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# The Optimum Solution of Degenerate Transportation Problem

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**Abstract**: The Solution of a Transportation Problem is obtained in two phases. The first phase is finding the initial basic feasible solution by using various methods. The optimal solution is obtained either by using stepping stone method or by MODI method in the second phase. Here we proposed the MODI method with modifications to solve the degenerate transportation problem. This is also illustrated with numerical example.

**Key Words**: Transportation problem, degeneracy, difference cost,optimum solution.

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## I. INTRODUCTION:

The Transportation model is a special case of Linear programming models, widely used in the areas of inventory control, employment scheduling, aggregate planning and personal assignment among other.It can be solved by the regular simplex metod. Due to its special structure of the model, the stepping stone method (charnes and cooper ,1954) ws developed for the efficiency reason. While the simplex method is not suitable for the transportation problem, especially for those large scale transportation problems.

Other research results can be found from Ford and Fulerson (1956), Balinski and Gomory(1964), Muller-Merback (1966), Grigoriadis and Walker (1968), Glover etal (1974), Shafoat and Goyal (1988) and Arsham and Kahn (1989). A brief review on this area was presented by Gass (1990).

The stepping stone method is very popularly used to solve transportation problem. It is a technique for moving from an initial feasible solution to an optimal solution by evaluating all non- basic cells that ie empty cells. It adopts the path tracing approach to evaluate an empty cell. Another way to evaluate empty cells s the modification distribution method (MODI). MODI method is similar to the stepping stone method.

One serious problem of the stepping stone method is the degeneracy, that is too few basic cells in a feasible solution. Some researchers carried out to solve degeneracy problem ( Goyal 1984 and Shafaat and Goyal, 1988).

The simplex degeneracy doesn't cause any serious difficulty, but it can cause computational problem in transportation technique. In stepping stone method it will not be possible to make closed paths for each and every vacant cell and hence net evaluations of all the vacant cells cannot be calculated. If MODI method is applied, it will not be possible to find all the dual variables $u_i, v_j$ ,since the number of allocated cells and their $c_{ij}$ values is not enough. It is thus necessary to identify a degenerate transportation problem and take appropriate steps to avoid computational difficulty.Degeneracy can occur in the initial solution or during some subsequent iteration.

**Degeneracy in the initial solution**: Normally, while finding the initial solution, any allocation made either satisfies supply or demand, but not both. If however,both supply and demand are satisfied simultaneously row as well as columns are cancelled simultaneously and the number of allocations become less than
$m + n - 1$.

**Degeneracy during subsequent iteration:**
Sometimes even if the starting feasible solution is non- degenerate, degeneracy may develop later at some subsequent iteration. This happens when the selection of the entering variable (least value in the closed loop that has been assigned a negative sign), causes two or more current basic variables to become zero.

**Optimality test:**
The optimality test for given Basic feasible solution of the transportation problem may be summarized as follows;

i) If all $d_{ij} \geq 0$ solution under test is optimal
ii) Alternate optimal solution exists, if none is negative but any is zero.

iii) Solution under test is not optimal, if any $d_{ij}$ is negative, then further improvement is required. Where $d_{ij} = c_{ij} - u_i - v_j$

**MODI's Algorithm**:

1. First, construct a transportation table entering the origin capacities $a_i$,the destination requirements $b_j$ and the costs$c_{ij}$ .
2. Find an initial basic feasible solution.
3. For all the basic variables $x_{ij}$ , solve the system of equation $c_{ij} + (u_i + v_j) = 0$for all $i,j$ for which cell $(i,j)$ is in the basis, starting initially with some $u_i = -c_{ij}$ and entering successfully the values of $u_i, v_j$ in the transportation table.
4. Compute the cost differences $d_{ij} = c_{ij} - u_i - v_j$
5. Apply the optimality test, if atleast one $d_{ij} < 0$ (negative) select the variable $x_{rs}$ (having the most negative $d_{rs}$ ) to enter the basis.
6. Let the variables $x_{rs}$ enter the basis. Allocate an unknown quantity $\theta$ to the cell $(r,s)$. Then construct a loop that starts and ends at the cell $(r,s)$ and connects some of the basic cells. The amount $\theta$ is added to and subtracted from the transition cells of the loop in such a manner that availabilities and requirements remain satisfied.
7. Assign the largest possible value to $\theta$ in such a way that the value of atleast one basic variable becomes zero and other basic variables remain non negative. The basic cell, whose allocation has been made zero, will leave the basis.
8. Now return to step 3 and then repeat the process until an optimum basic feasible solution is obtained.

In MODI method, if degeneracy occurs, assign $\varepsilon$ to the empty cell. To resolve the degeneracy, allocate an extremely small amount of goods to one or more of the empty cells so that a number of occupied cells becomes $m + n - 1$.The cell containing this extremely small allocation is considered to be an occupied cell, subject to the following assumptions

i) $\varepsilon < x_{ij}, x_{ij} > 0, ii) x_{ij} + \epsilon = x_{ij} - \epsilon \;\; iii) \epsilon + 0 = \epsilon$

**Proposed Method:**

1. In the case of degeneracy, we face the problem in two steps of MODI 's algorithm:

i) In finding the dual variables $u_i, v_j$ successively in the transportation table. In such a case we consider $u_i = -c_{ij}$ , $v_j = 0$ for the basic cell.

**ii)** Construct the loop,starts at the most negative cell and ends with the same cell connectingsome basiccells in such a way that at-least one of the basic cell should leave the basis in the MODI's algorithm.

**Numerical Example**:

**Table 1**

| $S_i/D_j$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | source |
|---|---|---|---|---|---|---|
| $S_1$ | 4 | 7 | 3 | 8 | 2 | 4 |
| $S_2$ | 1 | 4 | 7 | 3 | 8 | 7 |
| $S_3$ | 7 | 2 | 4 | 7 | 7 | 9 |
| $S_4$ | 4 | 8 | 2 | 4 | 7 | 2 |
| Destination | 8 | 3 | 7 | 2 | 2 | |

Step1:The initial feasible solution by North – West corner rule is in the following table . Degeneracy occurs in the initial stage. Now apply Proposed method to find the optimal solution. Non –basic cells are denoted by (.)

**Table 2**

| $S_i/D_j$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $u_i$ |
|---|---|---|---|---|---|---|
| $S_1$ | 4(4) | 7(0) | 3(-1) | 8(1) | 2(-2) | -4 |
| $S_2$ | 1(4) | 4(3) | 7(6) | 3(-1) | 8(7) | -1 |
| $S_3$ | 7(3) | 2(-5) | 4(7) | 7(2) | 7(3) | -4 |
| $S_4$ | 4(-3) | 8(-2) | 2(-5) | 4(-6) | 7(2) | -7 |
| $v_j$ | 0 | -3 | 0 | -3 | 0 | |

The most negative cost difference is in the cell (4,4) Now construct the loop by using the cells (4,4),(4,5),(3,5) and (3,4) and $\theta = \min(2,2) = 2$

| $S_i/D_j$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $u_i$ |
|---|---|---|---|---|---|---|
| $S_1$ | 4(4) | 7(0) | 3(-1) | 8(4) | 2(-5) | -4 |
| $S_2$ | 1(4) | 4(3) | 7(6) | 3(2) | 8(4) | -1 |
| $S_3$ | 7(3) | 2(-5) | 4(7) | 7(3) | 7(2) | -4 |
| $S_4$ | 4(0) | 8(1) | 2(-2) | 4(2) | 7(0) | -4 |
| $v_j$ | 0 | -3 | 0 | 0 | -3 | |

The most negative cost difference is in the cell (3,2), construct the loop by using the cells (3,2),(3,5),(1,5),(1,1),(2,1),(2,2) and (3,2). $\theta = \min(2,4,3) = 2$

| $S_i/D_j$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $u_i$ |
|---|---|---|---|---|---|---|
| $S_1$ | 4(2) | 7(0) | 3(-6) | 8(4) | 2(2) | -4 |
| $S_2$ | 1(6) | 4(1) | 7(1) | 3(2) | 8(9) | -1 |
| $S_3$ | 7(8) | 2(2) | 4(7) | 7(8) | 7(10) | 1 |
| $S_4$ | 4(0) | 8(1) | 2(-7) | 4(2) | 7(5) | -4 |
| $v_j$ | 0 | -3 | -5 | 0 | 2 | |

The most negative difference is in the cell (4,3), construct the closed loop (4,3),(4,4),(3,4) and (4,3). $\theta = \min(2,4,) = 2$

| $S_i/D_j$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $u_i$ |
|---|---|---|---|---|---|---|
| $S_1$ | 4(2) | 7(0) | 3(-6) | 8(-4) | 2(2) | -4 |
| $S_2$ | 1(6) | 4(1) | 7(1) | 3(-6) | 8(9) | -1 |
| $S_3$ | 7(8) | 2(2) | 4(5) | 7(2) | 7(10) | 1 |
| $S_4$ | 4(7) | 8(8) | 2(2) | 4(-1) | 7(5) | 3 |
| $v_j$ | 0 | -3 | -5 | -8 | 2 | |

The most negative is in the cell (1,4) and construct the loop with the cells (1,4),(1,1),(2,1),(2,2)(3,2),(3,3)and (1,4). $\theta = \min(2,1,5) = 1$

| $S_i/D_j$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $u_i$ |
|---|---|---|---|---|---|---|
| $S_1$ | 4(1) | 7(6) | 3(1) | 8(2) | 2(2) | -4 |
| $S_2$ | 1(7) | 4(6) | 7(7) | 3(0) | 8(9) | -1 |
| $S_3$ | 7(2) | 2(3) | 4(4) | 7(2) | 7(4) | -5 |
| $S_4$ | 4(1) | 8(8) | 2(2) | 4(-1) | 7(6) | -3 |
| $v_j$ | 0 | 3 | 1 | -2 | 2 | |

The most negative is in the cell (4,4). Construct the closed loop with the cells (4,4),(4,3),(3,3),(3,4) and (4,4). $\theta = \min(2,2) = 2$

| $S_i/D_j$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $u_i$ |
|---|---|---|---|---|---|---|
| $S_1$ | 4(1) | 7(6) | 3(1) | 8(3) | 2(2) | -4 |
| $S_2$ | 1(7) | 4(6) | 7(7) | 3(1) | 8(9) | -1 |
| $S_3$ | 7(2) | 2(3) | 4(6) | 7(1) | 7(4) | -5 |
| $S_4$ | 4(1) | 8(8) | 2(0) | 4(2) | 7(6) | -3 |
| $v_j$ | 0 | 3 | 1 | -1 | 2 | |

Since all non basic cells have non negative cost differences . The optimal solution is $x_{11} = 1, x_{13} = 1, x_{15} = 2, x_{21} = 7, x_{32} = 3, x_{33} = 6, x_{43} = 0, x_{44} = 2$
The total cost is 56.

## II. CONCLUSION:

In MODI method if degeneracy occurs, then we take the small number $\varepsilon$ to avoid the two problems finding the $u_i, v_j$ values and construction of loop for the most negative cost difference. The suggested method ensures improvement of the solution or recognition of its optimality, thereby avoiding unnecessary iterations that resulting in shifting of the $\varepsilon$ from an independent cell to another.

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