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Pairwise Separated Sets in Bitopological Spaces

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Abstract. Main aim of present research work is to introduce the concept of weak pairwise separated sets and separated sets for bitopological spaces. Various results related to pairwise separated sets will be discussed. Furthermore, the concept of weak pairwise connected bitopological space is introduced.
Keywords:-Bitopological spaces, pairwise closed, pairwise separated sets, pairwise open.

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I. INTRODUCTION AND PRELIMINARIES

In 1963, a new concept of bitopological spaces is introduced by J. C. Kelly [1]. This newly introduced concept of bitopological spaces is used by Kelly [1] to study non-symmetric functions which generate two arbitrary topologies on X. Furthermore, in the same research paper, the notations of separation axioms in topological space are generalized to bitopological spaces and these separation properties in bitopological spaces are named as pairwise Hausdorff, pairwise regular and pairwise normal. Many other researchers like Kim [2], Fletcher [3] et al., Patty [4], Pervin [5], Reilly [6], Saegrove [7] carried out further research in the area of compactness, connectedness, total disconnectedness and other separation properties of bitopological spaces.

In particular, the concept of connectedness in bitopological spaces is introduced by Pervin [5] and the same concept, under the name of pairwise connectedness, is further investigated by Reilly [6].

In this research study, main objective is to introduce the concept of weak pairwise separated sets and pairwise separated sets for bitopological spaces. Then various results involving newly introduced pairwise separated sets will be investigated. Further, the concept of weak pairwise connected bitopological space will be introduced in terms of weak pairwise separated sets.

A triplet (X, τ_1, τ_2), where τ_1, τ_2 are arbitrary topologies on X, is called a bitopological space on X. For any subset A of (X, τ_1, τ_2), τ_1–cl(A) and τ_2–cl(A) denote closure of A with respect to τ_1 and τ_2 respectively. Further, τ_1–open (τ_1–closed) and τ_2–open (τ_2–closed) will be used to denote open (closed) set in a bitopological space (X, τ_1, τ_2), with respect to τ_1 and τ_2 respectively. Any subset of a bitopological space (X, τ_1, τ_2) is said to be pairwise open (closed) if and only if it is open (closed) with respect to τ_1 and τ_2.

Definition 1 [5, 6]. Bitopological space (X, τ_1, τ_2) is called pairwise connected if and only if X cannot be expressed as the union of two non-empty disjoint set A and B such that $A\cap(\tau_1\text{-cl}(B))=\phi$ and $(\tau_2\text{-cl}(A))\cap B=\phi$.

Definition 2 [6]. Bitopological space (X, τ_1, τ_2) is said to be pairwise disconnected if and only if X can be expressed as the union of two non-empty disjoint set A and B such that $A\cap(\tau_1\text{-cl}(B))=\phi$ and $(\tau_2\text{-cl}(A))\cap B=\phi$.

II. PAIRWISE SEPARATED SETS IN A BITOPOLOGICAL SPACE

This research manuscript starts with the introduction of weak pairwise separated sets and pairwise separated sets in bitopological spaces.

Definition (Weak Pairwise Separated Sets) 3. Two non-empty subsets A and B of bitopological space (X,τ_1 ,τ_2) are weak pairwise separated sets if and only if $A\cap(\tau_1\text{-cl}(B))=\phi$ and $(\tau_2\text{cl}(A))\cap B=\phi$ or $A\cap(\tau_2\text{-cl}(B))=\phi$ and $(\tau_1\text{-cl}(A))\cap B=\phi$.

Definition (Pairwise Separated Sets) 4. Two non-empty subsets A and B of bitopological space (X,τ_1 ,τ_2) are pairwise separated sets if and only if $A\cap(\tau_1\text{-cl}(B))=\phi$ and $(\tau_2\text{-cl}(A))\cap B=\phi$.
From thes definitions it is clear that pairwise separated sets are always weak pairwise separated but converse is not always true. Also it is trivial that pairwise separated seta are disjoint. Proceeding theorems deal with generalizations of results for separated set of traditional topology to bitopological spaces.

Theorem 1. Subsets of pairwise separated sets are also pairwise separated.

Proof. Let A and B are pairwise separated sets in (X, τ_1, τ_2) and $C\subseteq A$, $D\subseteq B$. Then, $\tau_2\text{-cl}(C)\subseteq \tau_2\text{-cl}(A)$ and $\tau_1\text{-cl}(D)\subseteq \tau_1\text{-cl}(B)$. Evidently, $C\cap(\tau_1\text{-cl}(D))\subseteq A\cap(\tau_1\text{-cl}(B))=\phi$ and $(\tau_2\text{-cl}(C))\cap D\subseteq (\tau_2\text{-cl}(A))\cap B=\phi$. Thus, C and D are pairwise separated.

Theorem 2. Two pairwise closed subsets of a bitopological space are pairwise separated if and only if sets are disjoint.

Proof. Suppose that A and B are pairwise separated in (X, τ_1, τ_2). As $A\subseteq \tau_2\text{-cl}(A)$ and $B\subseteq \tau_1\text{-cl}(B)$. Therefore, $A\cap B\subseteq A\cap \tau_1\text{-cl}(B)=\phi$.Clearly, A and B are disjoint.
Conversely, let $A\cap B=\phi$. Since, A and B are pairwise closed therefore, $A\cap(\tau_1\text{-cl}(B))=\phi$ and $(\tau_2\text{-cl}(A))\cap B=\phi$. Thus, A and B are pairwise separated.

Theorem 3. Two pairwise open subsets of a bitopological space are pairwise separated if and only if sets are disjoint.

Proof. Suppose that A and B are pairwise separated in (X, τ_1, τ_2). As $A\subseteq \tau_2\text{-cl}(A)$ and $B\subseteq \tau_1\text{-cl}(B)$. Therefore, $A\cap B\subseteq A\cap (\tau_1\text{-cl}(B))=\phi$. Clearly, A and B are disjoint.
Conversely, let A and B are disjoint. If feasible, let $A\cap(\tau_1\text{-cl}(B))\neq\phi$. Therefore, there exists $x\in A$ and $x\in\tau_1\text{-cl}(B)$. Since, A is pairwise open so it is a τ_1-open set containing A. Consequently, $A\cap B\neq\phi$. It is contradiction with given fact. Thus, $A\cap(\tau_1\text{-cl}(B))=\phi$. Similarly, $(\tau_2\text{-cl}(A))\cap B=\phi$. Hence, desired result follows.

Theorem 4. If A and B are pairwise separated set in (X, τ_1, τ_2) then, $A\cup B$ pairwise closed set means A and B are pairwise closed sets.

Proof. We have, $A\cap(\tau_1\text{-cl}(B))=\phi$ and $(\tau_2\text{-cl}(A))\cap B=\phi$. Now, $A\cup B= \tau_1\text{-cl}(A\cup B)= (\tau_1\text{-cl}(A))\cup (\tau_1\text{-cl}(B))=\phi$. Then, $\tau_1\text{-cl}(A)\subseteq (\tau_1\text{-cl}(A))\cup (\tau_1\text{-cl}(B))$ implies $\tau_1\text{-cl}(A)\subseteq A\cup B$. Further, $\tau_1\text{-cl}(A)= (A\cup B)\cap (\tau_1\text{-cl}(A))=(A\cap(\tau_1\text{-cl}(A))\cup(B\cap(\tau_1\text{-cl}(A))= (A\cap(\tau_1\text{-cl}(A))=A$. Also it can be proved $\tau_2\text{-cl}(A)=A$. Similarly, we can prove that B is pairwise closed.

Theorem 5. If A and B are pairwise separated set in (X, τ_1, τ_2) then, $A\cup B$ pairwise open set means A and B are pairwise open sets.

Proof. Evidently, $X\text{-}(\tau_1\text{-cl}(A)$ is τ_1-open. Consider $A\cup B$ to be τ_1-open. Therefore, $(A\cup B)\cap(X\text{-}(\tau_1\text{-cl}(A))$ is τ_1-open. Now,
$(A\cup B)\cap(X\text{-}(\tau_1\text{-cl}(A))=[(A\cap(X\text{-}(\tau_1\text{-cl}(A))]\cup[(B\cap(X\text{-}(\tau_1\text{-cl}(A))]=$
$[(A\cap X)\text{-}(A\cap(\tau_1\text{-cl}(A)))]\cup[(B\cap X)\text{-}(B\cap(\tau_1\text{-cl}(A)))]=(A\text{-}A)\cup(B\text{-}\phi)=B$. Consequently, B is τ_1-open. On the same line, we can prove that B is τ_2-open. Similarly, A is pairwise open.

Theorem 6. A and B are pairwise separated set in (X, τ_1, τ_2) if and only if $A\cap B=\phi$ and A, B are pairwise open and pairwise closed in (Y, τ_1^Y, τ_2^Y), where $Y=A\cup B$.

Proof. Suppose that A and B are pairwise separated in (X, τ_1, τ_2). Therefore, $A\cap(\tau_1\text{-cl}(B))=\phi$ and $(\tau_2\text{-cl}(A))\cap B=\phi$. We have, $\tau_2^Y\text{-cl}(A)=(\tau_2\text{-cl}(A))\cap Y=(\tau_2\text{-cl}(A))\cap(A\cup B)=[(\tau_2\text{-cl}(A))\cap A]\cup[(\tau_2\text{-cl}(A))\cap B]=A$. Evidently, A is τ_2^Y-closed. Same way it can be proved that A is τ_1^Y-closed. Similarly, B is pairwise closed in (Y, τ_1^Y, τ_2^Y). Furthermore, since $A\cap B=\phi$ and $Y=A\cup B$ therefore, A and B are pairwise open in (Y, τ_1^Y, τ_2^Y).
Conversely, suppose that given condition holds. Now, $\tau_2^Y\text{-cl}(A)=A$ and $\tau_1^Y\text{-cl}(B)=B$. Therefore, $A\cap B=\phi \Rightarrow (\tau_2^Y\text{-cl}(A))\cap B=\phi$ and $A\cap(\tau_1^Y\text{-cl}(B))=\phi$. Hence, A and B are pairwise separated in (Y, τ_1^Y, τ_2^Y).

Theorem 7. If (Y, τ_1^Y, τ_2^Y) be a subspace of a bitopological space (X, τ_1, τ_2) then, $A, B \subseteq Y$ are pairwise separated in (Y, τ_1^Y, τ_2^Y) if and only if they are pairwise separated in (X, τ_1, τ_2).

Proof. If A and B are pairwise separated in (Y, τ_1^Y, τ_2^Y) then, $(\tau_2^Y\text{-cl}(A))\cap B=\phi$ and $A\cap(\tau_1^Y\text{-cl}(B))=\phi$. Therefore, $[(\tau_2\text{-cl}(A))\cap Y]\cap B=\phi$ and $A\cap[(\tau_1\text{-cl}(B))\cap Y]=\phi$. Evidently, $(\tau_2\text{-cl}(A))\cap B=\phi$ and $A\cap(\tau_1\text{-cl}(B))=\phi$. Thus, A and B are pairwise separated in (X, τ_1, τ_2).
Conversely, let A and B are pairwise separated in (Y, τ_1, τ_2) then, $(\tau_2\text{-cl}(A))\cap B=\phi$ and $A\cap(\tau_1\text{-cl}(B))=\phi$, i.e., $(\tau_2\text{-cl}(A))\cap(B\cap Y)=\phi$ and $(A\cap Y)\cap(\tau_1\text{-cl}(B))=\phi$. Evidently, $[(\tau_2\text{-cl}(A))\cap Y]\cap B=\phi$ and $A\cap[(\tau_1\text{-cl}(B))\cap Y]=\phi$. Consequently, A and B are pairwise separated in (Y, τ_1^Y, τ_2^Y).

By using the concept of weak pairwise separated sets and pairwise separated sets, the definition of weak pairwise disconnected and pairwise disconnected bitopological space [5, 6] can be expressed as follows:

Definition (Weak Pairwise Disconnected Bitopological space) 5. A bitopological space (X, τ_1, τ_2) is said to be weak pairwise disconnected if and only if X can be expressed as union of two weak pairwise separated sets.

It is trivial that a bitopological space which is not weak pairwise disconnected is weak pairwise connected.

Definition (Pairwise Disconnected Bitopological space) 6. A bitopological space (X, τ_1, τ_2) is said to be pairwise disconnected if and only if X can be expressed as union of two pairwise separated sets.

It is evident that a bitopological space which is not pairwise disconnected is pairwise connected [5, 6]. From definitions it is clear that every pairwise connected [5, 6] bitopological space is weak pairwsie connected but converse is not always true.

III. CONCLUSION

In this research manuscript, corresponding to traditional topology, idea of pairwise separated sets in bitopological spaces are discussed. Further, concept of pairwise separated sets in bitopological spaces is used to accomplish related results and pairwise connectedness & pairwise disconnectedness is expressed in terms of pairwise separated sets.

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