac{a}{b} \sum_{i=1}^n (1 + \frac{b}{a}) \ln(1 + \frac{b}{a}) -$$

$$\frac{a}{b} \sum_{i=1}^n (q_i + \frac{b}{a} p_i) \ln(q_i + \frac{b}{a} p_i) + \frac{a}{b} \sum_{i=1}^n (q_i + \frac{b}{a} p_i) q_i \ln q_i (1 + \frac{b}{a}), \quad b > 0, 0 < a \leq 1 \quad (3.28)$$

It is a measure of inaccuracy for two parametric measure of entropy

$$-\sum_{i=1}^n p_i \ln p_i + \frac{a}{b} \sum_{i=1}^n (1 + \frac{b}{a} p_i) \ln(1 + \frac{b}{a} p_i) - \frac{a}{b} (1 + \frac{b}{a}) \ln(1 + \frac{b}{a}) p_i, \quad b > 0, 0 < a \leq 1$$

And (3.27) is the generating function corresponding to two parametric measure of inaccuracy

$$-\sum_{i=1}^n p_i q_i \ln q_i + \frac{a}{b} \sum (1 + \frac{b}{a} p_i) \ln(1 + \frac{b}{a} p_i) - \frac{a}{b} (1 + \frac{b}{a}) \ln(1 + \frac{b}{a}), \quad b > 0, 0 < a \leq 1$$

$$h'_{\frac{a}{b}}(0) = -\sum_{i=1}^n p_i \ln q_i + \frac{1}{b} \sum_{i=1}^n (1 + bp_i) \ln(1 + bp_i) - \frac{1}{b} \sum_{i=1}^n (1 + b) \ln(1 + b) -$$

$$\frac{1}{b} \sum_{i=1}^n (q_i + bp_i) \ln(q_i + bp_i) + \frac{1}{b} \sum_{i=1}^n (q_i + bp_i) q_i \ln q_i (1 + b), \quad b > 0 \quad (3.29)$$

$$h'_{\frac{a}{b}}(0) = -\sum_{i=1}^n p_i \ln q_i + \frac{1}{b} \sum_{i=1}^n (1 + bp_i) \ln(1 + bp_i) - \frac{1}{b} \sum_{i=1}^n (1 + b) \ln(1 + b) -$$

$$\frac{1}{b} \sum_{i=1}^n (q_i + bp_i) \ln(q_i + bp_i) + \frac{1}{b} \sum_{i=1}^n (q_i + bp_i) q_i \ln q_i (1 + b),$$

$$\sum_{i=1}^n p_i \ln p_i + \frac{1}{a} \sum_{i=1}^n (1 + ap_i) \ln(1 + ap_i) - \frac{1}{a} (1 + a) \ln(1 + a) p_i, \quad a > 0 \quad (3.30)$$

[IX] Let,

$$h_{a,b,k}(t) = -\sum_{i=1}^n p_i q_i^t + \frac{b}{a^k} \sum (1 + \frac{a}{b} p_i) (1 + \frac{a}{b} q_i)^t - \frac{b}{a^k} \sum_{i=1}^n p_i (1 + \frac{a}{b})^{t+1}, \quad b > 0, 0 < a \leq 1 \quad (3.31)$$

$$h_{a,b,k}'(t) = -\sum_{i=1}^n p_i q_i^t \ln q_i + \frac{b}{a^k} \sum (1 + \frac{a}{b} p_i) (1 + \frac{a}{b} q_i)^t \ln(1 + \frac{a}{b} q_i) - \frac{b}{a^k} (1 + \frac{a}{b})^{t+1} \ln(1 + \frac{a}{b}), \quad b > 0, 0 < a \leq 1 \quad (3.32)$$

$$h_{a,b,k}'(0) = -\sum_{i=1}^n p_i \ln q_i + \frac{b}{a^k} \sum (1 + \frac{a}{b} p_i) \ln(1 + \frac{a}{b} q_i) - \frac{b}{a^k} (1 + \frac{a}{b}) \ln(1 + \frac{a}{b}), \quad (3.33)$$

$$h_{a,1,2}'(0) = -\sum_{i=1}^n p_i \ln q_i + \frac{1}{a^2} [\sum (1 + ap_i) \ln(1 + aq_i) - (1 + a) \ln(1 + a)], \quad a > 0 \quad (3.34)$$

which is measure of inaccuracy corresponding to Kapur's [5] measures of entropy

From (3.33),(3.23),(3.24) and (3.25), we can write as

$$h_{b,a,1}'(0) = h'_{\frac{b}{a}}(0)$$

$$h_{a,1,1}'(0) = h'_{\frac{1}{a}}(0)$$

$$h_{a,1,1}'(0) = h'_{\frac{1}{a}}(0)$$

$$h_{1,1,1}'(0) = h'_{\frac{1}{1}}(0)$$

$$h_{1,-1,1}'(0) = h'_{-\frac{1}{1}}(0)$$

Since, the generating function $h_{a,b,k}(t)$ includes several well know results of generating function for measure of inaccuracy

IV. CONCLUDING REMARKS

- In the available literature of information theory, we have observed various inaccuracy measures, each with its own advantages disadvantages and limitations. However, our main objective lies in the development of those measures which can find applications in a variety of disciplines.
- The researchers from biological sciences also examine Shannon's [2] or Renyi's [7] measures for examining diversity in different species. If we have a variety of information measures, we will have a number of options and conducive one can be selected.
- We have observed that derivative at 1 and 0 of proposed generating functions incorporates several renowned results.
- We have examined the results for a random variable taking a finite set of discrete values. The analysis can easily be extended to the various cases provided the number of values taken is infinite or variable is continues, subjected to the series and integrals which emerge all are convergent.
- We have discussed the measure of one, two, and three parametric measures of inaccuracy satisfying all the properties determined in the section two.
- Although, here we have discussed for one and two probability distribution the discussion can easily be expanded to multi-variate distribution.
- We recommend to give such generating functions for most measures of inaccuracy corresponding to measure of entropy and capable to have generating functions for all the measures.

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