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Studies on Some Extended Properties on Some Sequence Spaces of Fuzzy Real Numbers

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Abstract: In this article we study with counter examples about different properties of convergent, null and bounded double sequence spaces of fuzzy real numbers. Those will include different properties like sequence algebra, convergence free etc. We prove some inclusion results too.

Keywords: Orlicz Function, Fuzzy real number, Sequence Algebra.

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I. INTRODUCTION

Throughout, a double sequence is denoted by $<X_{nk}>$, a double infinite array of elements X_{nk}, where each X_{nk} is a fuzzy real number.

The initial works on double sequences is found in Bromwich [2]. Later on it was studied by Hardy [4], Moricz [7], Basarir and Sonalcan [1], Tripathy and Sarma [12], Sarma [10] and many others. Hardy [4] introduced the notion of regular convergence for double sequences.

The concept of paranormed sequences was studied by Nakano [8] and Simmons [10] at the initial stage. Later on it was studied by many others.

Sequences of fuzzy real numbers relative to the paranormed sequence spaces is studied by Choudhury and Tripathy [3].

An *Orlicz function M* is a mapping $M : [0, \infty) \to [0, \infty)$ such that it is *continuous*, *non-decreasing* and *convex* with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$, as $x \to \infty$.

Let D denote the set of all closed and bounded intervals $X = [a_1, a_2]$ on R, the real line. For $X, Y \in D$ we define

$$d(X, Y) = \max(|a_1 - b_1|, |a_2 - b_2|),$$

where $X = [a_1, a_2]$ and $Y = [b_1, b_2]$. It is known that (D, d) is a complete metric space.

A *fuzzy real number* X is a fuzzy set on R, *i.e.* a mapping $X : R \to I$ (=[0,1]) associating each real number t with its grade of membership $X(t)$.

The α - *level* set $[X]^\alpha$ of the fuzzy real number X, for $0 < \alpha \le 1$, defined as $[X]^\alpha = \{ t \in R : X(t) \ge \alpha \}$.

A fuzzy real number X is said to be *upper-semi continuous* if, for each $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$, for all $a \in I$ is open in the usual topology of R.

A *fuzzy real number X* is called *convex* if $X(t) \ge X(s) \wedge X(r) = \min(X(s), X(t))$, where $s < t < r$.

If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number X is called *normal.*

The set of all upper-semi continuous, normal, convex fuzzy real numbers is denoted by $R(I)$ and throughout the article, by a fuzzy real number we mean that the number belongs to $R(I)$.

The set R of all real numbers can be embedded in $R(I)$. For $r \in R$, $\bar{r} \in R(I)$ is defined by

$$\bar{r}(t) = \begin{cases} 1, & \text{for } t = r, \\ 0, & \text{for } t \ne r \end{cases}$$

A fuzzy real number X is called *non-negative* if $X(t) = 0$, for all $t < 0$. The set of all non-negative fuzzy real numbers is denoted by $R^*(I)$.

Let $\bar{d} : R(I) \times R(I) \to R$ be defined by

$$\bar{d}(X, Y) = \sup_{0 \le \alpha \le 1} d\left([X]^\alpha, [Y]^\alpha\right).$$

Then $\bar{d}$ defines a metric on $R(I)$.

The additive identity and multiplicative identity in $R(I)$ are denoted by $\bar{0}$ and $\bar{1}$ respectively.

II. DEFINITIONS AND PRELIMINARIES

A double sequence (X_{nk}) of fuzzy real numbers is said to be *convergent in Pringsheim's sense* to the fuzzy real number L if, for every $\varepsilon > 0$, there exists $n_0, k_0 \in N$ such that $\bar{d}\,(X_{nk}, L) < \varepsilon$, for all $n \geq n_0$, $k \geq k_0$.

A double sequence (X_{nk}) of fuzzy real numbers is said to be *regularly convergent* if it convergent in Pringsheim's sense and the following limits exist:

$$\lim_n \bar{d}(X_{nk}, L_k) = 0, \text{ for some } L_k \in R(I), \text{ for each } k \in N,$$

and $$\lim_k \bar{d}(X_{nk}, J_n) = 0, \text{ for some } J_n \in R(I), \text{ for each } n \in N.$$

A fuzzy real number sequence (X_k) is said to be *bounded* if $\sup_k |X_k| \leq \mu$, for some $\mu \in R^*(I)$.

Throughout the article $_2w_F$, $(_2\ell_\infty)_F$, $_2c_F$, $(_2c_0)_F$, $_2c_F^R$ and $(_2c_0^R)_F$ denote the classes of *all*, *bounded*, *convergent in Pringsheim's sense*, *null in Pringsheim's sense*, *regularly convergent* and *regularly null* fuzzy real number sequences respectively.

A double sequence space E_F is said to be *solid* (or *normal*) if $<Y_{nk}> \in E_F$, whenever $|Y_{nk}| \leq |X_{nk}|$, for all $n, k \in N$, for some $<X_{nk}> \in E_F$.

A double sequence space E_F is said to be *sequence algebra* if $(X_{nk} \otimes Y_{nk}) \in E_F$, whenever $(X_{nk}), (Y_{nk}) \in E_F$.

A double sequence space E_F is said to be *convergence free* if $(Y_{nk}) \in E_F$, whenever $(X_{nk}) \in E_F$ *and* $X_{nk} = \bar{0}$ implies $Y_{nk} = \bar{0}$.

We study different properties of the following sequence spaces those are defined by Sarma [10].

Let $p = <p_{nk}>$ be a sequence of strictly positive real numbers.

$$_2\ell_\infty(M, p) = \left\{ < X_{nk} > \in {}_2w_F : \lim_{n,k} \left\{ M\left(\frac{\bar{d}(X_{nk}, \bar{0})}{\rho} \right) \right\}^{p_{nk}} < \infty \right\}$$

$$_2c_F(M, p) = \{ < X_{nk} > \in {}_2w_F : \lim_{n,k} \left\{ M\left(\frac{\bar{d}(X_{nk}, L)}{\rho} \right) \right\}^{p_{nk}} = 0, \text{ for some } L \in R(I) \}$$

For $L = \bar{0}$ we get the class $(_2c_F)_0(M, p)$.

Also a fuzzy sequence $<X_{nk}> \in {}_2c_F^R(M, p)$ if $<X_{nk}> \in {}_2c_F(M, p)$ and the following limits exist:

$$\lim_n \left\{ M\left(\frac{\bar{d}(X_{nk}, L_k)}{\rho} \right) \right\}^{p_{nk}} = 0, \text{ for some } L_k \in R(I)$$

$$\lim_k \left\{ M\left(\frac{\bar{d}(X_{nk}, J_n)}{\rho} \right) \right\}^{p_{nk}} = 0, \text{ for some } J_n \in R(I)$$

III. MAIN RESULTS

Theorem 3.1. *Let* $0 < q_{ij} \leq p_{ij} < \infty$, *for all* $i, j \in N$. *Then* $Z(M, p) \subseteq Z(M, q)$ *for* $Z = {}_2c_F, (_2c^R)_F, (_2c_F)_0, (_2c_0^R)_F$.

Proof. Consider the sequence space $_2c_F(M, p)$ and $_2c_F(M, q)$. Let $<X_{nk}> \in {}_2c_F(M, p)$.

Then $\{\bar{d}(X_{nk}, L)\}^{p_{nk}} < \varepsilon$, for all $n \geq n_0$, $k \geq k_0$.

The result follows from the inequality $\{\bar{d}(X_{nk}, L)\}^{q_{nk}} \leq \{\bar{d}(X_{nk}, L)\}^{p_{nk}}$ with the help of non decreasing property of M.

The following result is proved in Sarma [10].

Theorem 3.2. *Let* $<p_{nk}>$ *be bounded. Then the classes of sequences* $(_2\ell_\infty)_F(M, p)$, $_2c_F^R(M, p), (_2c_F^R)_0(M, p)$ *are complete metric spaces with respect to the metric defined by,*

$$f(X, Y) = \inf\left\{ r^{\frac{p_{nk}}{J}} > 0 : \sup_{n,k} M\left(\frac{\bar{d}(X_{nk}, Y_{nk})}{r}\right) \le 1 \right\}, \quad \text{where } J = \max(1, 2^{H-1})$$

Property 3.3. *The spaces* $({}_2\ell_\infty)_F(M,p)$, ${}_2c_F(M,p)$, $({}_2c_0)_F(M,p)$, $({}_2c^R)_F(M,p)$ and $({}_2c_0^R)_F(M,p)$ *are not convergence free.*

The result follows from the following example.

Example 3.3. Consider the sequence space ${}_2c_F(M,p)$. Consider $M(x) = x$. Let $p_{1k} = 1$ for all $k \in N$, $p_{nk} = 3$, otherwise. Consider the sequence $\langle X_{nk}\rangle$ defined by,

$$X_{1k} = \bar{0},$$

and for other values $$X_{nk}(t) = \begin{cases} t+2, & \text{for } -2 \le t \le -1, \\ -nt(n+1)^{-1} + (n+1)^{-1}, & \text{for } -1 \le t \le n^{-1}, \\ 0, & \text{otherwise.} \end{cases}$$

Let the sequence $\langle Y_{nk}\rangle$ be defined by,

$$Y_{1k} = \bar{0}$$

and for other values $$Y_{nk}(t) = \begin{cases} 1, & \text{for } 0 \le t \le 1, \\ (n-t)(n-1)^{-1}, & \text{for } 1 \le t \le n, \\ 0, & \text{otherwise.} \end{cases}$$

Then $\langle X_{nk}\rangle \in {}_2c_F(M,p)$ but $\langle Y_{nk}\rangle \notin {}_2c_F(M,p)$. Hence the space ${}_2c_F(M,p)$ is not convergence free. Similarly the other spaces are also not convergence free.

Property 3.4. $Z(M,p) \subseteq ({}_2\ell_\infty)_F(M,p)$, *for* $Z = ({}_2c^R)_F$, $({}_2c_0^R)_F$. *The inclusions are strict.*

Proof. The result follows by the property that all regular convergence sequences are bounded.

Theorem 3.7. *The spaces* $({}_2\ell_\infty)_F(M,p)$, $({}_2c_0)_F(M,p)$, $({}_2c_0^R)_F(M,p)$ *are sequence algebras.*

Proof. Consider the sequence space $({}_2c_0)_F(M,p)$. Let $\langle X_{nk}\rangle, \langle Y_{nk}\rangle \in ({}_2c_0)_F(M,p)$. Then the result follows immediately from the inequality:

$$\{\bar{d}(X_{nk}Y_{nk}, \bar{0})\}^{p_{nk}} \le \{\bar{d}(X_{nk}, \bar{0})\}^{p_{nk}} \{\bar{d}(Y_{nk}, \bar{0})\}^{p_{nk}}.$$

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