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Concepts of Stabilityeigen Values for Predator –Prey Relationship

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Abstract: In this paper Predator prey model for scientific problem has been framed. Fox rabbit Predator –Prey model for the differential equations is described. In order to check the system's stability, Eigen values are required for that a set of equilibriums factors are discussed.

Key words: Predator- Prey, Eigen Values ,Lotka –Voltera Differential equations.

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I. INTRODUCTION:

For the motive of describing the relations between predators and prey we decided to imagine the relationship between Foxes and Rabbits. In this model, F will usuallysignify the inhabitantsof the Foxes and R will represent the inhabitants of the Rabbits. The population growing of the Foxes is dependent on Rabbits and of natural reasons. The population growth of the Rabbits is dependent on the number of Foxes that die from natural reasons.

1.2 Construction of the scientific problem:

Predator- Prey models are utilized by scientists to forecast or make clear trends in animal populations. There are many instances in nature where one species of animal feeds on another kind of animal, which in turn feeds on another thing. The first species is called Predator and the second is called the Prey.

Theoretically, the Predator can terminate all the prey so that the later become vanished. However if this happens the predator will also become extinct since; as we assume, it depends on the prey for its presence.

What actually happens in nature is that a cycle develops where at some time prey may be lavish and predators few. Because of the plenty of prey, the predator population grows and reduces the population of prey.

This outcome in a reduction of predators and resultant increase of prey and the cycle maintains. An essential problem of ecology, the science which studies the interactions of animals and their environment is to analyze the demand of co-occurrenceof the two species [6].

II. PREDATOR- PREY MODEL: RABBITS AND FOXES.

We will create a mathematical model which describes the relationships between predator and prey in the eco-system, where the predators are foxes and the Prey are rabbits.
In order for this model to work we must create a few assumptions.

III. ASSUMPTIONS:

Rabbits only die by being eaten by foxes and of natural causes.
Foxes only die from natural causes
The relations between foxes and rabbits can be defined by a function.

IV. DIFFERENTIAL EQUATIONS FOR PREDATOR- PREY MODELS.

One of the most interesting applications of structure of different equations is the predator prey problem. In this paper we will consideration an environment containing two related populations – a prey population such as foxes and a predator population such as rabbits. Clearly it is reasonable to expect that the two populations react in such a way as to influence each other's size, without loss of generality we will focuses on systems of two different equations[1][2].

V. LOTKA – VOLTRA MODEL:

System **Initial conditions**

$dR/dt = aR - bR^2 - cRF$; $R(0)= R^0$

$\frac{dF}{dt} = -pF + dFR$; $F(0) = F^0$

R: The population of the rabbit at time t;
F: The population of the Foxes at time t;
R^0: The initial size of the rabbit population;
F^0: The initial size of the fox inhabitants
a:Imitationlevel of rabbits
b:Death rate of rabbits;
C: Proportional to the number of rabbits thata fox can eat.
D: Amount of energy that a rabbit supplies to the intense fox;
p: death rate of foxes.[4].

The model $\boldsymbol{dR/dt = aR - bR^2 - cRF}$;

R'(t) , the growth rate of the rabbit population is stimulated in steps with the primary differential equation by three different terms.

It is positively stimulated by the current rabbit population size, as shown by the termaR, where a is a constant; non negative real number and aR is the birth rate of the rabbits.

It is negatively stimulated by the natural death rate of the rabbit, as shown by the term$-bR^2$; where b is a constant.non- negative real number and bR^2 is the natural death rate of the rabbits.

It is also negatively stimulated by the death rate of the rabbit due to consumption by foxes as shown by the term$-cRF$, where c is a constant non –negative real number and cRF is the death rate of the rabbits due to consumption by Foxes.

The model$\boldsymbol{\frac{dF}{dt} = -pF + dFR}$;

F `(t) is the growing rate of the Foxes population is stimulated according to the second differential equation by two different terms.

It is negatively stimulated by the current fox population size as shown by the term $-pF$,where p is a constant non- negative real number and F is the Fox population.

It is positively stimulated by the fox- rabbit contacts as shown by the term dFR, where dis a constant non-negative real number, F is the fox population and R is the rabbit population.

VI. COMPUTATIONOF EQUILIBRIUM POINTS:

Once the initial equations are understood, the next step is to find the equilibrium points. These are shown by the following computations[3].

Let $X = \frac{dR}{dt} = R(a - bR - cF)$ and

$$Y = \frac{dF}{dt} = F(-p + dR)$$

To compute equilibrium points, we have solve $\frac{dR}{dt} = 0$ and$\frac{dF}{dt} = 0$

Since $\frac{dR}{dt} = 0$ when $R = 0$ or $(a - bR - cF) = 0$

Solution: {R= $(a - cF/b$}.

$\frac{dF}{dt} = 0$When$F = 0$ or $-p + dR = 0$

Solution :{ R= p/d }.

Now we find all the combinations:

One of our equilibrium points is (0,0)

When F = 0; R= a/b.Therefor one of our equilibrium points is (a/b ,0)

For R= $\frac{a-cF}{b}$ $and\ R = \frac{p}{d}$;

$\frac{p}{d} = (a - cF)/b$

Solution $\{-\frac{p}{b} + ad/dc\}$

Thus one of our equilibrium points is

$(\frac{p}{d}, \frac{-pb + ad}{dc})$

Therefore the Equilibrium points are (0,0), (a/b, 0) and $(-pb + ad)/dc$.

VII. STABILITY OF THE EQUILIBRIUM POINTS:

Now to study the stability of the equilibrium points[5], we first need to find the Jacobin matrix which is

J (R, F) = $\begin{bmatrix} dX/dR & dX/dF \\ dY/dR & dY/dF \end{bmatrix}$

=$\begin{bmatrix} a-2bR-cF & -cR \\ dF & -p+dR \end{bmatrix}$

(i). Stability of (0, 0)

J (0,0)=$\begin{vmatrix} a-\lambda & 0 \\ 0 & -p-\lambda \end{vmatrix}$

The solutions are {λ=a}, {λ= - p}

Here the one Eigen value is negative and one is positive. Therefore the stability of (0,0) is semi-stable .

(ii). Stability of (a/b ,0)

J$(a/b, 0)$ = $\begin{bmatrix} -a-\lambda & -ca/b \\ 0 & -p+\frac{ad}{b}-\lambda \end{bmatrix}$

=($-a-\lambda$) $(-p+\frac{ad}{b}-\lambda)$

The solutions are { λ= $-a$}, {λ=$-p+\frac{ad}{b}$}. If λ = $(-p+\frac{ad}{b})$ < 0 ,the solutions are stable. If λ = $(-p+\frac{ad}{b})$ >0, the solutions are semi-stable.

(iii). Stability of ($\frac{-pb+ad}{dc}$)

J ($\frac{p}{d}$, $(-pb+\frac{ad}{dc})$ = $\begin{vmatrix} a-\frac{2bp}{d}-c(ad-pb)/cd-\lambda & -cp \\ ad-pb/c & -\lambda \end{vmatrix}$

= $\begin{vmatrix} -\frac{bp}{d}-\lambda & -cp/d \\ ad-pb/c & -\lambda \end{vmatrix}$

= λpb+λ^2d-p^2b+pad/d

The solutions are λ=$-pb+i\sqrt{-p2b2-4dpb+4pad2}$/2d and

λ=$-pb-i\sqrt{-p2b2-4dp2b+4ad2}$/2d

Since both Eigen values of the real parts are negative. Therefore the solutions are stable.

VIII. CONCLUSION:

This Lotka- Volterra Predator –Prey model is a rudimentary model of the complex ecology of this world.This model is an excellent tool to teach principles involved in ecology and to show some rather counter-initiative results. It also shows a special relationship between biology and mathematics.

To this end, it is natural to seek a mathematical formulation of this Predator –Prey problems and to use it to forecast the behavior of populations of various species at different times.

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