

## Melting Effect of MHD Micropolar Fluid Flow over a Radiative Vertical Surface towards Stagnation Point

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**Abstract:** Steady laminar flow of a micropolar fluid towards a stagnation point on a vertical melting surface is investigated. The external velocity is normal to the wall, and the wall temperature is assumed to vary linearly with distance from the stagnation point. The transformed nonlinear ordinary differential equations describing the flow are solved numerically by Runge-Kutta fourth order along with shooting technique. The velocity, angular velocity and temperature profiles of fluid flow for different values of the governing parameters, is presented. The effects of the melting and material parameters with radiation effect on the flow and heat transfer characteristics are examined thoroughly. The results show that both the material and melting parameters reduce the heat transfer rate on the fluid–solid interface.

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### I. INTRODUCTION

The flow and heat transfer behavior of certain fluids such as polymeric fluids, fluids with certain additives, colloidal fluids, paints, lubricating oils, liquid crystals, animal blood and real fluids with suspensions, cannot be explained on the basis of the Newtonian and non-Newtonian fluid flow theory. In these fluids there are several constitutive equations, which do not obey the Newtonian laws. To get over such a difficulty, Eringen [1] originated the theory of microfluids, which deals with a class of fluids, demonstrate certain microscopic effects arising from the local structure and micro-motions of the fluid elements, these fluids can endorse stress moment and are regulated by the spin inertia. Later, Eringen [2] produced a subclass of these fluids, known as micropolar fluids, where the microrotation effects and microrotation inertia exist but do not support stretch. They can support couple stress and body couples only. These micro structural fluids include polymers, suspensions, rheological materials, etc., for which the micropolar theory is an excellent model. An excellent review of micropolar fluids and their applications were provided by Ariman et al. [3].

The analysis of stagnation point flow towards a vertical plate has encountered a great attention of research interest due to its large applications in industries and practical applications. Some of the applications are cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, solar central receivers exposed to wind currents, and many hydrodynamic processes. Chamka [4] for instance, has studied the mixed convection flow near the stagnation point of a vertical semi-infinite permeable surface in the presence of a magnetic field. Ramachandran et al. [5] has investigated the nonmagnetic effect for impermeable surface on both arbitrary wall temperature and arbitrary surface heat flux variations. In which they found that a reverse flow developed in the buoyancy opposing flow region, and dual solutions exist for a certain range of the buoyancy parameter. This problem was then extended by Devi et al. [6] to the unsteady case, where they analyzed the similar results as in Ramachandran et al. [5]. It is worth mentioning that the stagnation-point flows have also been studied in many flow situations, likes in the papers by Chiam [7] has discussed stagnation point flow towards a stretching plate. They extended their work as heat transfer with variables conductivity in a stagnation point flow toward a stretching sheet by Chiam et al. [8]. Bhattacharya et al [9] investigated dual solutions in boundary layer stagnation-point flow and mass transfer with chemical reaction past a stretching/shrinking sheet and they also presented dual solutions in unsteady stagnation-point flow over a shrinking sheet [10]. Bhattacharyya and Layek [11] obtained effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. Bhattacharyya et al. [12] investigate stagnation-point flow and heat transfer over an exponentially shrinking sheet, Bhattacharyya et al. [13] presented slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet. Ishak et al. [14] investigated MHD mixed convection flow near the stagnation point on a vertical permeable surface and Wang [15] analyzed stagnation slip flow and heat transfer on a moving plate.

The study of magneto-hydrodynamic flow for electrically conducting fluid past heated surface has attracted the interest of many researches in view of its important applications in many engineering problems such as plasma studies, petroleum industries MHD power generations, the boundary layer control in

aerodynamics, cooling of nuclear reactors, and crystal growth. Until recently this study has been largely concerned with flow and heat transfer characteristics in various physical situations. Watanabe et al. [16] discussed the heat transfer in the thermal boundary layer of magneto-hydrodynamic flow over a flat plate. Rahman et al. [17] investigated the magnetohydrodynamic convective flow of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation or absorption. Alam et al. [18] presented the effects of thermophoresis and chemical reaction on unsteady hydromagnetic free convection and mass transfer flow past an impulsively started infinite inclined porous plate in the presence of heat generation/absorption. Alam et al. [19] analyzed MHD free convective heat and mass transfer flow past an inclined surface with heat generation. Alan1 et al. [20] discussed the effects of variable suction and thermophoresis on steady MHD combined free forced convective heat and mass transfer flow along a semi-infinite permeable inclined plate in the presence of thermal radiation. Molla et al. [21] obtained the natural convection flow along a heated wavy surface with a distributed heat source as given in Vajravelu et al. [22]. Moharmmadein and Gorla [23] investigated heat transfer in a micropolar fluid over a stretching sheet with viscous dissipation and internal heat generation.

Many processes in engineering areas occur at high temperature, and knowledge on radiation heat transfer becomes very important for design of reliable equipment, missiles, nuclear plants, gas turbines and various propulsion devices or aircraft, satellites and space vehicles. Sharma et.al [24-25] has presented the radiation effect with simultaneous thermal and mass diffusion in MHD mixed convection flow from a vertical surface. Perdikis and Repatis [26] investigated the heat transfer of a micropolar fluid in the presence of radiation. Elbashbeshby and Bazid [27] have obtained the radiation effects on the mixed convection flow of micropolar fluid. Moreover, when the radiative heat transfer takes place, the fluid involved can be electrically conducted in the sense that it is ionized owing to high operating temperature.

Phase change heat transfer of the type modeled in the present investigation finds important applications in melting of permafrost, magma solidification, and preparation of semiconductor materials. The similarity between the melting problems and diffusional mass transfer or transpiration cooling problems was first noticed by Yen and Tien [28]. Epstein and Cho [29] presented laminar film condensation on a vertical melting surface for 1D and 2D systems based on the Nusselt method to examine the melting rate. The problem of steady laminar boundary-layer flow and heat transfer from a warm laminar liquid flow to a melting surface moving parallel to a constant free stream was studied by Ishak et al. [30]. The dual solutions exist when a solid surface and free stream move in the opposite directions. Yacob, Ishak, and Pop [31] obtained the effect of melting heat transfer on stagnation-point flow of a micropolar fluid towards a horizontal linearly stretching/shrinking sheet. Recently transient mixed convective heat transfer with melting effect from the vertical plate in a liquid saturated porous medium was studied by Cheng et al. [32]. Melting effect on steady laminar flow of a micropolar fluid over a stagnation point on a vertical surface was investigated by Rakesh et al. [33] they obtained the material and melting parameters reduce the heat transfer rate on the fluid–solid interface. More recently melting heat transfer in boundary layer stagnation point flow of MHD micropolar fluid towards a stretching/ shrinking surface analyzed by Khilap et al. [34]. Melting heat transfer effects on stagnation point flow of micropolar fluid with variable dynamic viscosity and thermal conductivity at constant vortex viscosity [35] investigated by Adegbe et al.

The aim of the present chapter is to analyzed radiation effect on steady laminar flow of a micropolar fluid towards a stagnation point on a vertical melting surface of a magnetohydrodynamic (MHD) micropolar fluid. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically using the Runge-Kutta method with shooting technique. The effects of various governing parameters on the fluid velocity, temperature and angular velocity are shown in figures and analyzed in detail.

## **II. MATHEMATICAL FORMULATION OF PROBLEM**

A steady, two-dimensional flow of an incompressible, radiating, electrically conducting micropolar fluid near the stagnation point on a vertical plate with prescribed surface heat flux is considered as shown in figure 1. It is assumed that the velocity of the flow external to the boundary layer  $U(x)$  and the surface heat flux  $q_w(x)$  of the plate are proportional to the distance from the stagnation point i.e.  $U(x) = ax$  is velocity of the flow external to the boundary layer and heat flux  $q_w(x) = bx$  is proportional to the distance  $x$  from the stagnation point, where  $a$  and  $b$  are constants.  $T_\infty > T_w$ ,  $T_w$  is temperature of melting surface at wall of surface and  $T_\infty$  the temperature in the free stream. Under all these assumption along with the Boussinesq and boundary layer approximations, the system of equations that models the flow is given by

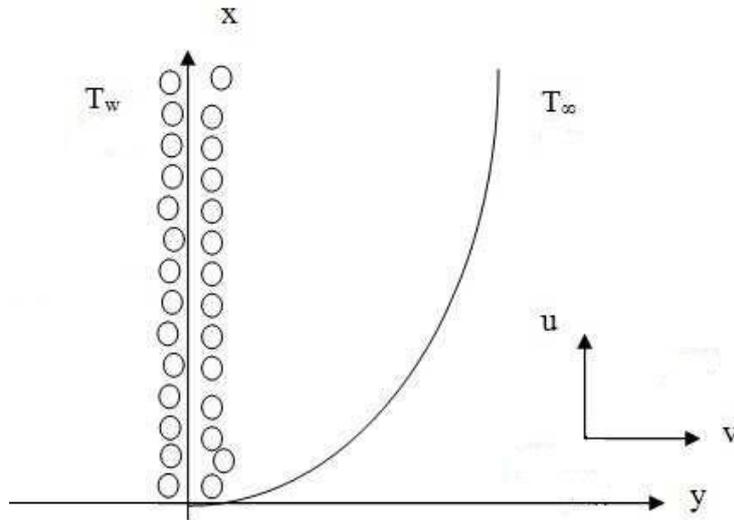


Figure 1 Flow Geometry

Equation of Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots (1)$$

Equation of Linear Momentum

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\partial U}{\partial x} + \frac{\mu + \kappa}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial \omega}{\partial y} - \frac{\sigma B_0^2}{\rho} (U - u) + g\beta(T - T_\infty) \quad \dots (2)$$

Equation of Angular Momentum

$$\rho j \left( u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} \right) = \gamma \frac{\partial^2 \omega}{\partial y^2} - \kappa \left( 2\omega + \frac{\partial u}{\partial y} \right) \quad \dots (3)$$

Equation of Energy

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{a}{k} \frac{\partial q_r}{\partial y} \quad \dots (4)$$

The boundary condition for velocity, angular velocity and temperature profiles are

$$y = 0; \quad T = T_w, \quad u = 0, \quad \omega = -\frac{1}{2} \frac{\partial u}{\partial y}, \quad \dots (5)$$

$$y \rightarrow \infty; \quad T = T_\infty, \quad u = U(x), \quad \omega = 0 \quad \dots (6)$$

And

$$k \left( \frac{\partial T}{\partial y} \right)_{y=0} = \rho [l + c_s(T_m - T_0)] v(x, 0) \quad \dots (7)$$

As previous published literature we consider that  $\gamma = \left( \mu + \frac{\kappa}{2} \right) = \mu \left( 1 + \frac{K}{2} \right) j$ , where  $j = \frac{\nu}{a}$  is the reference length and  $K = \frac{\kappa}{\mu}$  is the micropolar or material parameter. We note that  $m$  is a constant such  $0 \leq m \leq 1$  the case when  $m = 0$  shows  $\omega = 0$  at the surface it indicate flow of concentration of microelement closed to the wall surface unable to rotate. This called strong concentration which indicates that no microrotation near the wall. In case  $m = 0.5$ , it indicates that the vanishing of anti-symmetric part of the stress tensor and denote weak concentration and case  $m = 1$  is used for the modeling of turbulent boundary layer flows. Equation (6) proves that the heat conducted to the melting surface is equal to the heat of melting plus the sensitive heat requires raising the temperature of the solid surface  $T_0$  to its melting temperature  $T_w$ .

Here  $u$  and  $v$  are components of velocity along the  $x$  and  $y$  axis respectively. Further,  $\mu$  is dynamic viscosity,  $\nu = \frac{\mu}{\rho}$  kinematic viscosity of fluid,  $\kappa$  is vortex viscosity,  $\sigma$  is electrical conductivity of the fluid,  $\rho$  is fluid density,  $T$  is fluid temperature,  $j$  is micro inertia density,  $\omega$  is microrotation component,  $\gamma$  is spin gradient viscosity,  $\alpha$  is thermal diffusivity.

Using Roseland's approximation for radiation, we obtain the radiative heat flux  $q_r$  modeled as

$$q_r = -\left(\frac{4\sigma^*}{3k_1}\right) \frac{\partial T^4}{\partial y} \quad \dots (8)$$

Where  $\sigma^*$  the Stefan-Boltzmann constant is,  $k_1$  is the absorption coefficient. We assume that temperature variation within the flow is small such that  $T^4$  may be expand as linear combination of the temperature, we expand  $T^4$  in a Taylor's series about to  $T_\infty^4$  as follows:

$$T^4 = T_\infty^4 + 4T_\infty^3 (T - T_\infty) + 6T_\infty^2 (T - T_\infty)^2 + \dots \dots \dots \dots \dots \quad \dots (9)$$

Expanding  $T^4$  about  $T_\infty$  and neglecting higher order terms we get

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4,$$

Now Eq. (8) Differentiating with respect to  $y$  its reduces to

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty \sigma^*}{3k_1} \frac{\partial^2 T}{\partial y^2}$$

Then equation (4) becomes

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\frac{\alpha}{\rho c_p} + \frac{16\alpha\sigma^*T_\infty^3}{3kk_1\rho c_p}\right) \frac{\partial^2 T}{\partial y^2} \quad \dots (10)$$

The equation of continuity (1) is fulfilled by introducing the stream function  $\psi$  such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad \dots (11)$$

By introducing the following similarity transformation:

$$\eta = \left(\frac{\alpha}{\nu}\right)^{1/2} y, \quad f(\eta) = \frac{\psi}{(\alpha\nu)^{1/2} x}, \quad \theta(\eta) = \frac{T-T_w}{T_\infty-T_w}, \quad h(\eta) = \frac{\omega}{\alpha(\alpha\nu)^{1/2} x} \quad \dots (12)$$

$$(1 + K)f'''' + ff'' + 1 - f'^2 + Kh' + M_n f' + \lambda\theta = 0 \quad \dots (13)$$

$$\left(1 + \frac{K}{2}\right)h'' + fh' - f'h + K(2h + f'') = 0 \quad \dots (14)$$

$$(3R_a + 4)\theta'' + 3R_a Pr f\theta' = 0 \quad \dots (15)$$

The boundary conditions (5) and (6) becomes

$$f'(0) = 0, \quad h(0) = -\frac{1}{2}f''(0), \quad Prf(0) + M\theta'(0) = 0, \quad \theta(0) = 0 \quad \dots (16)$$

$$f'(\infty) \rightarrow 1, \quad h(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 1 \quad \dots (17)$$

Where prime denote differentiation with respect to  $\eta$ ,  $R_a = \frac{\alpha k_1}{4\sigma^* T_\infty^3}$  is the thermal radiation parameter.  $\lambda = \frac{Gr_x}{Re_x^2}$  is buoyancy parameter,  $Gr_x = \frac{g\beta(T_m - T_\infty)x^2}{\nu^2}$  is the local Reynolds number, Magnetic parameter  $M_n = \frac{\sigma B_0^2}{\rho\alpha}$  and  $Pr = \frac{\nu}{\alpha}$  is the Prandtl number. It is worth mentioning that the cases  $\lambda < 0$  and  $\lambda > 0$  correspond to the opposing and assisting flows, respectively, and for the pure forced convection flow  $\lambda = 0$ . In boundary condition eq. (15),  $M$  is the dimensionless melting parameter defined as

$$M = \frac{c_f(T_\infty - T_m)}{l + c_s(T_m - T_0)} \quad \dots (18)$$

This parameter is a combination of the Stefan numbers  $c_f(T_\infty - T_m)/l$  and

$c_s(T_\infty - T_0)/l$  for the liquid and solid phases respectively.

The skin friction coefficient  $C_f$  and the local Nusselt number  $Nu_x$  are defined as follows

$$C_f = \frac{\tau_w}{\rho U^2/2}, \quad Nu_x = \frac{xq_w}{k(T_\infty - T_m)} \quad \dots (19)$$

Where the wall shear stress  $\tau_w$  and the heat flux  $q_w$  are expressed as

$$\tau_w = \left[(\mu + \kappa) \frac{\partial u}{\partial y} + \kappa\omega\right]_{y=0}, \quad q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0} \quad \dots (20)$$

Using the similarity variables (11), we get

$$\frac{1}{2} C_f Re_x^{1/2} = \left(1 + \frac{K}{2}\right) f''(0), \quad \frac{Nu_x}{Re_x^{1/2}} = -\theta'(0) \quad \dots (21)$$

### III. NUMERICAL SOLUTION

The set of coupled non-linear governing boundary layer equations (1) - (4) together with the boundary conditions (5)-(6) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential equations (1) - (4) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size  $h=0.0001$  is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation fluid velocity, angular velocity and temperature are also sorted out and their numerical values are presented.

### IV. RESULT AND DISCUSSION

The governing equations (13) - (15) subject to the boundary conditions (16) – (17) are solved by using MATLAB computer programming for different values of step size  $\Delta\eta$  and found that there is a negligible, change in the velocity, temperature, angular velocity, local Nusselt number and skin friction coefficient for values of  $\Delta\eta < 0.001$ . Therefore present paper we have set step- size  $\Delta\eta=0.001$ . In order to get a clear insight of the physical problem, the velocity, temperature and angular velocity have been discussed by assigning numerical values to the parameters encountered in the problem. The distribution of the velocity, microrotation and temperature functions with the variation of material parameter, magnetic parameter, radiation parameter and Grashof number parameter has been shown graphically in Figures 1-16. In order to validate the numerical result obtained, we compared our results with those reported by Yacob et al. [31] and Khilap et al.[34] as shown in a Table 1; and they found to be in a favorable agreement.

**Table 1** Comparison between  $f''(0)$  and  $\theta'(0)$  calculated by the present method Yacob et al. [31] and Khilap et al. [35] for various values of  $M_n, \varepsilon, K$  and  $M$ .

$M_n$	$\varepsilon$	$M$	$K$	$q_w$	Yacob et al.[31]		Khilap et al.[34]		Present result	
					$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$
0	0	0	0	0	1.232588	0.570465	1.232588	0.570466	1.232589	0.570467
			1	0	1.006404	0.544535	1.006404	0.544535	1.006403	0.544534
		1	0	0	1.037003	0.361961	1.037003	0.361962	1.037003	0.361963
			1	0	0.879324	0.347892	0.879324	0.347892	0.879325	0.347891

The effect of variation of the magnetic parameter  $M_n$  on the velocity  $f'(0)$ , angular velocity  $g(\eta)$  and temperature profiles  $\theta(\eta)$  is presented in Figures (2) – (4) respectively. Here Magnetic parameter varies as  $M_n = 0, 0.5, 1$  and other parameters  $R_a = 1, K = 1, N = 0.5, Pr = 1, M = 1, \lambda = 0.5$  are fixed. It is well known that the application of a uniform magnetic field normal to the flow direction gives rise to a force called Lorentz. This force has the tendency to slow down the velocity of the fluid and angular velocity of microrotation in the boundary layer Figure (3) shows the effects of Magnetic parameter  $M$  on the microrotation. It is seen that as the magnetic parameter increases the microrotation increases near the plate and the trend gets reversed away from the plate and to increase its temperature. This is obvious from the decreases in the velocity profiles, angular velocity of microrotation profiles, while temperature profiles increases, presented in Figures (2) – (4).

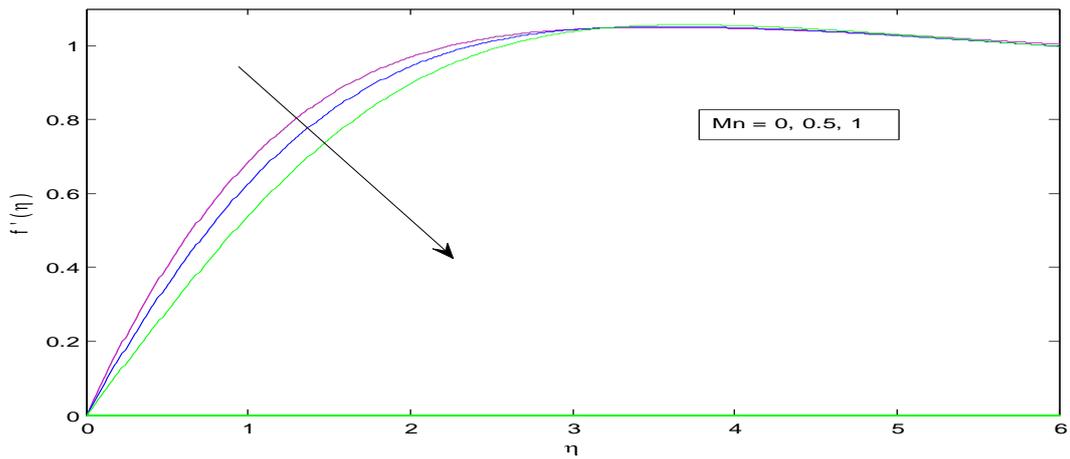


Figure (2) Velocity profiles  $f'$  against  $\eta$  for various values of Magnetic Parameter  $M$

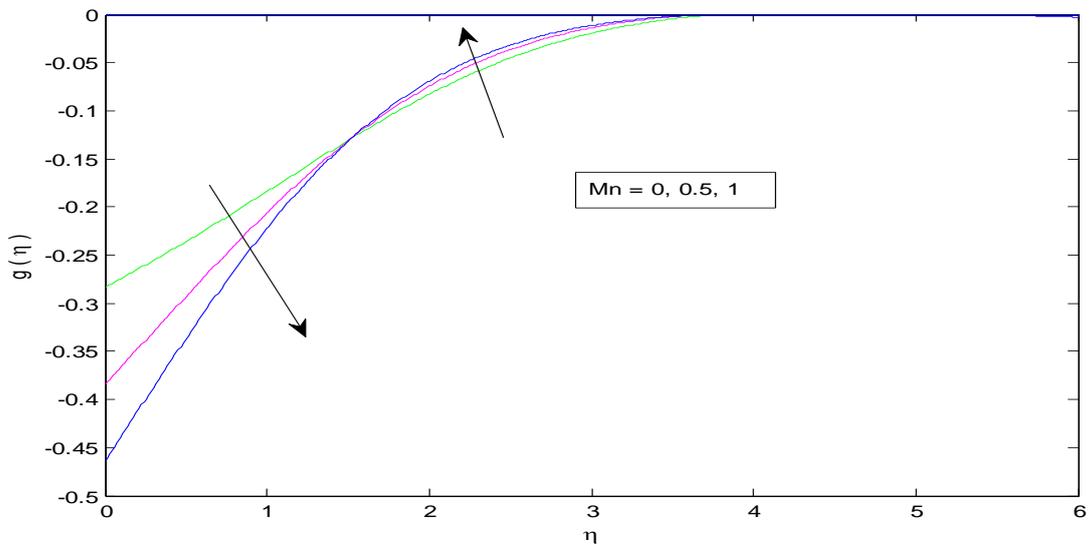


Figure (3) Angular velocity profiles  $g$  against  $\eta$  for various values of Magnetic Parameter  $M$

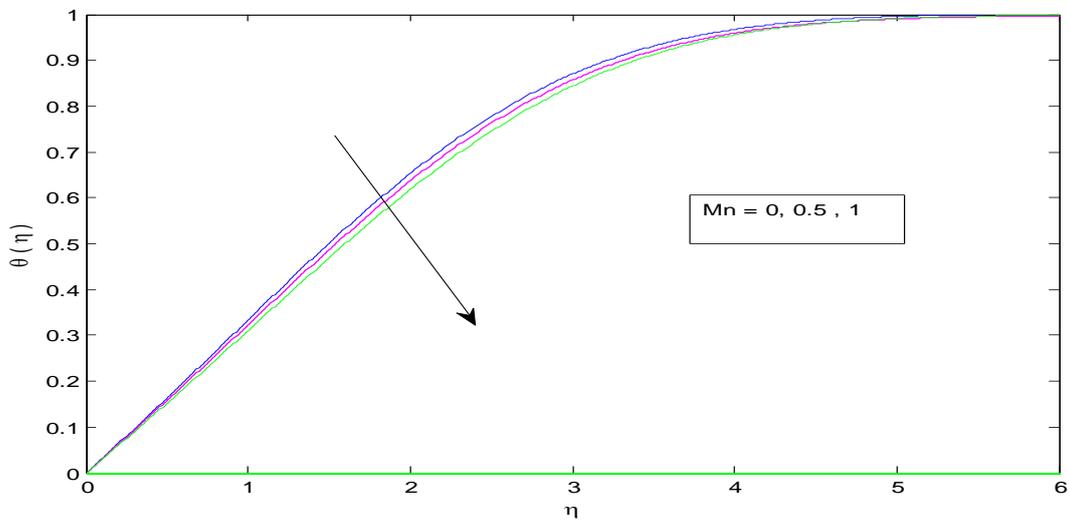


Figure (4) Temperature profiles  $\theta$  against  $\eta$  for various values of Magnetic Parameter  $M$

The effects of the thermal buoyancy parameter  $\lambda$  on the velocity field shown in Figure (5) for various values of buoyancy parameter for various values of  $\lambda = 0, 0.5, 1$  and when  $M_n = 0.5, K = 1, N = 0.5, Pr = 1, M = 1, R_a = 1, \lambda = 0.5$  are fixed. The thermal buoyancy parameter signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. The flow is accelerated due to the enhancement in buoyancy force corresponding to an increase in the thermal buoyancy parameter. It is noticed that the thermal buoyancy parameter influences the velocity field almost in the boundary layer when compared to far away from the plate. It is seen that as the thermal buoyancy parameter increases, the velocity field increases. Figure (6) shows that the variation of the angular velocity with the buoyancy parameter  $\lambda = 0, 0.5, 1$  and when  $M_n = 0.5, R_a = 1, N = 0.5, Pr = 1$ . It is noticed that the angular velocity increases with an increase in the buoyancy parameter. Figure (7) depicts the temperature profiles for different values of the buoyancy parameter. It is noticed that the temperature decreases with an increase in the thermal buoyancy parameter.

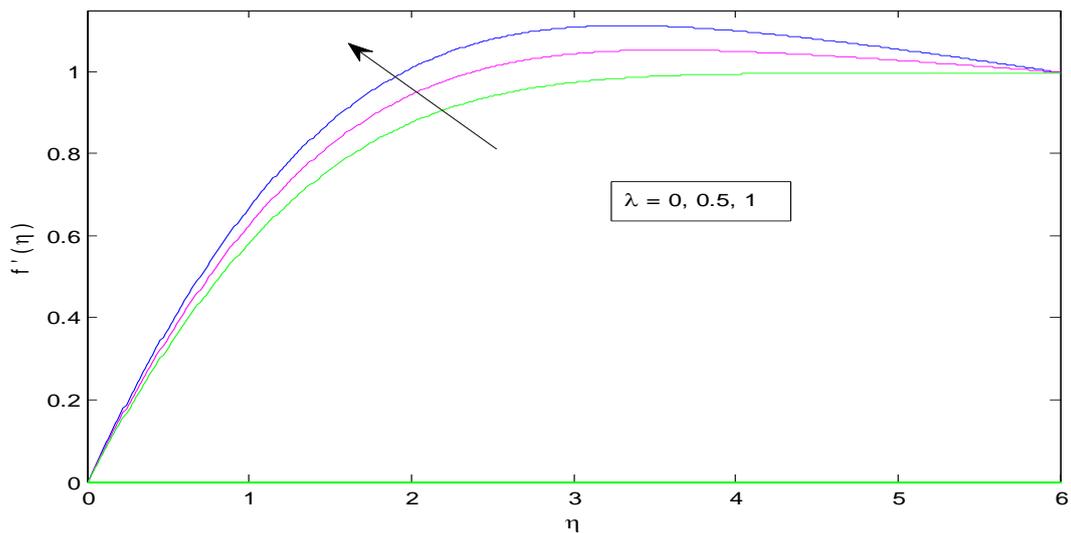


Figure (5) Velocity profiles  $f'$  against  $\eta$  for various values of Buoyancy Parameter  $\lambda$

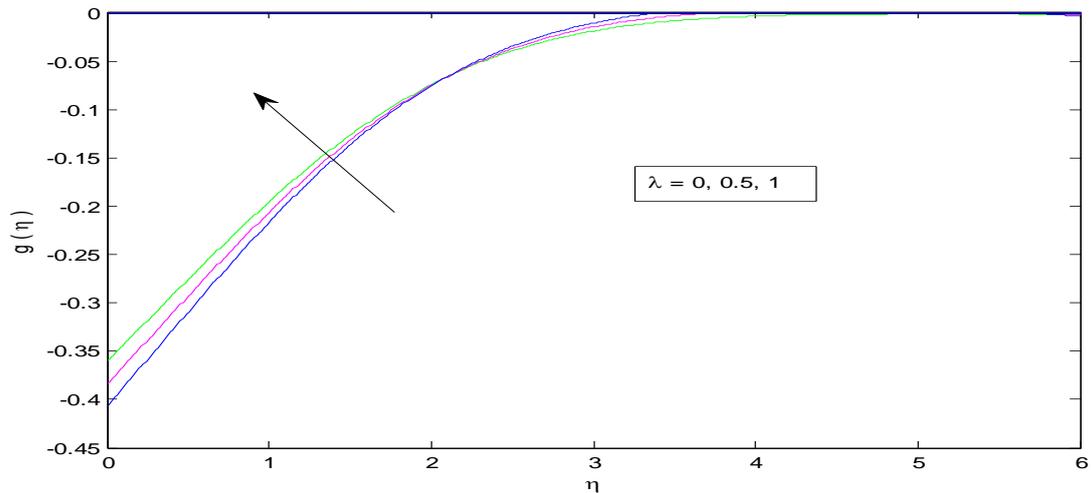
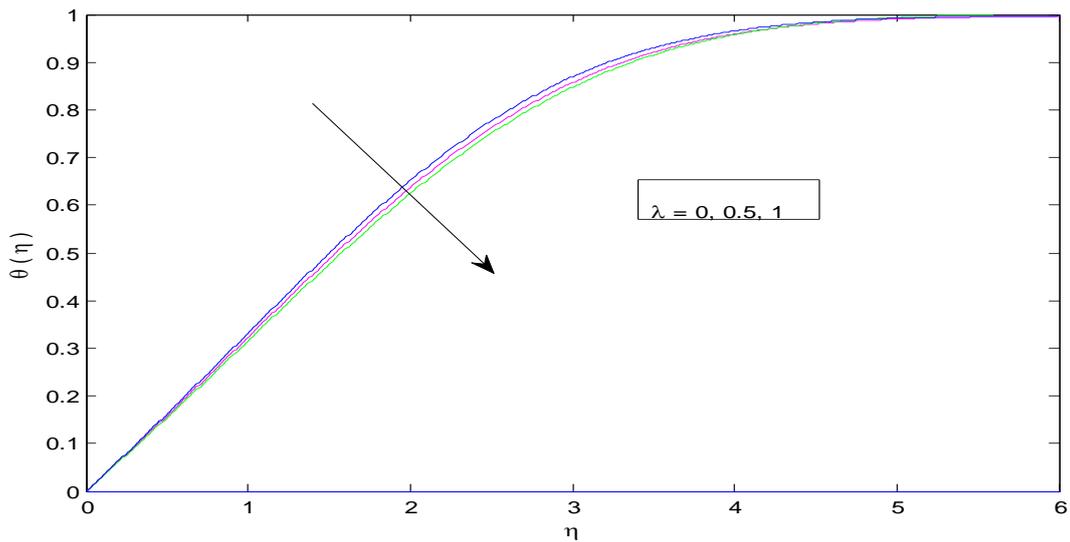
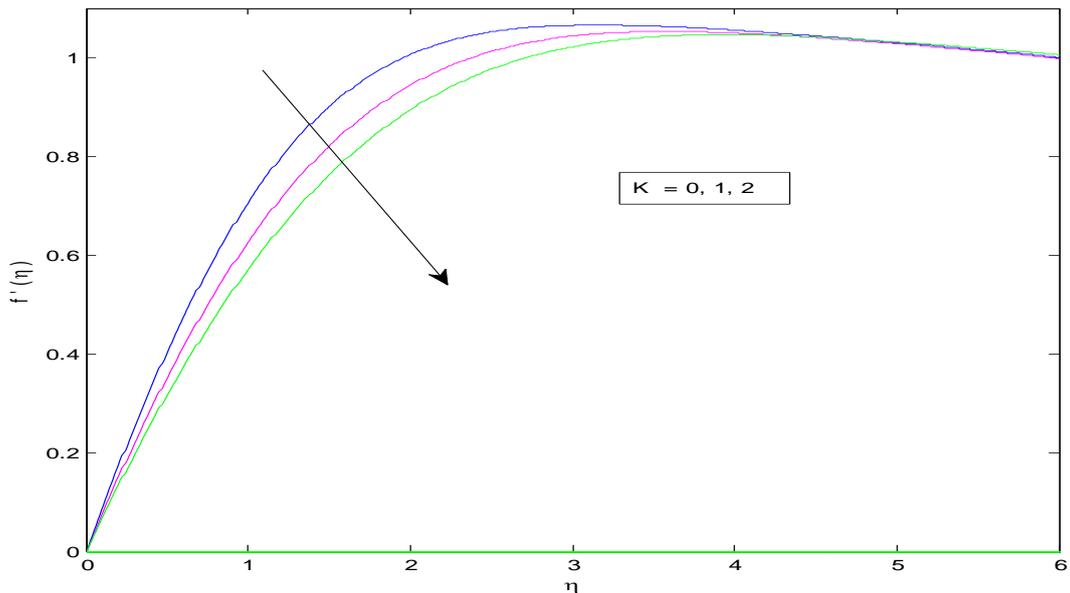


Figure (6) Angular velocity profiles  $g$  against  $\eta$  for various values of Buoyancy Parameter  $\lambda$



**Figure (7) Temperature profiles  $\theta$  against  $\eta$  for various values of Buoyancy Parameter  $\lambda$**

Figure (8) illustrates the effect of Micropolar parameter  $K$  on the velocity. It is noticed that micropolar parameter  $K = 0, 1, 2$  and when  $M_n = 0.5, \lambda = 0.5, R_a = 1, N = 0.5, M = 1, Pr = 1$  are fixed. As the material parameter increases, the velocity decreases at the all points. Figure (9) depicts the variation of the angular velocity with the material parameter ( $K$ ). It is noticed that as the material parameter increases the angular velocity decreases. Figure (10) shows the effect of the material parameter ( $K$ ) on the temperature. It is noticed that as the material parameter increases, the temperature decreases. Figure (8)-(10) gives the velocity, angular velocity and temperature profiles for different values of the material parameter. It is seen that an increase in this parameter decreases all the characteristics.



**Figure (8) Velocity profile  $f'$  against  $\eta$  for various values of Micropolar Parameter  $K$**

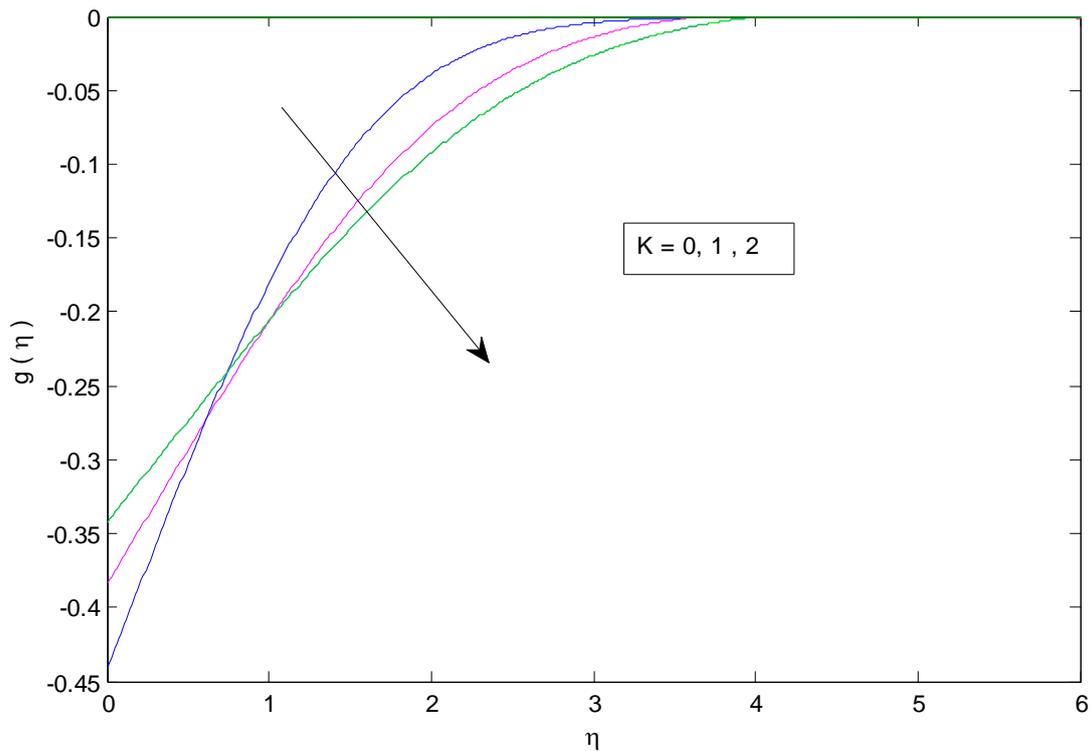


Figure (9) Angular velocity profiles  $g$  against  $\eta$  for various values of Micropolar parameter  $K$

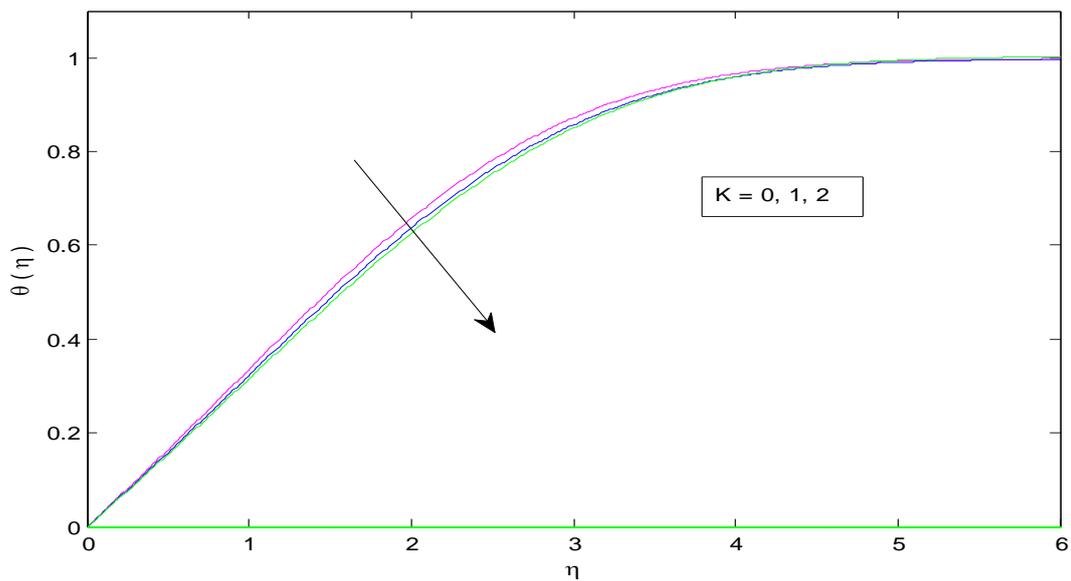


Figure (10) Temperature Profiles  $\theta$  against  $\eta$  for various values of Micropolar Parameter  $K$

The effect of the radiation parameter on the dimensionless velocity, angular velocity component and dimensionless temperature is shown in Figures (11), (12) and (13) respectively. Figure (11) shows that velocity component  $f'(0)$  slightly decreases with an increase in the radiation parameter. The effect of the radiation parameter  $R_a = 1, 2, 3$  and when  $M_n = 0.5, K = 1, N = 0.5, Pr = 1, M = 1, \lambda = 0.5$  are fixed then the angular velocity is illustrated in Figure (12). It is observed that as the radiation parameter increases, the angular velocity decreases slightly. From Figure (13) it is seen that the temperature decreases as the radiation parameter increases. This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.

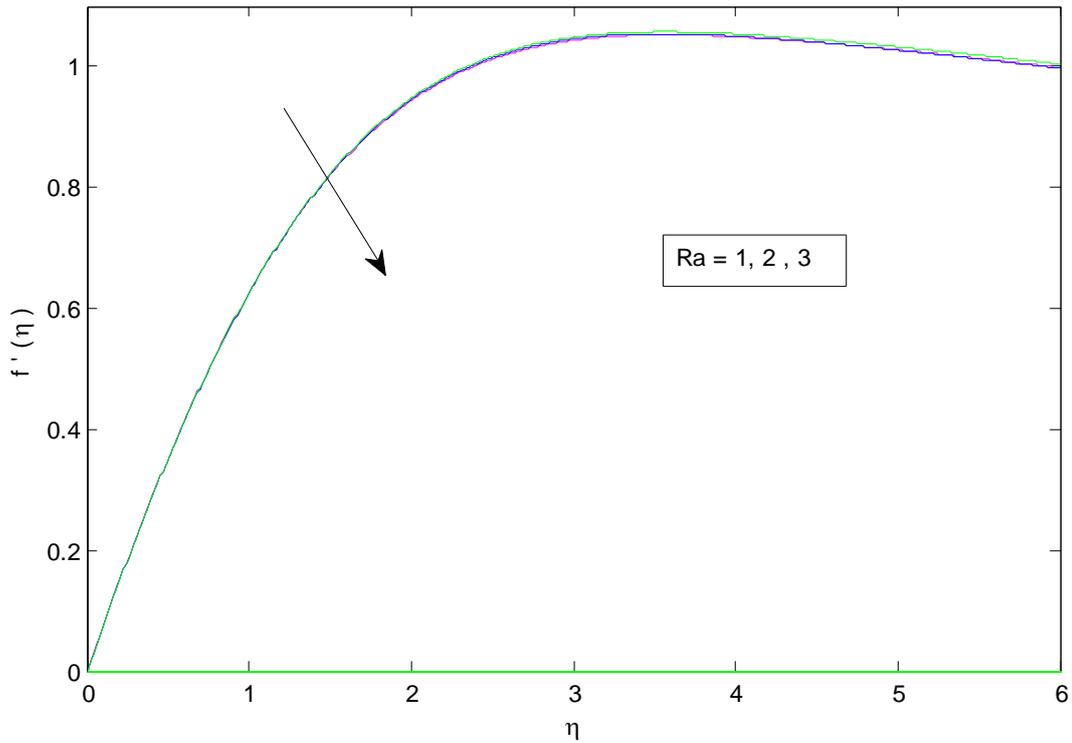


Figure (11) Velocity profile  $f'$  against  $\eta$  for various values of Radiation Parameter  $R_a$

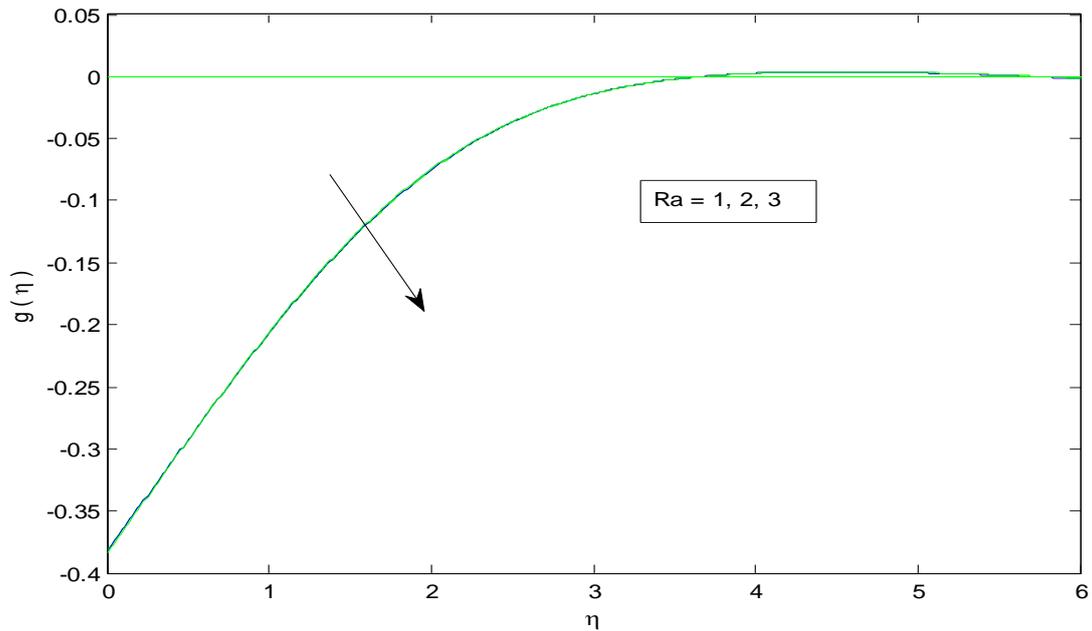


Figure (12) Angular velocity profiles  $g$  against  $\eta$  for various values of Radiation Parameter  $R_a$

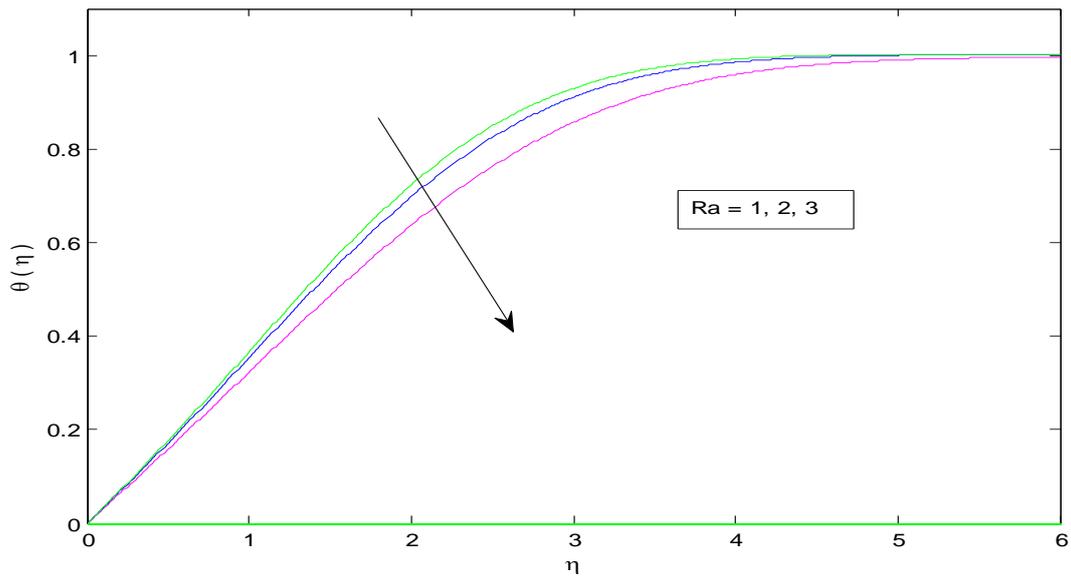


Figure (13) Temperature Profiles  $\theta$  against  $\eta$  for various values of Radiation Parameter  $R_a$

The effect of the radiation parameter on the dimensionless velocity, angular velocity component and dimensionless temperature is depicts in Figures (14), (15) and (16) respectively. It is noticed that the temperature, velocity, and angular velocity profiles for different values of the Melting parameter  $M = 0, 1, 2$  and when  $M_n = 0.5, \lambda = 0.5, R_a = 1, N = 0.5, Pr = 1, K = 1$  are fixed. It is seen that an increase in this parameter decreases all the characteristics.

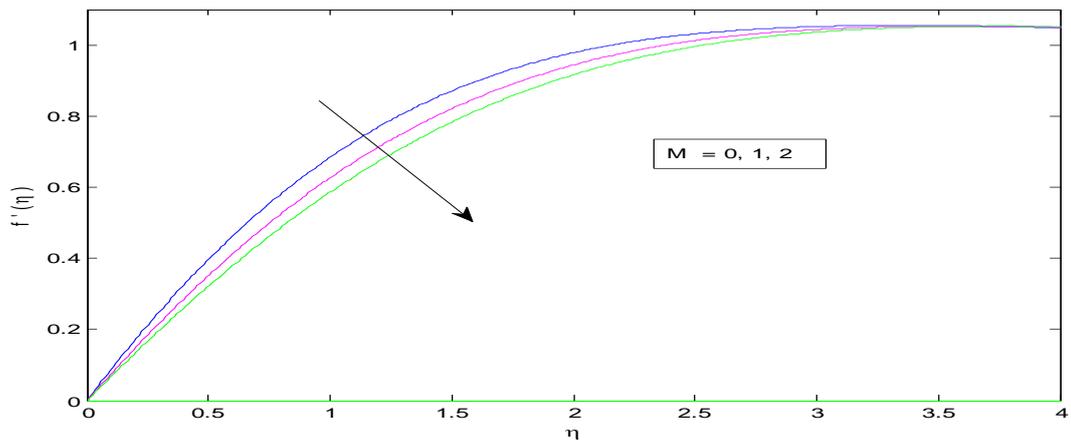


Figure (14) Velocity profile  $f'$  against  $\eta$  for various values of Melting Parameter  $M$

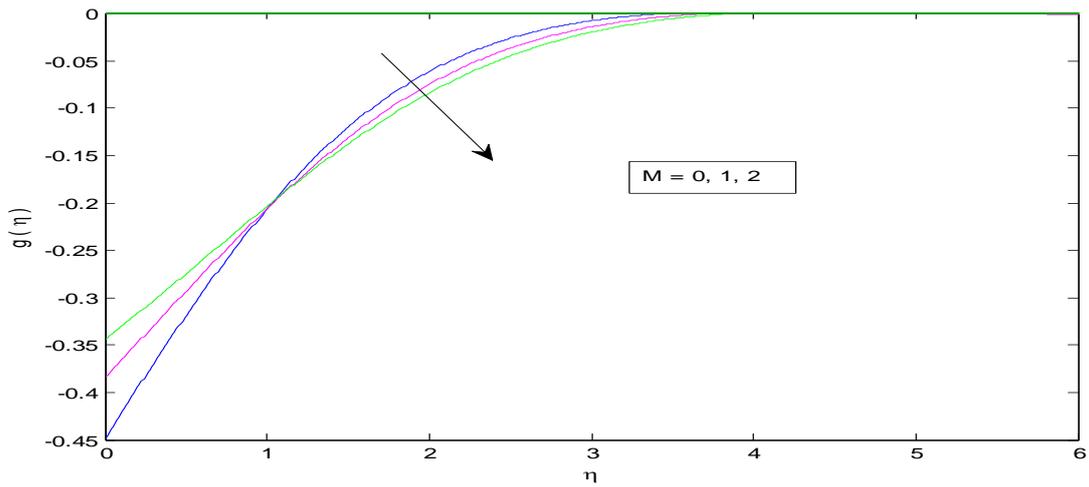


Figure (15) Angular velocity profiles  $g$  against  $\eta$  for various values of Melting Parameter  $M$

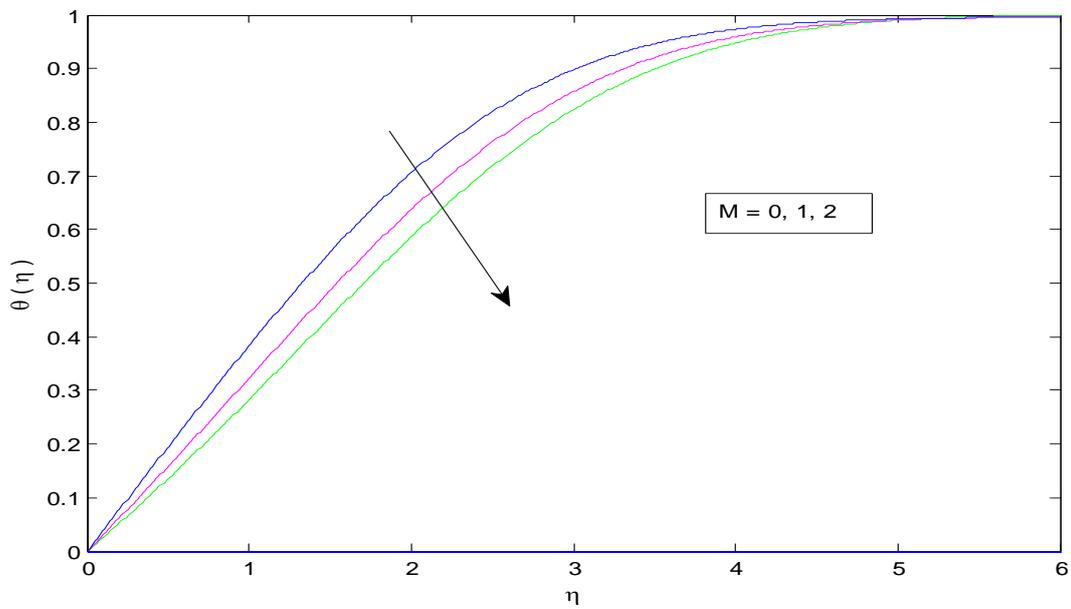


Figure (16) Temperature Profiles  $\theta$  against  $\eta$  for various values of Melting Parameter  $M$

Figure (17) Illustrates the dimensionless velocity component  $f'(0)$  for different values of the Prandtl number  $Pr = 0.71, 1, 7$  and when  $M_n = 0.5, \lambda = 0.5, R_a = 1, N = 0.5, M = 1$  are fixed. The graph shows that the effect of increasing values of the Prandtl number results in a increasing velocity.

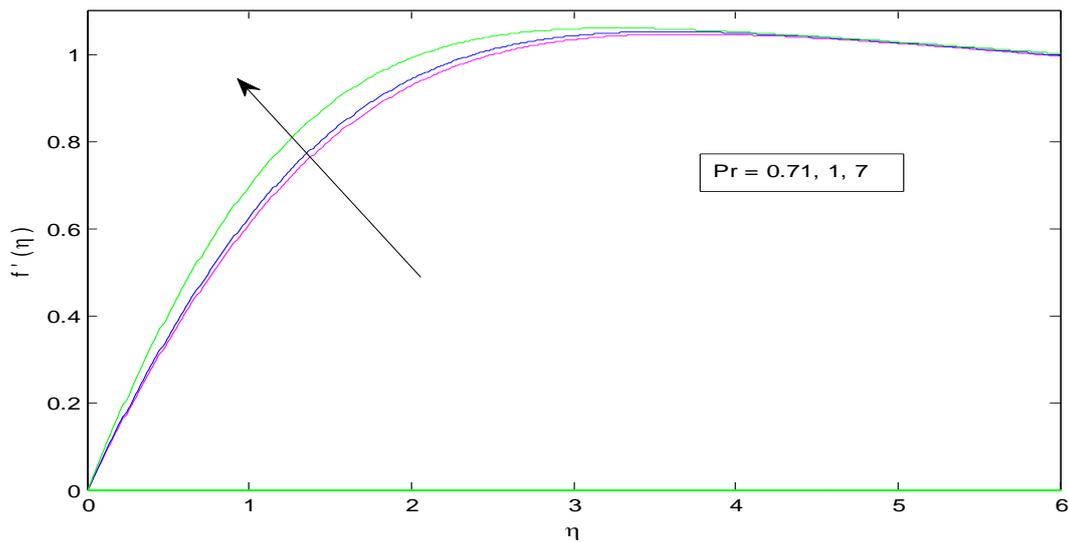


Figure (16) Velocity profiles  $f'$  against  $\eta$  for various values of Prandtl Number Parameter  $Pr$

## V. CONCLUSIONS

The effect of melting phenomenon on the steady laminar stagnation-point flow and heat transfer from a warm micropolar fluid to a vertical surface has been investigated. The transformed nonlinear ordinary differential equations were solved numerically using a Runge- Kutta method by using shooting technique. The velocity profiles increases as buoyancy parameter and Prandtl number increases. The velocity profiles decreases with increasing the value of magnetic parameter, micropolar parameter, Radiation parameter and the Melting parameters. The angular velocity profiles increasing with Magnetic parameter, Buoyancy parameter and Radiation parameter while reverse effects shown with Melting parameter and micropolar parameter. The temperature profile is increasing with increasing the values of Magnetic parameter and decreases with the increases the values of Melting parameter, Radiation parameter, Micropolar parameter and buoyancy parameter.

## VI. NOMENCLATURE

$a$	Constant ( $m^{-1}$ )
$B_0$	Magnetic field
$C_f$	Skin-friction coefficient
$C_p$	Specific heat at constant pressure ( $J kg^{-1}K^{-1}$ )
$C_s$	Heat capacity of solid surface
$g$	Dimensionless angular velocity
$Gr_x$	Grashof number
$j$	Micro inertia density ( $m^2$ )
$K$	Micropolar or material parameter
$k$	Thermal conductivity of the fluid ( $W m^{-1}K^{-1}$ )
$k_1$	Mean absorption coefficient
$M_n$	Magnetic parameter
$n$	Constant
$Nu$	Nusselt number
$Pr$	Prandtl number
$T$	Temperature (K)
$T_s$	Temperature of the solid medium
$u, v$	Dimensionless velocities along $x$ and $y$ direction respectively
$x, y$	Axial and perpendicular co-ordinate (m)
$Re_x$	Reynolds number
$R_a$	Radiation Parameter
$f$	Dimensionless stream function

### Greek symbols

$\Psi$	Stream function
$\alpha$	Thermal diffusivity

$\gamma$	Spin-gradient viscosity (N s)
$\mu$	Dynamic viscosity (Pa s)
$\sigma$	Electrical conductivity of the fluid
$\sigma^*$	Stefan-Boltzmann Constant
$\theta$	Dimensionless temperature
$\rho$	Density
$\nu$	Kinematic viscosity
$\kappa$	Vortex viscosity
$\omega$	Component of microrotation ( rad s <sup>-1</sup> )
$\lambda$	Buoyancy Parameter
$\tau_w$	Surface heat flux
$\eta$	Non dimensionless distance

### Subscripts

M	Condition at the melting surface
$\infty$	free stream condition.
s	Solid medium

### Superscripts

'	Derivative with respect to $\eta$
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