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Characteritistic Features of Fuzzy Matrix, Fuzzy Soft Matrix and Intuitionistic Fuzzy Soft Matrix

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Abstract: This paper discusses about some features and properties of fuzzy matrix, fuzzy soft matrix and Intuitionistic fuzzy soft matrix with examples.
Keyword: Fuzzy matrix, fuzzy soft matrix, Intuitionistic fuzzy soft matrix.

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I. INTRODUCTION :

Fuzzy matrix are now a very rich topic in modeling uncertain situations occurred in science, automata theory, logic of binary relations, medical diagonis etc.

Fuzzy matrices defined first time by Thomson in 1977 [1] and discussed about the convergence of the powers of a fuzzy matrix. The theories of fuzzy matrix were developed by Kim and Roush [2] as an extension of Boolean Matrices. In 1965, Zadeh [3] first introduced the concept of fuzzy set theory. In 1999, soft set theory was firstly introduced by Molodtsov [4] as a general mathematical tool for dealing with uncertain, fuzzy not clearly defined objects. In 2001, Maji, Biswas and Roy [5] studied the theory of soft sets initiated by Molodtsov [4] and developed several basic notions of soft set theory. In 2009, Ahmad and Kharal [6] developed the result of Maji [5]. In 2012, Cagman and Enginogu [7] defined fuzzy soft matrices and constructed decision making problem. In 2012, Rajarajeswari and Dhanalakshmi [8] introduced the application of similarity between two fuzzy soft set based on distance. In 2012 Neog, sut extended fuzzy soft matrix theory and its application.

In 2014, Dr.N.Sarala and Mrs.I.Jannathul Firthouse extended intuitionistic fuzzy soft matrix in obesity problems.

In this paper, we proposed the properties of fuzzy matrix, fuzzy soft matrix and intuitionistic fuzzy soft matrix.

II. PROPERTIES OF FUZZY MATRIX

Addition and Multiplication of Fuzzy Matrices Using reference Function.

2.1 Addition of fuzzy matrices:

Two fuzzy matrices are conformable for addition if the matrices are of same order. That is to say, when we wish to find addition of two matrices, the number of rows and columns of both the matrices should be same.

If A and B be two matrices of same order then their addition can be defined as follows

$$A + B = \{max\,(a_{ij}, b_{ij}), min\,(r_{ij}, r'_{ij})\}$$

where a_{ij} stands for the membership function of fuzzy matrix A for the ith row and jth column and r_{ij} is the corresponding reference function and b_{ij} stands for the membership function of the fuzzy matrix B for the ith row and jth column where r'_{ij} represents the corresponding reference function.

Again we can see that if we define the addition of two fuzzy matrices in the aforesaid manner and thereafter if we use the new definition of complementation of fuzzy matrices which is in accordance with the definition of complementation of fuzzy sets as introduced by Baruah [2,3,4] and later on used in the works of Dhar [5,6,7,8,9 & 10]. We would be able to arrive at the following result ;

$$(A + B)^c = A^c + B^c$$

Example 2.1

Let A and B be two fuzzy matrices of order 3.

$$A = \begin{bmatrix} 0.3 & 0.7 & 0.8 \\ 0.4 & 0.5 & 0.3 \\ 0.6 & 0.1 & 0.4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.8 & 0.5 & 0.2 \\ 0.5 & 1 & 0.8 \end{bmatrix}$$

be two fuzzy matrices of order 3.

Then $A + B = [C_{ij}]$

where C_{11} = {max(a_{11}, b_{11}), min($r_{11}, r_{11}^{'}$)}
= {max(0.3,1), min(0,0)} = (1,0)

C_{12} = {max(a_{12}, b_{12}), min($r_{12}, r_{12}^{'}$)}
= {max(0.7,0.2), min(0,0)} = (0.7,0)

C_{13} = {max(a_{13}, b_{13}), min($r_{13}, r_{13}^{'}$)}
= {max(0.8,0.3), min(0,0)} =(0.8,0)

Proceeding in the above manner, we get

$$A + B = \begin{bmatrix} (1,1) & (1,0.7) & (1,0.8) \\ (1,0.8) & (1,0.5) & (1,0.3) \\ (1,0.6) & (1,0) & (0.8,0) \end{bmatrix}$$

Again we have

$$A^c = \begin{bmatrix} (1,0.3) & (1,0.7) & (1,0.8) \\ (1,0.4) & (1,0.5) & (1,0.3) \\ (1,0.6) & (1,0.1) & (1,0.4) \end{bmatrix}$$

$$B^c = \begin{bmatrix} (1,1) & (1,0.2) & (1,0.3) \\ (1,0.8) & (1,0.5) & (1,0.2) \\ (1,0.5) & (1,1) & (1,0.8) \end{bmatrix}$$

be the two fuzzy complement matrices. Proceeding similarly we get the sum of these two matrices as

$$A^c + B^c = \begin{bmatrix} (1,1) & (1,0.7) & (1,0.8) \\ (1,0.8) & (1,0.5) & (1,0.3) \\ (1,0.6) & (1,1) & (1,0.8) \end{bmatrix}$$

Again proceeding in similarly way,

$$(A + B)^c = \begin{bmatrix} (1,1) & (1,0.7) & (1,0.8) \\ (1,0.8) & (1,0.5) & (1,0.3) \\ (1,0.6) & (1,1) & (1,0.8) \end{bmatrix}$$

Hence we have the following result $(A + B)^c = A^c + B^c$.

2.2 **Multiplication of fuzzy matrices :**

Now after finding addition of fuzzy matrices, we shall try to find the multiplication of two fuzzy matrices. The product of two fuzzy matrices under usual matrix multiplication is not a fuzzy matrix. Let A and B be a two fuzzy matrices. Then the product AB to be defined if the number of columns of the first fuzzy matrix A is equal to the number of rows of the second fuzzy matrix B.

In the process of finding multiplication of fuzzy matrices, if this condition is satisfied then the multiplication of two fuzzy matrices A and B, will be defined and can be represented in the following form:

$$AB = \{\max min\ (a_{ij}, b_{ij}), \min max\ (r_{ij}, r_{ij}^{'})\}$$

Example 2.2

$$A^c = \begin{bmatrix} (1,0.3) & (1,0.7) & (1,0.8) \\ (1,0.4) & (1,0.5) & (1,0.3) \\ (1,0.6) & (1,0.1) & (1,0.4) \end{bmatrix}$$

$$B^c = \begin{bmatrix} (1,1) & (1,0.2) & (1,0.3) & (1,0.6) \\ (1,0.8) & (1,0.5) & (1,0.2) & (1,0.6) \\ (1,0.5) & (1,1) & (1,0.8) & (1,0.7) \end{bmatrix}$$

Then the product would be defined as

$$A^c B^c = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix}$$

Let C_{11} = {max min(a_{11}, b_{11}),min, max($r_{11}, r_{11}^{'}$)}
= [max{min(1,1), min(1,1), min(1,1)},
min{max(0.3,1), min(0.7,0.8),min(0.8,0.5)}]
= {max(1,1,1), min(1,0.8,0.8)} = (1,0.8)

C_{12} = {max min(a_{12}, b_{12}),min, max($r_{12}, r_{12}^{'}$)}
= [max{min(1,1), min(1,1), min(1,1)},
min{max(0.3,0.2), max(0.7,0.5), max(0.8,1)}]
= {max(1,1,1), min(0.3,0.7,1)} = (1,0.3)

C_{13} = {max min(a_{13}, b_{13}),min, max$(r_{13}, r_{13}^{'})$}
= [max{min(1,1), min(1,1), min(1,1)},
min{max(0.3,0.3), max(0.7,0.2), max(0.8,0.8)}]
= {max(1,1,1), min(0.3,0.7,0.8)} = (1,0.3)

C_{14} = {max min(a_{14}, b_{14}),min, max$(r_{14}, r_{14}^{'})$}
= [max{min(1,1), min(1,1), min(1,1)},
min{max(0.3,0.6), max(0.7,0.9), max(0.8,0.7)}]
= {max(1,1,1), min(0.6,0.9,0.8)} = (1,0.6)

proceeding similarly we get the product of the two matrices as

$$A^c B^c = \begin{bmatrix} (1,0.8) & (1,0.3) & (1,0.3) & (1,0.6) \\ (1,0.5) & (1,0.4) & (1,0.4) & (1,0.6) \\ (1,0.5) & (1,0.5) & (1,0.2) & (1,0.6) \end{bmatrix}$$

Multiplication of matrices in the aforesaid manner would lead us to write some properties of fuzzy matrices about which we shall discuss in the following section. But before proceeding further, we would like to mention one thing that since we have defined complementation of a fuzzy matrix, it would be helpful if we try to establish the properties with the help of complementation of fuzzy matrices. In the following section, we have cited some numerical examples for the purpose of showing the properties of multiplication of fuzzy matrices.

2.3 Properties of Fuzzy Matrix Multiplication

In this section, we shall consider some of properties of multiplication of fuzzy matrices.

Property 1:

Multiplication of fuzzy matrices is associative, if conformability is assured that is A(BC) = (AB)C if A, B, C are $m \times n, n \times p, p \times q$ matrices respectively. The same result would hold if we consider the complementation fuzzy matrices in our manner. Here we would like to cite an example with the complementation of fuzzy matrices illustration for illustration purposes.

Example 2.3

$A = \begin{bmatrix} 0.1 & 0.3 \\ 0.5 & 0.7 \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0.2 \\ 0.7 & 0.8 \end{bmatrix}$ and $C = \begin{bmatrix} 0.1 & 1 \\ 0.9 & 0.6 \end{bmatrix}$ be three fuzzy matrices then their complement would be defined as

$$A^c = \begin{bmatrix} (1,0.1) & (1,0.3) \\ (1,0.5) & (1,0.7) \end{bmatrix}$$

$$B^c = \begin{bmatrix} (1,0.5) & (1,0.2) \\ (1,0.7) & (1,0.8) \end{bmatrix}$$

and

$$C^c = \begin{bmatrix} (1,0.1) & (1,1) \\ (1,0.9) & (1,0.6) \end{bmatrix}$$

Then, we have

$$A^c B^c = \begin{bmatrix} (1,0.5) & (1,0.2) \\ (1,0.5) & (1,0.5) \end{bmatrix}$$

and

$$B^c C^c = \begin{bmatrix} (1,0.5) & (1,0.6) \\ (1,0.7) & (1,0.8) \end{bmatrix}$$

Consequently, we get

$$A^c (B^c C^c) = \begin{bmatrix} (1,0.5) & (1,0.6) \\ (1,0.7) & (1,0.8) \end{bmatrix}$$

Consequently, we get

$$A^c (B^c C^c) = (A^c B^c) C^c$$

Property 2.4

Multiplication of fuzzy matrices is distributive with respect to addition of fuzzy matrices. That is, A(B+C) = AB+AC, where A, B, C are m×n, n×p, p×q matrices respectively. Here we shall show the following if the complementation of the matrices are considered.

$$A^c (B^c + C^c) = A^c B^c + A^c C^c$$

Property 2.5

Multiplication of fuzzy matrices is not always commutative. That is to say that whenever A^c B^c and $B^c A^c$ exist and are matrices of same type, it is not necessary that

$$A^c B^c = \begin{bmatrix} (1,0.5) & (1,0.2) \\ (1,0.5) & (1,0.5) \end{bmatrix}$$

and Similarly $B^c A^c = \begin{bmatrix} (1,0.5) & (1,0.2) \\ (1,0.5) & (1,0.5) \end{bmatrix}$

which shows that

$$A^c B^c \neq B^c A^c$$

III Properties of a fuzzy soft matrix

Proposition : 3.1

Let $\tilde{A} = [\alpha_{ij}^{\tilde{A}}] \in FSM_{m\times n}$. Then (i) $\left[[\tilde{A}]^0\right]^0 = [\tilde{A}]$, (ii) $\tilde{A} \,\tilde{\cup}\, \tilde{A} = \tilde{A}$, (iii) $\tilde{A} \,\tilde{\cap}\, \tilde{A} = \tilde{A}$, (iv) $\tilde{A} \,\tilde{\cup}\, [\tilde{0}] = \tilde{A}$, (v) $\tilde{A} \,\tilde{\cap}\, [\tilde{0}] = \tilde{0}$, (iv) $[\tilde{0}]^0 = [\tilde{1}]$.

Proof :

(i) $\left[[\tilde{A}]^0\right]^0 = [1-(1-\alpha_{ij}^{\tilde{A}})] = [\alpha_{ij}^{\tilde{A}}] = \tilde{A}$

(ii) $\tilde{A} \,\tilde{\cup}\, \tilde{A} = [max\{\alpha_{ij}^{\tilde{A}}, \alpha_{ij}^{\tilde{A}}\}] = [\alpha_{ij}^{\tilde{A}}] = \tilde{A}$

(iii) $\tilde{A} \,\tilde{\cap}\, \tilde{A} = [min\{\alpha_{ij}^{\tilde{A}}, 0\}] = [\alpha_{ij}^{\tilde{A}}] = \tilde{A}$

(iv) $\tilde{A} \,\tilde{\cap}\, [\tilde{0}] = [min\{\alpha_{ij}^{\tilde{A}}, 0\}] = [\tilde{0}]$

(v) $[\tilde{0}]^0 = [1-0] = [\tilde{1}]$

Proposition : 3.2

Let $\tilde{A} = [\alpha_{ij}^{\tilde{A}}], \tilde{B} = [\beta_{ij}^{\tilde{A}}] \in FSM_{m\times n}$. Then (i) $[\tilde{A} \,\tilde{\cup}\, \tilde{B}] = [\tilde{A}]^0 \,\tilde{\cap}\, [\tilde{B}]^0$, (ii) $[\tilde{A} \,\tilde{\cap}\, \tilde{B}] = [\tilde{A}]^0 \,\tilde{\cup}\, [\tilde{B}]^0$.

Proof:

(i) For all i and j

$[\tilde{A} \,\tilde{\cup}\, \tilde{B}]^0 = [max\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}]^0 = [1 - max\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}]$

$= [min\{1-\alpha_{ij}^{\tilde{A}}, 1-\beta_{ij}^{\tilde{B}}\}] = [\tilde{A}]^0 \,\tilde{\cap}\, [\tilde{B}]^0$

(ii) For all i and j

$[\tilde{A} \,\tilde{\cap}\, \tilde{B}]^0 = [min\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}]^0 = [1 - min\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}]$

$= [max\{1-\alpha_{ij}^{\tilde{A}}, 1-\beta_{ij}^{\tilde{B}}\}] = [\tilde{A}]^0 \,\tilde{\cup}\, [\tilde{B}]^0$

Proposition : 3.3

Let $\tilde{A} = [\alpha_{ij}^{\tilde{A}}], \tilde{B} = [\beta_{ij}^{\tilde{B}}], \tilde{C} = [\gamma_{ij}^{\tilde{c}}] \in FSM_{m\times n}$. Then (i) $\tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C}) = (\tilde{A} \,\tilde{\cup}\, \tilde{B}) \,\tilde{\cap}\, (\tilde{A} \,\tilde{\cup}\, \tilde{C})$, (ii) $\tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C}) = (\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cup}\, (\tilde{A} \,\tilde{\cap}\, \tilde{C})$.

Proof:

(i) $\tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C}) = [\alpha_{ij}^{\tilde{A}}] \,\tilde{\cup}\, [min\{\beta_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{c}}\}]$

$= [\max(\alpha_{ij}^{\tilde{A}}, min\{\beta_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{c}}\})]$

$(\tilde{A} \,\tilde{\cup}\, \tilde{B}) \,\tilde{\cap}\, (\tilde{A} \,\tilde{\cup}\, \tilde{C}) = \left[[max\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}] \,\tilde{\cap}\, [max\{\alpha_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{c}}\}]\right]$

$= [min(max\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}), max\{\alpha_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{c}}\}]$

$= [min(\alpha_{ij}^{\tilde{A}}, max\{\beta_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{c}}\})]$

$= [max(\alpha_{ij}^{\tilde{A}}, min\{\beta_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{c}}\})]$

Hence, $\tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C}) = (\tilde{A} \,\tilde{\cup}\, \tilde{B}) \,\tilde{\cap}\, (\tilde{A} \,\tilde{\cup}\, \tilde{C})$.

(ii) $\tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C}) = [\alpha_{ij}^{\tilde{A}}] \,\tilde{\cap}\, [max\{\beta_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{c}}\}]$

$= [\min(\alpha_{ij}^{\tilde{A}}, max\{\beta_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{c}}\})]$

$(\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cup}\, (\tilde{A} \,\tilde{\cap}\, \tilde{C}) = \left[[min\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}] \,\tilde{\cup}\, [min\{\alpha_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{c}}\}]\right]$

$= [max(min\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}), min\{\alpha_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{c}}\}]$

$= [max(\alpha_{ij}^{\tilde{A}}, min\{\beta_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{c}}\})]$

$= [min(\alpha_{ij}^{\tilde{A}}, max\{\beta_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{c}}\})]$

Hence, $\tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C}) = (\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cup}\, (\tilde{A} \,\tilde{\cap}\, \tilde{C})$.

Example : 3.3

(i) Let $\tilde{A} = [\alpha_{ij}^{\tilde{A}}], \tilde{B} = [\beta_{ij}^{\tilde{B}}], \tilde{C} = [\gamma_{ij}^{\tilde{c}}] \in FSM_{2\times 2}$,

Where $\tilde{A} = \begin{bmatrix} (0.5,0) & (0.4,0) \\ (0.1,0) & (0.3,0) \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} (0.3,0) & (0.6,0) \\ (0.8,0) & (0.1,0) \end{bmatrix}$ and

$\tilde{C} = \begin{bmatrix} (0.7,0) & (0.1,0) \\ (0.2,0) & (0.5,0) \end{bmatrix}$

$\tilde{B} \,\tilde{\cap}\, \tilde{C} = \begin{bmatrix} (0.3,0) & (0.1,0) \\ (0.2,0) & (0.1,0) \end{bmatrix}$; $\tilde{A} \,\tilde{\cup}\, \tilde{B} = \begin{bmatrix} (0.5,0) & (0.6,0) \\ (0.8,0) & (0.3,0) \end{bmatrix}$

$\tilde{A} \,\tilde{\cup}\, \tilde{C} = \begin{bmatrix} (0.7,0) & (0.4,0) \\ (0.2,0) & (0.5,0) \end{bmatrix}$

$\tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C}) = \begin{bmatrix} (0.5,0) & (0.4,0) \\ (0.2,0) & (0.3,0) \end{bmatrix}$

$(\tilde{A} \,\tilde{\cup}\, \tilde{B}) \,\tilde{\cap}\, (\tilde{A} \,\tilde{\cup}\, \tilde{C}) = \begin{bmatrix} (0.5,0) & (0.4,0) \\ (0.2,0) & (0.3,0) \end{bmatrix}$

Hence $\tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C}) = (\tilde{A} \,\tilde{\cup}\, \tilde{B}) \,\tilde{\cap}\, (\tilde{A} \,\tilde{\cup}\, \tilde{C})$.

(ii) The above said matrices may be considered for the

$\tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C}) = (\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cup}\, (\tilde{A} \,\tilde{\cap}\, \tilde{C})$

$(\tilde{B} \,\tilde{\cup}\, \tilde{C}) = \begin{bmatrix} (0.7,0) & (0.6,0) \\ (0.8,0) & (0.5,0) \end{bmatrix}; \tilde{A} \,\tilde{\cap}\, \tilde{B} = \begin{bmatrix} (0.3,0) & (0.4,0) \\ (0.1,0) & (0.1,0) \end{bmatrix}$

$\tilde{A} \,\tilde{\cap}\, \tilde{C} = \begin{bmatrix} (0.5,0) & (0.1,0) \\ (0.1,0) & (0.3,0) \end{bmatrix}$

$\tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C}) = \begin{bmatrix} (0.5,0) & (0.4,0) \\ (0.1,0) & (0.3,0) \end{bmatrix}$

$(\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cup}\, (\tilde{A} \,\tilde{\cap}\, \tilde{C}) = \begin{bmatrix} (0.5,0) & (0.4,0) \\ (0.1,0) & (0.3,0) \end{bmatrix}$

Hence $\tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C}) = (\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cup}\, (\tilde{A} \,\tilde{\cap}\, \tilde{C})$.

Proposition : .3.5

Let $\tilde{A} = [\alpha_{ij}^{\tilde{A}}], \tilde{B} = [\beta_{ij}^{\tilde{B}}], \tilde{C} = [\gamma_{ij}^{\tilde{c}}] \in FSM_{m\times n}$. Then (i) $\tilde{A} \,\tilde{\cup}\, \tilde{B} = \tilde{B} \,\tilde{\cup}\, \tilde{A}$, (ii) $(\tilde{A} \,\tilde{\cup}\, \tilde{B}) \,\tilde{\cup}\, \tilde{C} = \tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C})$, (iii) $\tilde{A} \,\tilde{\cup}\, [\tilde{1}] = [\tilde{1}]$.

Proof:

(i) For all i and j

$\tilde{A} \,\tilde{\cup}\, \tilde{B} = [max\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}] = [max\{\beta_{ij}^{\tilde{B}}, \alpha_{ij}^{\tilde{A}}\}] = \tilde{B} \,\tilde{\cup}\, \tilde{A}$

(ii) $(\tilde{A} \,\tilde{\cup}\, \tilde{B}) \,\tilde{\cup}\, \tilde{C} = [(max\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\})] \,\tilde{\cup}\, [\gamma_{ij}^{\tilde{c}}]$

$= [max(max\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}), \gamma_{ij}^{\tilde{c}}]$

$= \tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C})$

(iii) $\tilde{A} \,\tilde{\cup}\, [\tilde{1}] = [(max\{\alpha_{ij}^{\tilde{A}}, 1\})] = [\tilde{1}]$.

Proposition : 3.6

Let $\tilde{A} = [\alpha_{ij}^{\tilde{A}}], \tilde{B} = [\beta_{ij}^{\tilde{B}}], \tilde{C} = [\gamma_{ij}^{\tilde{c}}] \in FSM_{m\times n}$. Then (i) $\tilde{A} \,\tilde{\cap}\, \tilde{B} = \tilde{B} \,\tilde{\cap}\, \tilde{A}$, (ii) $(\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cap}\, \tilde{C} = \tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C})$, (iii) $\tilde{A} \,\tilde{\cap}\, [\tilde{1}] = [\tilde{A}]$.

Proof:

(i) For all i and j

$\tilde{A} \,\tilde{\cap}\, \tilde{B} = [min\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\}] = [min\{\beta_{ij}^{\tilde{B}}, \alpha_{ij}^{\tilde{A}}\}] = \tilde{B} \,\tilde{\cap}\, \tilde{A}$

(ii) $(\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cap}\, \tilde{C} = [(min\{\alpha_{ij}^{\tilde{A}}, \beta_{ij}^{\tilde{B}}\})] \,\tilde{\cap}\, [\gamma_{ij}^{\tilde{c}}]$

$= [min(\alpha_{ij}^{\tilde{A}}, min\{\beta_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{c}}\})]$

$= \tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C})$

(iii) $\tilde{A} \,\tilde{\cup}\, [\tilde{1}] = [(min\{\alpha_{ij}^{\tilde{A}}, 1\})] = [\tilde{A}]$.

Proposition : 3.7

Let $\tilde{A} = [\alpha_{ij}^{\tilde{A}}], \tilde{B} = [\beta_{ij}^{\tilde{B}}], \tilde{C} = [\gamma_{ij}^{\tilde{c}}] \in FSM_{m\times n}$,where $[\alpha_{ij}^{\tilde{A}}] = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})]$, $[\beta_{ij}^{\tilde{B}}] = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})]$, $[\gamma_{ij}^{\tilde{c}}] = [(x_{ij}^{\tilde{c}}, y_{ij}^{\tilde{c}})]$. Then the following results hold.

(i) $\tilde{A} + \tilde{B} = \tilde{B} + \tilde{A}$ (Commutative Law)

(ii) $(\tilde{A} + \tilde{B}) + \tilde{C} = \tilde{A} + (\tilde{B} + \tilde{C})$ (Associative Law)

Proof:

(i) Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})], \tilde{B} = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})]$

Now $\tilde{A} + \tilde{B} = [(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}), (min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\})]$

$= [(max\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{A}}\}), (min\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{A}}\})]$

$= \tilde{B} + \tilde{A}$.

(ii) Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})], \tilde{B} = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})], \tilde{C} = [(x_{ij}^{\tilde{c}}, y_{ij}^{\tilde{c}})]$

Now $(\tilde{A} + \tilde{B}) + \tilde{C} = [(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}), (min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\})] + [(x_{ij}^{\tilde{c}}, y_{ij}^{\tilde{c}})]$

$= [(max(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}), x_{ij}^{\tilde{c}}), (min(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}), y_{ij}^{\tilde{C}})]$

$= \left[\left(max\left(x_{ij}^{\tilde{A}}, (x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}})\right)\right), (min(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}), y_{ij}^{\tilde{C}})\right]$

$= \tilde{A} + (\tilde{B} + \tilde{C})$.

Example : 3.7

(i) Let $\tilde{A} = \begin{bmatrix} (0.5,0) & (0.2,0) \\ (0.4,0) & (0.1,0) \end{bmatrix}$; $\tilde{B} = \begin{bmatrix} (0.1,0) & (0.6,0) \\ (0.3,0) & (0.7,0) \end{bmatrix}$; and

$\tilde{C} = \begin{bmatrix} (0.8,0) & (0.1,0) \\ (0.5,0) & (0.4,0) \end{bmatrix}$

$\tilde{A} + \tilde{B} = \begin{bmatrix} (0.5,0) & (0.6,0) \\ (0.4,0) & (0.7,0) \end{bmatrix}$, $\tilde{B} + \tilde{A} = \begin{bmatrix} (0.5,0) & (0.6,0) \\ (0.4,0) & (0.7,0) \end{bmatrix}$

Hence $\tilde{A} + \tilde{B} = \tilde{B} + \tilde{A}$

(ii) The above said matrices may be considered for the $(\tilde{A} + \tilde{B}) + \tilde{C} = \tilde{A} + (\tilde{B} + \tilde{C})$

$\tilde{A} + \tilde{B} = \begin{bmatrix} (0.5,0) & (0.6,0) \\ (0.4,0) & (0.7,0) \end{bmatrix}$, $\tilde{B} + \tilde{A} = \begin{bmatrix} (0.5,0) & (0.6,0) \\ (0.4,0) & (0.7,0) \end{bmatrix}$

$$(\tilde{A} + \tilde{B}) + \tilde{C} = \begin{bmatrix} (0.5,0) & (0.6,0) \\ (0.4,0) & (0.7,0) \end{bmatrix} + \begin{bmatrix} (0.8,0) & (0.1,0) \\ (0.5,0) & (0.4,0) \end{bmatrix} = \begin{bmatrix} (0.8,0) & (0.6,0) \\ (0.5,0) & (0.7,0) \end{bmatrix}$$

$$\tilde{A} + (\tilde{B} + \tilde{C}) = \begin{bmatrix} (0.5,0) & (0.2,0) \\ (0.4,0) & (0.1,0) \end{bmatrix} + \begin{bmatrix} (0.8,0) & (0.6,0) \\ (0.5,0) & (0.7,0) \end{bmatrix} = \begin{bmatrix} (0.8,0) & (0.6,0) \\ (0.5,0) & (0.7,0) \end{bmatrix}$$

Hence $(\tilde{A} + \tilde{B}) + \tilde{C} = \tilde{A} + (\tilde{B} + \tilde{C})$.

Proposition : 3.8

The product of two fuzzy soft matrices $\tilde{A}$ and $\tilde{B}$ representing fuzzy soft sets over the same initial universe is defined only when the matrices are square matrices. Then $\tilde{A}.\tilde{B}$ and $\tilde{B}.\tilde{A}$ exist and matrices are of same type, it is not necessary that $\tilde{A}.\tilde{B} = \tilde{B}.\tilde{A}$. Let the following illustration may be taken into account for proof.

Example : 3.8

Let $\tilde{A} = \begin{bmatrix} (0.4,0) & (0.3,0) \\ (0.6,0) & (0.5,0) \end{bmatrix}$; $\tilde{B} = \begin{bmatrix} (0.2,0) & (0.1,0) \\ (0.1,0) & (0.7,0) \end{bmatrix}$

be two fuzzy soft square matrices representing two fuzzy soft sets defined over the same initial universe.

Then $\tilde{A}.\tilde{B} = \begin{bmatrix} (0.2,0) & (0.3,0) \\ (0.2,0) & (0.5,0) \end{bmatrix}$, $\tilde{B}.\tilde{A} = \begin{bmatrix} (0.2,0) & (0.2,0) \\ (0.6,0) & (0.5,0) \end{bmatrix}$

From the example, we obtained that $\tilde{A}.\tilde{B} \neq \tilde{B}.\tilde{A}$.

If the product $\tilde{A}.\tilde{B}$ is defined then $\tilde{B}.\tilde{A}$ may not be defined.

Example : 3.9

If the product $\tilde{A}.\tilde{B}$ is defined then $\tilde{B}.\tilde{A}$ may not be defined.

Let $\tilde{A} = \begin{bmatrix} (0.5,0) & (0.1,0) & (0.4,0) & (0.8,0) \\ (0.2,0) & (0.6,0) & (0.1,0) & (0.3,0) \\ (0.4,0) & (0.3,0) & (0.7,0) & (0.2,0) \end{bmatrix}$;

$\tilde{B} = \begin{bmatrix} (0.3,0) & (0.4,0) \\ (0.5,0) & (0.1,0) \\ (0.2,0) & (0.1,0) \\ (0.6,0) & (0.7,0) \end{bmatrix}$

We have $\tilde{A}.\tilde{B} = \begin{bmatrix} (06,0) & (0.7,0) \\ (0.5,0) & (0.3,0) \\ (0.3,0) & (0.4,0) \end{bmatrix}$

But here $\tilde{B}.\tilde{A}$ is not defined as number of columns in $\tilde{B} = 2$, whereas number of row in $\tilde{A} = 3$.

Proposition : 3.10

Let $\tilde{A} = [\alpha_{ij}^{\tilde{A}}], \tilde{B} = [\beta_{ij}^{\tilde{B}}] \in FSM_{m\times n}$, where $[\alpha_{ij}^{\tilde{A}}] = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})]$, $[\beta_{ij}^{\tilde{B}}] = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})]$. Then the following results hold.

(i) $(\tilde{A}^T)^T = \tilde{A}$

(ii) $(\tilde{A} + \tilde{B})^T = \tilde{A}^T + \tilde{B}^T$

Proof:

(i) Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})]$

Here $\tilde{A}^T = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})]^T = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})]$

$\tilde{A}^T = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right]^T = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right] = \tilde{A}$

Hence $\left(\tilde{A}^T\right)^T = \tilde{A}$

(ii) $\tilde{A} = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right], \tilde{B} = \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right], \tilde{C} = \left[\left(x_{ij}^{\tilde{c}}, y_{ij}^{\tilde{c}}\right)\right]$

Here $\tilde{A} + \tilde{B} = \left[\left(max\left(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\right), min\left(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\right)\right)\right]^T$

$= \left[\left(max\left(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\right), min\left(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\right)\right)\right]$

$= \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right) + \left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right]$

$= \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right]^T + \left[x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right]^T$

$= \tilde{A}^T + \tilde{B}^T$

Hence $\left(\tilde{A} + \tilde{B}\right)^T = \tilde{A}^T + \tilde{B}^T$.

Example : 3.10

Let $\tilde{A} = \begin{bmatrix} (0.2,0) & (0.5,0) & (0.3,0) \\ (0.4,0) & (0.7,0) & (0.1,0) \\ (0.6,0) & (0.1,0) & (0.5,0) \end{bmatrix}$ and $\tilde{B} = \begin{bmatrix} (0.1,0) & (0.3,0) & (0.4,0) \\ (0.2,0) & (0.1,0) & (0.7,0) \\ (0.5,0) & (0.6,0) & (0.3,0) \end{bmatrix}$

$\left(\tilde{A}^T\right) = \begin{bmatrix} (0.2,0) & (0.4,0) & (0.6,0) \\ (0.5,0) & (0.7,0) & (0.1,0) \\ (0.3,0) & (0.1,0) & (0.5,0) \end{bmatrix}$;

$\left(\tilde{A}^T\right)^T = \begin{bmatrix} (0.2,0) & (0.5,0) & (0.3,0) \\ (0.4,0) & (0.7,0) & (0.1,0) \\ (0.6,0) & (0.1,0) & (0.5,0) \end{bmatrix} = \tilde{A}$

Hence $\left(\tilde{A}^T\right)^T = \tilde{A}$.

(ii) The above said matrices may be considered for the $\left(\tilde{A} + \tilde{B}\right)^T = \tilde{A}^T + \tilde{B}^T$

$\tilde{B}^T = \begin{bmatrix} (0.1,0) & (0.2,0) & (0.5,0) \\ (0.3,0) & (0.1,0) & (0.6,0) \\ (0.4,0) & (0.7,0) & (0.3,0) \end{bmatrix}$;

$\tilde{A} + \tilde{B} = \begin{bmatrix} (0.2,0) & (0.5,0) & (0.4,0) \\ (0.4,0) & (0.7,0) & (0.7,0) \\ (0.6,0) & (0.6,0) & (0.5,0) \end{bmatrix}$;

$\left(\tilde{A} + \tilde{B}\right)^T = \begin{bmatrix} (0.2,0) & (0.4,0) & (0.6,0) \\ (0.5,0) & (0.7,0) & (0.6,0) \\ (0.4,0) & (0.7,0) & (0.5,0) \end{bmatrix}$

$\tilde{A}^T + \tilde{B}^T = \begin{bmatrix} (0.2,0) & (0.4,0) & (0.6,0) \\ (0.5,0) & (0.7,0) & (0.6,0) \\ (0.4,0) & (0.7,0) & (0.5,0) \end{bmatrix}$

Hence $\left(\tilde{A} + \tilde{B}\right)^T = \tilde{A}^T + \tilde{B}^T$.

IV Properties of Intuitionistic Fuzzy Soft Matrix

Proposition : 4.1

Let $\tilde{A} = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right]$ be fuzzy soft square matrices each of order n. Then $tr\left(\tilde{A}\right) = tr\left(\tilde{A}^T\right)$, where $\tilde{A}^T$ is the transpose of $\tilde{A}$.

Proof :

Let $\tilde{A} = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right]$, then $\tilde{A}^T = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right]$

$$tr\left(\tilde{A}^T\right) = \left(max\left(x_{ii}^{\tilde{A}}\right), min\left(x_{ii}^{\tilde{A}}\right)\right) = tr\left(\tilde{A}\right)$$

Hence $tr\left(\tilde{A}\right) = tr\left(\tilde{A}^T\right)$.

Example : 4.1

Let $\tilde{A} = \begin{bmatrix} (0.4,0) & (0.2,0) & (0.7,0) \\ (0.5,0) & (0.3,0) & (0.8,0) \\ (0.1,0) & (0.6,0) & (0.1,0) \end{bmatrix}$;

$$\tilde{A}^T = \begin{bmatrix} (0.4,0) & (0.5,0) & (0.1,0) \\ (0.2,0) & (0.3,0) & (0.6,0) \\ (0.7,0) & (0.8,0) & (0.1,0) \end{bmatrix}$$

$$tr(\tilde{A}) = (max\{0.4,0.3,0.1\}, min\{0,0,0\}) = (0.4,0)$$

$$tr(\tilde{A}^T) = (max\{0.4,0.3,0.1\}, min\{0,0,0\}) = (0.4,0)$$

Hence $tr(\tilde{A}) = tr(\tilde{A}^T)$.

Proposition : 4.2

Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})], \tilde{B} = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})] \in IFSM_{m\times n}$. Then De Morgan's type results are true which can be written as : (a) $(\tilde{A} \tilde{\cup} \tilde{B})^0 = \tilde{A}^0 \tilde{\cap} \tilde{B}^0$ (b) $(\tilde{A} \tilde{\cap} \tilde{B})^0 = \tilde{A}^0 \tilde{\cup} \tilde{B}^0$.

Proof :

(a) For all i and j, we have

$$\begin{aligned}(\tilde{A} \tilde{\cup} \tilde{B})^0 &= [(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}})] \tilde{\cup} [(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}})]^0 \\ &= [max(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}), min(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}})]^0 \\ &= [min(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}), max(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}})] \\ &= [(y_{ij}^{\tilde{A}}, x_{ij}^{\tilde{A}})] \tilde{\cap} [(y_{ij}^{\tilde{B}}, x_{ij}^{\tilde{B}})] \\ &= \tilde{A}^0 \tilde{\cap} \tilde{B}^0\end{aligned}$$

(b) For all i and j, we have

$$\begin{aligned}(\tilde{A} \tilde{\cap} \tilde{B})^0 &= [(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}})] \tilde{\cap} [(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}})]^0 \\ &= [min(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}), max(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}})]^0 \\ &= [max(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}), min(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}})] \\ &= [(y_{ij}^{\tilde{A}}, x_{ij}^{\tilde{A}})] \tilde{\cup} [(y_{ij}^{\tilde{B}}, x_{ij}^{\tilde{B}})] \\ &= \tilde{A}^0 \tilde{\cup} \tilde{B}^0.\end{aligned}$$

Example 4.2

Let $\tilde{A} = \begin{bmatrix} (0.1,0.2) & (0.2,0.3) & (0.7,0.1) \\ (0.3,0.2) & (0.5,0.1) & (0.6,0.2) \\ (0.3,0.3) & (0.4,0.2) & (0.3,0.1) \\ (0.4,0.1) & (0.5,0.3) & (0.1,0.8) \end{bmatrix}$ and $\tilde{B} = \begin{bmatrix} (0.3,0.2) & (0.5,0.2) & (0.2,0.6) \\ (0.4,0.1) & (0.3,0.5) & (0.3,0.4) \\ (0.5,0.1) & (0.1,0.3) & (0.8,0.1) \\ (0.2,0.3) & (0.1,0.7) & (0.4,0.1) \end{bmatrix}$

Then

$\tilde{A} \tilde{\cup} \tilde{B} = \begin{bmatrix} (0.3,0.2) & (0.5,0.2) & (0.7,0.1) \\ (0.4,0.1) & (0.5,0.1) & (0.6,0.2) \\ (0.5,0.1) & (0.4,0.2) & (0.8,0.1) \\ (0.4,0.1) & (0.5,0.3) & (0.4,0.1) \end{bmatrix}$; $\tilde{A} \tilde{\cap} \tilde{B} = \begin{bmatrix} (0.1,0.2) & (0.2,0.3) & (0.2,0.6) \\ (0.3,0.2) & (0.3,0.5) & (0.3,0.4) \\ (0.3,0.3) & (0.1,0.3) & (0.3,0.1) \\ (0.2,0.3) & (0.1,0.7) & (0.1,0.8) \end{bmatrix}$

$\tilde{A}^0 = \begin{bmatrix} (0.2,0.1) & (0.3,0.2) & (0.1,0.7) \\ (0.2,0.3) & (0.1,0.5) & (0.2,0.6) \\ (0.3,0.3) & (0.2,0.4) & (0.1,0.3) \\ (0.1,0.4) & (0.3,0.5) & (0.8,0.1) \end{bmatrix}$ and $\tilde{B}^0 = \begin{bmatrix} (0.2,0.3) & (0.2,0.5) & (0.6,0.2) \\ (0.1,0.4) & (0.5,0.3) & (0.4,0.3) \\ (0.1,0.5) & (0.3,0.1) & (0.1,0.8) \\ (0.3,0.2) & (0.7,0.1) & (0.1,0.4) \end{bmatrix}$

(a) $(\tilde{A} \tilde{\cup} \tilde{B})^0 = \begin{bmatrix} (0.2,0.3) & (0.2,0.5) & (0.1,0.7) \\ (0.1,0.4) & (0.1,0.5) & (0.2,0.6) \\ (0.1,0.5) & (0.2,0.4) & (0.1,0.8) \\ (0.1,0.4) & (0.3,0.5) & (0.1,0.4) \end{bmatrix}$;

$\tilde{A}^0 \tilde{\cap} \tilde{B}^0 = \begin{bmatrix} (0.2,0.3) & (0.2,0.5) & (0.1,0.7) \\ (0.1,0.4) & (0.1,0.5) & (0.2,0.6) \\ (0.1,0.5) & (0.2,0.4) & (0.1,0.8) \\ (0.1,0.4) & (0.3,0.5) & (0.1,0.4) \end{bmatrix}$

Hence $(\tilde{A} \tilde{\cup} \tilde{B})^0 = \tilde{A}^0 \tilde{\cap} \tilde{B}^0$.

(b) $(\tilde{A} \tilde{\cap} \tilde{B})^0 = \begin{bmatrix} (0.2,0.1) & (0.3,0.2) & (0.6,0.2) \\ (0.2,0.3) & (0.5,0.3) & (0.4,0.3) \\ (0.3,0.3) & (0.3,0.1) & (0.1,0.3) \\ (0.3,0.2) & (0.7,0.1) & (0.8,0.1) \end{bmatrix}$;

$$\tilde{A}^0 \tilde{\cup} \tilde{B}^0 = \begin{bmatrix} (0.2,0.1) & (0.3,0.3) & (0.6,0.3) \\ (0.2,0.4) & (0.5,0.4) & (0.4,0.4) \\ (0.3,0.3) & (0.3,0.1) & (0.1,0.3) \\ (0.3,0.2) & (0.7,0.1) & (0.8,0.1) \end{bmatrix}$$

Hence $\left(\tilde{A} \tilde{\cap} \tilde{B}\right)^0 = \tilde{A}^0 \tilde{\cup} \tilde{B}^0$.

Proposition : 4.3

Let $\tilde{A} = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right], \tilde{B} = \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right] \in IFSM_{m\times n}$. Then (i) $\left(\tilde{A}^0 \tilde{\cap} \tilde{B}^0\right)^0 = \tilde{A} \tilde{\cup} \tilde{B}$ (ii) $\left(\tilde{A}^0 \tilde{\cup} \tilde{B}^0\right)^0 = \tilde{A} \tilde{\cap} \tilde{B}$.

Proof :

(a) For all i and j, we have

$$\begin{aligned}\left(\tilde{A}^0 \tilde{\cap} \tilde{B}^0\right)^0 &= \left(\left[\left(y_{ij}^{\tilde{A}}, x_{ij}^{\tilde{A}}\right)\right] \tilde{\cap} \left[\left(y_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\right)\right]\right)^0 \\ &= \left[min\left(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\right), max\left(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\right)\right]^0 \\ &= \left[max\left(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\right), min\left(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\right)\right] \\ &= \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right] \tilde{\cup} \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right] \\ &= \tilde{A} \tilde{\cup} \tilde{B}\end{aligned}$$

(b) For all i and j, we have

$$\begin{aligned}\left(\tilde{A} \tilde{\cup} \tilde{B}\right)^0 &= \left[\left(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\right)\right] \tilde{\cup} \left[\left(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\right)\right]^0 \\ &= \left[max\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\right), min\left(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\right)\right]^0 \\ &= \left[min\left(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\right), max\left(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\right)\right] \\ &= \left[\left(y_{ij}^{\tilde{A}}, x_{ij}^{\tilde{A}}\right)\right] \tilde{\cap} \left[\left(y_{ij}^{\tilde{B}}, x_{ij}^{\tilde{B}}\right)\right] \\ &= \tilde{A} \tilde{\cap} \tilde{B}\end{aligned}$$

Example 4.3 :

Let $\tilde{A}, \tilde{B} \in IFSM_{m\times n}$ as in the Example 4.2

(i) $$\left(\tilde{A}^0 \tilde{\cap} \tilde{B}^0\right) = \begin{bmatrix} (0.2,0.3) & (0.2,0.5) & (0.1,0.7) \\ (0.1,0.4) & (0.1,0.5) & (0.2,0.6) \\ (0.1,0.5) & (0.2,0.4) & (0.1,0.8) \\ (0.1,0.4) & (0.3,0.5) & (0.1,0.4) \end{bmatrix}$$

$$\left(\tilde{A}^0 \tilde{\cap} \tilde{B}^0\right)^0 = \begin{bmatrix} (0.3,0.2) & (0.5,0.2) & (0.7,0.1) \\ (0.4,0.1) & (0.5,0.1) & (0.6,0.2) \\ (0.5,0.1) & (0.4,0.2) & (0.8,0.1) \\ (0.4,0.1) & (0.5,0.3) & (0.4,0.1) \end{bmatrix}$$

$$\tilde{A} \tilde{\cup} \tilde{B} = \begin{bmatrix} (0.3,0.2) & (0.5,0.2) & (0.7,0.1) \\ (0.4,0.1) & (0.5,0.1) & (0.6,0.2) \\ (0.5,0.1) & (0.4,0.2) & (0.8,0.1) \\ (0.4,0.1) & (0.5,0.3) & (0.4,0.1) \end{bmatrix}$$

Hence $\left(\tilde{A}^0 \tilde{\cap} \tilde{B}^0\right)^0 = \tilde{A} \tilde{\cup} \tilde{B}$.

(ii) $$\left(\tilde{A}^0 \tilde{\cup} \tilde{B}^0\right) = \begin{bmatrix} (0.2,0.1) & (0.3,0.2) & (0.6,0.2) \\ (0.2,0.3) & (0.5,0.3) & (0.4,0.3) \\ (0.3,0.3) & (0.3,0.1) & (0.1,0.3) \\ (0.3,0.2) & (0.7,0.1) & (0.8,0.1) \end{bmatrix}$$

$$\left(\tilde{A}^0 \tilde{\cup} \tilde{B}^0\right)^0 = \begin{bmatrix} (0.1,0.2) & (0.2,0.3) & (0.2,0.6) \\ (0.3,0.2) & (0.3,0.5) & (0.3,0.4) \\ (0.3,0.3) & (0.1,0.3) & (0.3,0.1) \\ (0.2,0.3) & (0.1,0.7) & (0.1,0.8) \end{bmatrix}$$

$$\tilde{A} \tilde{\cap} \tilde{B} = \begin{bmatrix} (0.1,0.2) & (0.2,0.3) & (0.2,0.6) \\ (0.3,0.2) & (0.3,0.5) & (0.3,0.4) \\ (0.3,0.3) & (0.1,0.3) & (0.3,0.1) \\ (0.2,0.3) & (0.1,0.7) & (0.1,0.8) \end{bmatrix}$$

Hence $\left(\tilde{A}^0 \tilde{\cup} \tilde{B}^0\right)^0 = \tilde{A} \tilde{\cap} \tilde{B}$.

Proposition : 4.4

Let $\tilde{A} = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right], \tilde{B} = \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right], \tilde{C} = \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right] \in IFSM_{m\times n}$. Then (i) $\tilde{A} \tilde{\cup} \tilde{B} = \tilde{B} \tilde{\cup} \tilde{A}$, (ii) $\tilde{A} \tilde{\cap} \tilde{B} = \tilde{B} \tilde{\cap} \tilde{A}$ (iii)$\left(\tilde{A} \tilde{\cup} \tilde{B}\right) \tilde{\cup} \tilde{C} = \tilde{A} \tilde{\cup} \left(\tilde{B} \tilde{\cup} \tilde{C}\right)$, (iv) $\left(\tilde{A} \tilde{\cap} \tilde{B}\right) \tilde{\cap} \tilde{C} = \tilde{A} \tilde{\cap} \left(\tilde{B} \tilde{\cap} \tilde{C}\right)$

Proof :

(i) For all i and j, we have

$$\tilde{A} \tilde{\cup} \tilde{B} = [max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}]$$
$$= [max\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{A}}\}, min\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{A}}\}]$$
$$= \tilde{B} \tilde{\cup} \tilde{A}$$

Validity : since $0 \le x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}} \le 1, 0 \le y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}} \le 1, max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\} \le 1, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\} \le 1.$

(ii) For all i and j, we have

$$\tilde{A} \tilde{\cap} \tilde{B} = [min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, max\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}]$$
$$= [min\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{A}}\}, max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{A}}\}]$$
$$= \tilde{B} \tilde{\cap} \tilde{A}$$

Validity : same as (i)

(iii) For all i and j, we have

$$(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cup} \tilde{C} = [(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\})] \tilde{\cup} [x_{ij}^{\tilde{c}}, y_{ij}^{\tilde{c}}]$$
$$= [(max\{(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}), x_{ij}^{\tilde{C}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}y_{ij}^{\tilde{c}},)]$$
$$= [(max\{x_{ij}^{\tilde{A}}, (x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}})\}, min\{y_{ij}^{\tilde{A}}, (y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{c}})\})]$$
$$= \tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cup} \tilde{C})$$

Validity : since $0 \le x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}} \le 1, 0 \le y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{C}} \le 1, max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\} \le 1, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{C}}\} \le 1.$

(iv) For all i and j, we have

$$(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cap} \tilde{C} = [(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, max\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\})] \tilde{\cap} [x_{ij}^{\tilde{c}}, y_{ij}^{\tilde{c}}]$$
$$= [(min\{(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}), x_{ij}^{\tilde{C}}\}, max\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}y_{ij}^{\tilde{c}})]$$
$$= [(min\{x_{ij}^{\tilde{A}}, (x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}})\}, max\{y_{ij}^{\tilde{A}}, (y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{c}})\})]$$
$$= \tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cap} \tilde{C})$$

Validity : same as (iii)

Example 4.4 :

Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})], \tilde{B} = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})], \tilde{C} = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})] \in IFSM_{2\times2}$, where

$$\tilde{A} = \begin{bmatrix} (0.4,0.2) & (0.3,0.4) \\ (0.5,0.3) & (0.1,0.7) \end{bmatrix}; \tilde{B} = \begin{bmatrix} (0.2,0.3) & (0.3,0.4) \\ (0.8,0.1) & (0.5,0.2) \end{bmatrix}; \tilde{C} = \begin{bmatrix} (0.3,0.1) & (0.5,0.1) \\ (0.6,0.2) & (0.4,0.3) \end{bmatrix}$$

(i) $\tilde{A} \tilde{\cup} \tilde{B} = \begin{bmatrix} (0.4,0.2) & (0.3,0.4) \\ (0.8,0.1) & (0.5,0.2) \end{bmatrix}; \tilde{B} \tilde{\cup} \tilde{A} = \begin{bmatrix} (0.4,0.2) & (0.3,0.4) \\ (0.8,0.1) & (0.5,0.2) \end{bmatrix}$

Hence $\tilde{A} \tilde{\cup} \tilde{B} = \tilde{B} \tilde{\cup} \tilde{A}$

(ii) $\tilde{A} \tilde{\cap} \tilde{B} = \begin{bmatrix} (0.2,0.3) & (0.3,0.4) \\ (0.5,0.3) & (0.1,0.7) \end{bmatrix}; \tilde{B} \tilde{\cap} \tilde{A} = \begin{bmatrix} (0.2,0.3) & (0.3,0.4) \\ (0.5,0.3) & (0.1,0.7) \end{bmatrix}$

Hence $\tilde{A} \tilde{\cap} \tilde{B} = \tilde{B} \tilde{\cap} \tilde{A}$

(iii) $(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cup} \tilde{C} = \begin{bmatrix} (0.4,0.2) & (0.3,0.4) \\ (0.8,0.1) & (0.5,0.2) \end{bmatrix} \tilde{\cup} \begin{bmatrix} (0.3,0.1) & (0.5,0.1) \\ (0.6,0.2) & (0.4,0.3) \end{bmatrix} = \begin{bmatrix} (0.4,0.1) & (0.5,0.1) \\ (0.8,0.1) & (0.5,0.2) \end{bmatrix}$

$\tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cup} \tilde{C}) = \begin{bmatrix} (0.4,0.2) & (0.3,0.4) \\ (0.5,0.3) & (0.1,0.7) \end{bmatrix} \tilde{\cup} \begin{bmatrix} (0.3,0.1) & (0.5,0.1) \\ (0.8,0.1) & (0.5,0.2) \end{bmatrix} = \begin{bmatrix} (0.4,0.1) & (0.5,0.1) \\ (0.8,0.1) & (0.5,0.2) \end{bmatrix}$

Hence $(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cup} \tilde{C} = \tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cup} \tilde{C})$

(iv) $(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cap} \tilde{C} = \begin{bmatrix} (0.2,0.3) & (0.3,0.4) \\ (0.5,0.3) & (0.1,0.7) \end{bmatrix} \tilde{\cap} \begin{bmatrix} (0.3,0.1) & (0.5,0.1) \\ (0.6,0.2) & (0.4,0.3) \end{bmatrix} = \begin{bmatrix} (0.2,0.3) & (0.3,0.4) \\ (0.5,0.3) & (0.1,0.7) \end{bmatrix}$

$\tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cap} \tilde{C}) = \begin{bmatrix} (0.4,0.2) & (0.3,0.4) \\ (0.5,0.3) & (0.1,0.7) \end{bmatrix} \tilde{\cap} \begin{bmatrix} (0.2,0.3) & (0.3,0.4) \\ (0.6,0.2) & (0.4,0.3) \end{bmatrix} = \begin{bmatrix} (0.2,0.3) & (0.3,0.4) \\ (0.5,0.3) & (0.1,0.7) \end{bmatrix}$

Hence $(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cap} \tilde{C} = \tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cap} \tilde{C})$.

Proposition : 4.5

Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})], \tilde{B} = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})], \tilde{C} = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})] \in IFSM_{m\times n}$.

Then (i) $\tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cup} \tilde{C}) = (\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cup} (\tilde{A} \tilde{\cap} \tilde{C})$, (ii) $(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cup} \tilde{C} = (\tilde{A} \tilde{\cap} \tilde{C}) \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C})$ (iii)$\tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C}) = (\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cap} (\tilde{A} \tilde{\cup} \tilde{C})$, (iv) $(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cap} \tilde{C} = (\tilde{A} \tilde{\cap} \tilde{C}) \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C})$

Proof :

Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})], \tilde{B} = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})], \tilde{C} = [(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}})] \in IFSM_{m\times n}$. Then

(i) $\tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cup} \tilde{C}) = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})] \tilde{\cap} [max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{C}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}]$

$$= [min(x_{ij}^{\tilde{A}}, max\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\}), min(y_{ij}^{\tilde{A}}, max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\})]$$

$$(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cup} (\tilde{A} \tilde{\cap} \tilde{C}) = [(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{C}}\})] \tilde{\cup} [(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{c}}\}, max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{c}}\})]$$

$= \left[max\left(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\}\right), min\left(max\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}, max\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}\right)\right]$

$= \left[\left(min\left(x_{ij}^{\tilde{A}}, max\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\}\right), max\left\{y_{ij}^{\tilde{A}}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}\right\}\right)\right]$

Hence $\tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cup} \tilde{C}) = (\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cup} (\tilde{A} \tilde{\cap} \tilde{C})$.

(ii) $(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cup} \tilde{C} = \left[\left(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, max\{y_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}\right)\right] \tilde{\cup} \left[\left(x_{ij}^{\tilde{C}}, x_{ij}^{\tilde{C}}\right)\right]$

$= \left[max\left(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, x_{ij}^{\tilde{C}}\right), min\left(max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\}, y_{ij}^{\tilde{C}}\right)\right]$

$(\tilde{A} \tilde{\cup} \tilde{C}) \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C}) = \left[\left(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{C}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}\right)\right] \tilde{\cap} \left[\left(max\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{c}}\}, min\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{c}}\}\right)\right]$

$= \left[min\left(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{C}}\}, max\{y_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\}\right), max\left(min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}\right)\right]$

$= \left[\left(max\left(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, x_{ij}^{\tilde{C}}\right), min\{max\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}, y_{ij}^{\tilde{C}}\}\right)\right]$

Hence $(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cup} \tilde{C} = (\tilde{A} \tilde{\cup} \tilde{C}) \tilde{\cap} (\tilde{B} \tilde{\cup} \tilde{C})$.

(iii) $\tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C}) = \left[\{x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\}\right] \tilde{\cup} \left[min\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\}, max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{C}}\}\right]$

$= \left[max\left(\mu_{ij}^{\tilde{A}}, min\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\}\right), min\left(y_{ij}^{\tilde{B}}, max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{C}}\}\right)\right]$

$(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cup} (\tilde{A} \tilde{\cup} \tilde{C}) = \left[\left(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}\right)\right] \tilde{\cap} \left[\left(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{c}}\}, min\{x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{c}}\}\right)\right]$

$= \left[min\left(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{C}}\}\right), max\left(min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}\right)\right]$

$= \left[\left(min\left(x_{ij}^{\tilde{A}}, max\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\}\right), max\left\{y_{ij}^{\tilde{A}}, max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{C}}\}\right\}\right)\right]$

$= \left[\left(max\left(x_{ij}^{\tilde{A}}, max\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\}\right), min\left\{y_{ij}^{\tilde{A}}, max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{C}}\}\right\}\right)\right]$

Hence $\tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C}) = (\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cup} (\tilde{A} \tilde{\cup} \tilde{C})$.

(iv) $(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cap} \tilde{C} = \left[\left(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}\right)\right] \tilde{\cap} \{x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}$

$= \left[min\left(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, x_{ij}^{\tilde{C}}\right), max\left(min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}, y_{ij}^{\tilde{C}}\right)\right]$

$(\tilde{A} \tilde{\cap} \tilde{C}) \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C}) = \left[\left(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{C}}\}, max\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}\right)\right] \tilde{\cup} \left[\left(min\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{c}}\}, max\{y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{c}}\}\right)\right]$

$= \left[max\left(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{C}}\}, min\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}}\}\right), min\left(max\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}, max\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{C}}\}\right)\right]$

$= \left[\left(max\left(min\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, x_{ij}^{\tilde{C}}\right), max\left(min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}, y_{ij}^{\tilde{C}}\right)\right)\right]$

$= \left[\left(min\left(max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, x_{ij}^{\tilde{C}}\right), max\{min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}, y_{ij}^{\tilde{C}}\}\right)\right]$

Hence $(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cap} \tilde{C} = (\tilde{A} \tilde{\cap} \tilde{C}) \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C})$.

Proposition : 4.6

Let $\tilde{A} = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right] \in IFSM_{m\times n}, \tilde{B} = \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right] \in IFSM_{m\times n}, \tilde{C} = \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right] \in IFSM_{m\times n}$. Then (i) $\tilde{A} + \tilde{B} = \tilde{B} + \tilde{A}$, (ii) $(\tilde{A} + \tilde{B}) + \tilde{C} = \tilde{A} + (\tilde{B} + \tilde{C})$, (iii) $(\tilde{A} + \tilde{B})^T = \tilde{A}^T + \tilde{B}^T$

Proof :

(i) $\tilde{A} + \tilde{B} = \left[max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}\right]$

$= \left[max\{x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{A}}\}, min\{y_{ij}^{\tilde{B}}, x_{ij}^{\tilde{A}}\}\right]$

$= \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right] + \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right]$

$= \tilde{B} + \tilde{A}$

(ii) $(\tilde{A} + \tilde{B}) + \tilde{C} = \left[max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}\right] + \left[x_{ij}^{\tilde{C}}, y_{ij}^{\tilde{C}}\right]$

$= \left[max\{(x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}), x_{ij}^{\tilde{C}}\}, min\{(y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}), y_{ij}^{\tilde{C}}\}\right]$

$= \left[max\{x_{ij}^{\tilde{A}}, (x_{ij}^{\tilde{B}}, x_{ij}^{\tilde{C}})\}, min\{y_{ij}^{\tilde{A}}, (y_{ij}^{\tilde{B}}, y_{ij}^{\tilde{C}})\}\right]$

$= \tilde{A} + (\tilde{B} + \tilde{C})$

Hence $(\tilde{A} + \tilde{B}) + \tilde{C} = \tilde{A} + (\tilde{B} + \tilde{C})$

(iii) $(\tilde{A} + \tilde{B})^T = \left[max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}\right]^T$

$= \left[max\{x_{ij}^{\tilde{A}}, x_{ij}^{\tilde{B}}\}, min\{y_{ij}^{\tilde{A}}, y_{ij}^{\tilde{B}}\}\right]$

$= \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right) + \left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right]$

$= \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right]^T + \left[x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right]^T$

$= \tilde{A}^T + \tilde{B}^T$

Hence $(\tilde{A} + \tilde{B})^T = \tilde{A}^T + \tilde{B}^T$

Example 4.6 :

Let $\tilde{A} = \left[\left(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}}\right)\right], \tilde{B} = \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right], \tilde{C} = \left[\left(x_{ij}^{\tilde{B}}, y_{ij}^{\tilde{B}}\right)\right] \in IFSM_{2\times 2}$, where

$$\tilde{A} = \begin{bmatrix} (0.5,0.2) & (0.3,0.1) \\ (0.4,0.3) & (0.7,0.2) \end{bmatrix}; \tilde{B} = \begin{bmatrix} (0.1,0.6) & (0.5,0.3) \\ (0.2,0.4) & (0.3,0.4) \end{bmatrix}; \tilde{C} = \begin{bmatrix} (0.3,0.5) & (0.4,0.4) \\ (0.6,0.2) & (0.1,0.5) \end{bmatrix}$$

(i) $\tilde{A}+\tilde{B}=\begin{bmatrix}(0.5,0.2) & (0.3,0.1)\\(0.4,0.3) & (0.7,0.2)\end{bmatrix}_{2\times 2}+\begin{bmatrix}(0.1,0.6) & (0.5,0.3)\\(0.2,0.4) & (0.3,0.4)\end{bmatrix}_{2\times 2}$

$=\begin{bmatrix}(0.5,0.2) & (0.5,0.1)\\(0.4,0.3) & (0.7,0.2)\end{bmatrix}_{2\times 2}=\tilde{B}+\tilde{A}$

(ii) $(\tilde{A}+\tilde{B})+\tilde{C}=\begin{bmatrix}(0.5,0.2) & (0.5,0.1)\\(0.4,0.3) & (0.7,0.2)\end{bmatrix}_{2\times 2}+\begin{bmatrix}(0.3,0.5) & (0.4,0.4)\\(0.6,0.2) & (0.1,0.5)\end{bmatrix}_{2\times 2}=$
$\begin{bmatrix}(0.5,0.2) & (0.5,0.1)\\(0.6,0.2) & (0.7,0.2)\end{bmatrix}_{2\times 2}$

Hence $(\tilde{A}+\tilde{B})+\tilde{C}=\tilde{A}+(\tilde{B}+\tilde{C})$

(iii) $(\tilde{A}+\tilde{B})^T=\begin{bmatrix}(0.5,0.2) & (0.4,0.3)\\(0.5,0.1) & (0.7,0.2)\end{bmatrix}_{2\times 2}$

$\tilde{A}^T+\tilde{B}^T=\begin{bmatrix}(0.5,0.2) & (0.4,0.3)\\(0.3,0.1) & (0.7,0.2)\end{bmatrix}_{2\times 2}+\begin{bmatrix}(0.1,0.6) & (0.2,0.4)\\(0.5,0.3) & (0.3,0.4)\end{bmatrix}_{2\times 2}$

$=\begin{bmatrix}(0.5,0.2) & (0.4,0.3)\\(0.5,0.1) & (0.7,0.2)\end{bmatrix}_{2\times 2}$

Hence $(\tilde{A}+\tilde{B})^T=\tilde{A}^T+\tilde{B}^T$

Proposition : 4.7

Let $\tilde{A}=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]\in IFSM_{m\times n}$. Then (i) $(\tilde{A}^T)^T=\tilde{A}$, (ii) $\tilde{A}\,\tilde{\cup}\,\tilde{A}^T=\tilde{A}^T\,\tilde{\cup}\,\tilde{A}$, (iii) $\tilde{A}\,\tilde{\cap}\,\tilde{A}^T=\tilde{A}^T\,\tilde{\cap}\,\tilde{A}$.

Proof :

(i) Let $\tilde{A}=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]$

Here $\tilde{A}^T=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]^T=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]$

$(\tilde{A}^T)^T=\left([(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]^T\right)^T=(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})=\tilde{A}$

(ii) Let $\tilde{A}=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]$ and $\tilde{A}^T=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]$

$\tilde{A}\,\tilde{\cup}\,\tilde{A}^T=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]\,\tilde{\cup}\,(x_{ji}^{\tilde{A}},y_{ji}^{\tilde{A}})$

$=[(max\{x_{ij}^{\tilde{A}},x_{ji}^{\tilde{A}}\},min\{y_{ij}^{\tilde{A}},y_{ji}^{\tilde{A}}\}]$

$=[(max\{x_{ji}^{\tilde{A}},x_{ij}^{\tilde{A}},\},min\{y_{ji}^{\tilde{A}},y_{ij}^{\tilde{A}}\}]$

$=\tilde{A}^T\,\tilde{\cup}\,\tilde{A}$

Hence $\tilde{A}\,\tilde{\cup}\,\tilde{A}^T=\tilde{A}^T\,\tilde{\cup}\,\tilde{A}$

(iii) Let $\tilde{A}=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]$ and $\tilde{A}^T=[(x_{ji}^{\tilde{A}},y_{ji}^{\tilde{A}})]$

$\tilde{A}\,\tilde{\cap}\,\tilde{A}^T=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]\,\tilde{\cap}\,[(x_{ji}^{\tilde{A}},y_{ji}^{\tilde{A}})]$

$=[min\{(x_{ij}^{\tilde{A}},x_{ji}^{\tilde{A}})\},max\{x_{ij}^{\tilde{A}},y_{ji}^{\tilde{A}}\}]$

$=[min\{(x_{ij}^{\tilde{A}},x_{ji}^{\tilde{A}})\},max\{y_{ji}^{\tilde{A}},y_{ij}^{\tilde{A}}\}]$

$=\tilde{A}^T\,\tilde{\cap}\,\tilde{A}$

Hence $\tilde{A}\,\tilde{\cap}\,\tilde{A}^T=\tilde{A}^T\,\tilde{\cap}\,\tilde{A}$.

Example 4.7:

Let $\tilde{A}=\begin{bmatrix}(0.1,0.7) & (0.3,0.1)\\(0.5,0.2) & (0.1,0.5)\end{bmatrix},\in IFSM_{2\times 2},\tilde{A}^T=\begin{bmatrix}(0.1,0.7) & (0.5,0.2)\\(0.3,0.1) & (0.1,0.5)\end{bmatrix}$

(i) $(\tilde{A}^T)^T=\begin{bmatrix}(0.1,0.7) & (0.3,0.1)\\(0.5,0.2) & (0.1,0.5)\end{bmatrix}=\tilde{A}$

Hence $(\tilde{A}^T)^T=\tilde{A}$

(ii) $\tilde{A}\,\tilde{\cup}\,\tilde{A}^T=\begin{bmatrix}(0.1,0.7) & (0.5,0.1)\\(0.5,0.1) & (0.1,0.5)\end{bmatrix}=\tilde{A}^T\,\tilde{\cup}\,\tilde{A}$

Hence $\tilde{A}\,\tilde{\cup}\,\tilde{A}^T=\tilde{A}^T\,\tilde{\cup}\,\tilde{A}$

(iii) $\tilde{A}\,\tilde{\cap}\,\tilde{A}^T=\begin{bmatrix}(0.1,0.7) & (0.3,0.2)\\(0.3,0.2) & (0.1,0.5)\end{bmatrix}=\tilde{A}^T\,\tilde{\cap}\,\tilde{A}$

Hence $\tilde{A}\,\tilde{\cap}\,\tilde{A}^T=\tilde{A}^T\,\tilde{\cap}\,\tilde{A}$

Proposition : 4.8

Let $\tilde{A}=[(x_{ij}^{\tilde{A}},y_{ij}^{\tilde{A}})]\in IFSM_{m\times n}$, if k is a scalar such that $0\le k\le 1$, then (i) $(k\tilde{A})^T=k\tilde{A}^T$, (ii) $(k\tilde{A}^0)^T=k\tilde{A}^{0^T}$, (iii) $tr(k\tilde{A})=ktr\tilde{A}$, (iv) $tr(k\tilde{A}^0)=ktr\tilde{A}^0$

Proof :

(i) Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})] \in IFSM_{m\times n}$

We have $k\tilde{A} = [(kx_{ij}^{\tilde{A}}, ky_{ij}^{\tilde{A}})]$

Therefore $(k\tilde{A})^T = [(kx_{ij}^{\tilde{A}}, ky_{ij}^{\tilde{A}})]^T = k[(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})] = k\tilde{A}^T$

(ii) Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})] \in IFSM_{m\times m}$

We have $k\tilde{A}^0 = [(ky_{ij}^{\tilde{A}}, kx_{ij}^{\tilde{A}})]$

$(k\tilde{A}^0)^T = [(ky_{ij}^{\tilde{A}}, kx_{ij}^{\tilde{A}})] = [(ky_{ji}^{\tilde{A}}, kx_{ji}^{\tilde{A}})] = k[(y_{ij}^{\tilde{A}}, x_{ij}^{\tilde{A}})]^T = k(\tilde{A}^0)^T$

(iii) Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})] \in IFSM_{m\times m}$

We have $k\tilde{A} = [(kx_{ij}^{\tilde{A}}, ky_{ij}^{\tilde{A}})]$

$\therefore tr(k\tilde{A}) = \sum_{i=1}^{m}(kx_{ii}^{\tilde{A}} - ky_{ii}^{\tilde{A}}) = k\sum_{i=1}^{m}(x_{ii}^{\tilde{A}} - y_{ii}^{\tilde{A}}) = ktr\tilde{A}.$

(iv) Let $\tilde{A} = [(x_{ij}^{\tilde{A}}, y_{ij}^{\tilde{A}})] \in IFSM_{m\times m}$

We have $k\tilde{A}^0 = [(ky_{ij}^{\tilde{A}}, kx_{ij}^{\tilde{A}})]$

$\therefore tr(k\tilde{A}^0) = \sum_{i=1}^{m}(ky_{ii}^{\tilde{A}} - kx_{ii}^{\tilde{A}}) = k\sum_{i=1}^{m}(y_{ii}^{\tilde{A}} - x_{ii}^{\tilde{A}}) = ktr(\tilde{A}^0)$

III. CONCLUSION

In this paper, we have extended fuzzy matrix, fuzzy soft matrix and intuitionistic fuzzy matrix by employing different kinds of its properties with example.

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