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Propagation of Stoneley Waves at an Interface between Two Transversely Isotropic Micropolar Elastic Solid Media

Rahul Hooda[1], Asha Sangwan[2], Shankar Gulia[3]
[1]*Assistant Professor, Department of Mathematics, A.I.J.H.M. College, Rohtak-124001, Haryana, India*
[2]*Assistant Professor, Department of Mathematics, Govt. College, Sampla, Rohtak-124001, Haryana, India*
[3] *Research Scholar, Department of Mathematics, I.G.University, Meerpur(Rewari)-123401, Haryana, India*
Corresponding Author: Asha Sangwan[2]

Abstract: The aim of the present paper is to investigate the propagation of Stoneley waves at the interface of two dissimilar transversely isotropic micropolar solid media. The basic equations are solved to obtain the general surface wave solutions in the medium in x-z plane. Following radiation conditions in the media, the particular solutions are obtained, which satisfied the appropriate boundary conditions at an interface to obtain the secular equations of the Stoneley wave in media. Limiting case of Rayleigh wave is also deduced from the present investigation.

Keywords: Stoneley wave, Rayleigh wave, secular equation, transverse isotropy, micropolar.

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I. INTRODUCTION

The exact number of the layers beneath the earth's surface is not known. One has, therefore, to consider various appropriate models for the purpose of theoretical investigations. These models not only provide better information about the internal composition of the earth but also helpful in exploration of valuable materials beneath the earth surface. Mathematical modelling of surface wave propagation along with the free boundary of an elastic half-space or along the interface between two dissimilar elastic half-spaces has been subject of continued interest for many years. These waves are well known in the study of geophysics, ocean acoustics, SAW devices and non-destructive evaluation.

There is a rich literature available in surface waves in terms of classical elasticity [1-7]. Surface wave propagating along the free boundary of an elastic half-space, non-attenuated in their direction of propagation and damped normal to the boundary are known as Rayleigh waves in the literature, after their discovery by Rayleigh [5]. Stoneley [7] studied the existence of waves, which are similar to surface waves and propagating along the plane interface between two distinct elastic solid half-spaces in perfect contact. Stoneley waves can also propagate on interfaces either two elastic media or a solid medium and a liquid medium. Stoneley [7] derived the dispersion equation for the propagation of Stoneley waves. Since then a number of problems concerning the propagation of Stoneley waves along the solid-solid and fluid-soild boundary have been discussed by several researchers, including Strick and Ginzbarg [9], Lim and Musgrave [10], Chadwick and Currie [11], Abbudi and Barnet [12], Goda [13], Tajuddin [14], Abd-Alla [15].

Abd-Alla and Ahmed [16] studied the Stoneley and Rayleigh waves in an inhomogeneous orthotropic elastic medium under the influence of gravity field. Tomar and Singh [17] discussed the propagation of Stoneley waves at an interface between two microstretch elastic half-spaces. Markov [18] discussed propagation of Stoneley wave at the boundary of two fluid-saturated porous media. Singh and Renu [19] studied the surface wave propagation in an initially stressed transversely isotropic thermoelastic solid. Abd-Alla et al. [20] investigated the propagation of surface waves in a rotating fibre-reinforced viscoelastic anisotropic media of a higher order and fraction orders of nth order including time rate of strain with voids. Singh and Sindhu [21] studied the propagation of Rayleigh wave in micropolar piezoelectric medium. Kumar et.al. [22] discussed the propagation of Stoneley wave in transversely isotropic thermoelastic media.

In the present paper, we solved the governing equations of transversely isotropic micropolar medium analytically for surface wave solutions, where the particular solutions in the half-spaces are applied at required boundary conditions at the interface to obtain the secular equation for Stoneley wave in the present model. The secular equation for the Rayleigh wave is obtained as a limiting case.

II. BASIC EQUATIONS

Following Iesan [8], the equations of motion for a homogeneous transversely isotropic micropolar solid in the absence of body forces and body couples consists are

$$\sigma_{ji,j}=\rho \ddot{u}_i \tag{1}$$

$$m_{ik,i}+\varepsilon_{ijk}\sigma_{ij}=\rho j\ddot{\phi}_k \quad (i,j,k=1,2,3) \tag{2}$$

The constitutive equations are written as

$$\sigma_{ij}=A_{ijkl}e_{kl}+G_{ijkl}\psi_{kl} \tag{3}$$

$$m_{ij}=G_{klij}e_{kl}+B_{ijkl}\psi_{kl} \tag{4}$$

The geometrical equations are written as

$$e_{ij}=u_{j,i}+\varepsilon_{ijk}\phi_k\,,\ \ \psi_{ij}=\phi_{j,i} \tag{5}$$

where σ_{ij} is the stress tensor, ρ is the mass density, $\vec{u}$ is the displacement vector, $\vec{\phi}$ is the microrotation vector, j is the micro-inertia, m_{ij} is the couple stress tensor, ε_{ijk} is the alternating symbol, e_{ij} and ψ_{ij} are kinematic strain measures and A_{ijkl}, B_{ijkl} and G_{ijkl} are constitutive coefficients. Superposed dot denote partial differentiation with respect to the time t. Subscripts preceded by a comma denote partial differentiation with respect to the corresponding cartesian coordinate and the repeated index in the subscript implies summation.

III. FORMULATION OF THE PROBLEM

Let M and M' be two dissimilar homogeneous, transversely isotropic micropolar solid media. They are perfectly welded in contact as shown in Fig. 1. These two media extend to an infinite great distance from the origin and are separated by a plane horizontal boundary and M' is to be taken above M. The origin of the Cartesian coordinate system (x,y,z) is taken at any point on the plane interface and z-axis pointing vertically downwards into M is taken which is designated as $z \geq 0$.

We consider the possibility of a type of wave travelling in the x-direction in such a manner that the disturbance is largely confined to the neighbourhood of the boundary which implies that the wave is a surface wave. The present study is restricted to the plane strain parallel to x-z plane. For two-dimensional problem, the displacement vector $\vec{u}'$ and microrotation vector $\vec{\phi}'$ in medium M' and displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$ in medium M are taken as

$$\vec{u}'=\left(u_1',0,u_3'\right),\ \vec{\phi}'=\left(0,\phi_2',0\right),\ \vec{u}=\left(u_1,0,u_3\right)\ \text{ and }\ \vec{\phi}=\left(0,\phi_2,0\right) \tag{6}$$

Using eqs. (1) to (6), the equations of motion for transversely isotropic micropolar medium M are expressed as

$$A_{11}\frac{\partial^2 u_1}{\partial x^2}+\left(A_{13}+A_{56}\right)\frac{\partial^2 u_3}{\partial x\partial z}+A_{55}\frac{\partial^2 u_1}{\partial z^2}+K_1\frac{\partial \phi_2}{\partial z}=\rho\frac{\partial^2 u_1}{\partial t^2} \tag{7}$$

$$A_{66}\frac{\partial^2 u_3}{\partial x^2}+\left(A_{13}+A_{56}\right)\frac{\partial^2 u_1}{\partial x\partial z}+A_{33}\frac{\partial^2 u_3}{\partial z^2}+K_2\frac{\partial \phi_2}{\partial x}=\rho\frac{\partial^2 u_3}{\partial t^2} \tag{8}$$

$$B_{77}\frac{\partial^2 \phi_2}{\partial x^2}+B_{66}\frac{\partial^2 \phi_2}{\partial z^2}-\chi\phi_2-K_1\frac{\partial u_1}{\partial z}-K_2\frac{\partial u_3}{\partial x}=\rho j\frac{\partial^2 \phi_2}{\partial t^2} \tag{9}$$

where

$$K_1=A_{56}-A_{55},K_2=A_{66}-A_{56},\chi=K_2-K_1 \tag{10}$$

Similarly, we can get similar equations of motion for medium M' with all the parameters in medium M' are denoted by super script “dash”.

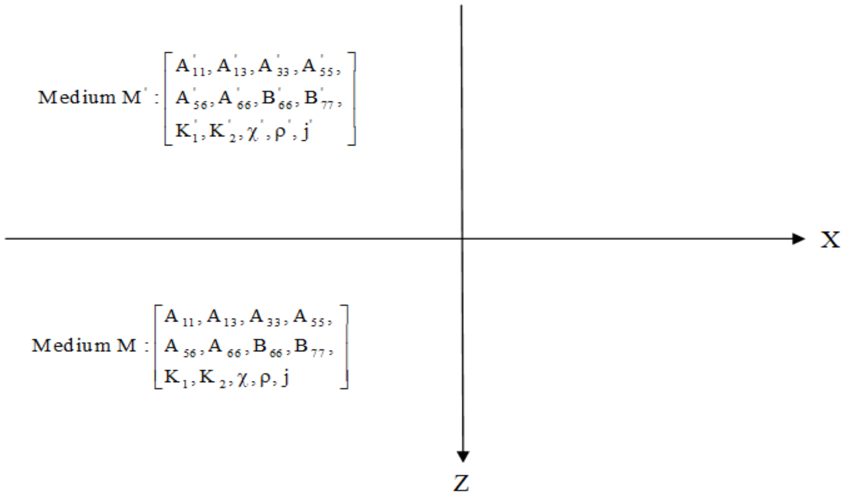

Fig 1. Geometry of the problem

IV. SOLUTION OF THE PROBLEM

We seek the surface wave solution of the eqs. (7) to (9) in x-z plane in the following form

$$\{u_1, u_3, \phi_2\} = \{\bar{u}_1(z), \bar{u}_3(z), \bar{\phi}_2(z)\}\exp.\{i\xi(x - ct)\} \tag{11}$$

where ξ is the wave number and c is phase velocity of the wave,

Making use of eq. (11) in eqs. (7-9) and applying the radiation conditions $u_1 \to 0,\ u_3 \to 0, \phi_2 \to 0\ as\ z \to \infty$, we obtain the following particular solutions for medium M

$$u_1 = \left(\sum_{j=1}^{3} A_j e^{-m_j z}\right) e^{i\xi(x-ct)} \tag{12}$$

$$u_3 = \left(\sum_{j=1}^{3} \zeta_j A_j e^{-m_j z}\right) e^{i\xi(x-ct)} \tag{13}$$

$$\phi_2 = \left(\sum_{j=1}^{3} \eta_j A_j e^{-m_j z}\right) e^{i\xi(x-ct)} \tag{14}$$

and similarly, for medium M'

$$u_1' = \left(\sum_{j=1}^{3} A_j' e^{m_j' z}\right) e^{i\xi(x-ct)} \tag{15}$$

$$u_3' = \left(\sum_{j=1}^{3} \zeta_j' A_j' e^{m_j' z}\right) e^{i\xi(x-ct)} \tag{16}$$

$$\phi_2' = \left(\sum_{j=1}^{3} \eta_j' A_j' e^{m_j' z}\right) e^{i\xi(x-ct)} \tag{17}$$

where the expressions for coupling coefficients $\zeta_j,\ \eta_j\ (j = 1,2,3)$ and the relations between $m_j\,(j = 1,2,3)$ are given as follow:

$$m_1^2 + m_2^2 + m_3^2 = \frac{S_1}{S_0}$$

$$m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2 = \frac{S_2}{S_0}$$

$$m_1^2 m_2^2 m_3^2 = \frac{S_3}{S_0}$$

where

$S_0 = A_{33}A_{55}B_{66},$

$S_1 = \left(A_{33}A_{55}P + A_{55}B_{66}N + A_{33}B_{66}L - B_{66}M^2\right)\xi^2 - A_{33}K_1^2,$

$S_2 = \left(A_{33}LP + A_{55}NP + B_{66}LN - PM^2\right)\xi^4 + \left(2K_1K_2M - K_1^2N - K_2^2A_{55}\right)\xi^2,\ S_3 = LNP\,\xi^6 - LK_2^2\,\xi^4,$

$L = \left(A_{11} - \rho c^2\right),\ M = \left(A_{13} + A_{56}\right),\quad N = \left(A_{11} - \rho c^2\right),\ P = \left(B_{77} - \rho jc^2 + \dfrac{\chi}{\xi^2}\right),$

The expressions for $\zeta_j,\ \eta_j\ \left(j = 1,2,3\right)$ are given as

$$\zeta_j = i\left[\frac{\left(\dfrac{m_j^2}{\xi^2}\dfrac{M}{A_{55}}\right) - \dfrac{K_2}{K_1}\left(\dfrac{m_j^2}{\xi^2} - \dfrac{L}{A_{55}}\right)}{\dfrac{K_2}{K_1}\dfrac{m_j}{\xi}\dfrac{M}{A_{55}} + \dfrac{m_j}{\xi}\dfrac{A_{33}}{A_{55}}\left(\dfrac{m_j^2}{\xi^2} - \dfrac{N}{A_{33}}\right)}\right],\qquad \left(j = 1,2,3\right)$$

$$\frac{\eta_j}{\xi} = \left[\frac{\left(\dfrac{m_j^2}{\xi^2} - \dfrac{L}{A_{55}}\right)\left(\dfrac{m_j^2}{\xi^2} - \dfrac{N}{A_{33}}\right) + \dfrac{m_j^2}{\xi^2}\dfrac{M^2}{A_{33}A_{55}}}{\dfrac{K_2}{A_{33}}\dfrac{m_j}{\xi}\dfrac{M}{A_{55}} + \dfrac{m_j}{\xi}\dfrac{K_1}{A_{55}}\left(\dfrac{m_j^2}{\xi^2} - \dfrac{N}{A_{33}}\right)}\right],\qquad \left(j = 1,2,3\right)$$

The quantities $\zeta_j^{'},\ \eta_j^{'},\ m_j^{'}(j = 1,2,3)$ are defined in the same way as their counterparts without superscript dash.

V. BOUNDARY CONDITIONS

The appropriate boundary conditions at an interface z=0 are continuity of components of displacement, continuity of component of microrotation, the continuity of normal and tangential force stress components and continuity of tangential couple stress component i.e.,

$$u_1 = u_1^{'},\ u_3 = u_3^{'},\ \phi_2 = \phi_2^{'},\ \sigma_{33} = \sigma_{33}^{'},\ \sigma_{31} = \sigma_{31}^{'},\ m_{32} = m_{32}^{'}, \tag{18}$$

where

$\sigma_{33}^{'} = A_{13}^{'}u_{1,1}^{'} + A_{33}^{'}u_{3,3}^{'}\quad \sigma_{33} = A_{13}u_{1,1} + A_{33}u_{3,3}$

$\sigma_{31}^{'} = A_{56}^{'}u_{3,1}^{'} + A_{55}^{'}u_{1,3}^{'} + \left(A_{56}^{'} - A_{55}^{'}\right)\phi_2^{'},$

$\sigma_{31} = A_{56}u_{3,1} + A_{55}u_{1,3} + \left(A_{56} - A_{55}\right)\phi_2,$

$m_{32}^{'} = B_{66}^{'}\phi_{2,3}^{'},\quad m_{32} = B_{66}\phi_{2,3}.$

Here, the symbols with superscript dash correspond to medium $M^{'}$

Boundary conditions imply the following equations

$$\sum_{j=1}^{3} A_j = \sum_{j=1}^{3} A_j^{'} \tag{19}$$

$$\sum_{j=1}^{3} \zeta_j A_j = \sum_{j=1}^{3} \zeta_j^{'} A_j^{'} \tag{20}$$

$$\sum_{j=1}^{3} \eta_j A_j = \sum_{j=1}^{3} \eta_j^{'} A_j^{'} \tag{21}$$

$$\sum_{j=1}^{3}\left(i\xi A_{13} - m_j\zeta_j A_{33}\right)A_j = \sum_{j=1}^{3}\left(i\xi A_{13}^{'} + m_j^{'}\zeta_j^{'}A_{33}^{'}\right)A_j^{'} \tag{22}$$

$$\sum_{j=1}^{3}\left\{i\xi\zeta_j A_{56} - m_j A_{55} + \left(A_{56} - A_{55}\right)\eta_j\right\}A_j = \sum_{j=1}^{3}\left\{i\xi\zeta_j^{'}A_{56}^{'} + m_j^{'}A_{55}^{'} + \left(A_{56}^{'} - A_{55}^{'}\right)\eta_j^{'}\right\}A_j^{'} \tag{23}$$

$$\sum_{j=1}^{3}\left(m_j\eta_j B_{66}\right)A_j = \sum_{j=1}^{3}\left(-m_j^{'}n_j^{'}B_{66}^{'}\right)A_j^{'} \tag{24}$$

VI. STONELEY WAVE

Elimination of constants A_j and $A_j^{'}\left(j = 1,2,3\right)$ from eqs.(19-24) gives the following secular equation for the surface wave in transversely isotropic micropolar solid media i.e

$$\det(a_{ij}) = 0;\qquad (i, j = 1,2,...,6) \tag{25}$$

where

$a_{11} = 1, a_{12} = 1, a_{13} = 1, a_{14} = -1, a_{15} = -1, a_{16} = -1,$

$a_{21}=\zeta_1, a_{22}=\zeta_2, a_{23}=\zeta_3, a_{24}=-\zeta_1', a_{25}=-\zeta_2', a_{26}=-\zeta_3',$

$a_{31}=\eta_1, a_{32}=\eta_2, a_{33}=\eta_3, a_{34}=-\eta_1', a_{35}=-\eta_2', a_{36}=-\eta_3',$

$$a_{4j}=\begin{cases}\left(i\xi A_{13}-m_j\zeta_j A_{33}\right), & (j=1,2,3)\\ -\left(i\xi A_{13}'+m_{j-3}'\zeta_{j-3}'A_{33}'\right), & (j=4,5,6)\end{cases}$$

$$a_{5j}=\begin{cases}\left\{i\xi\zeta_j A_{56}-m_j A_{55}+\left(A_{56}-A_{55}\right)\eta_j\right\}, & (j=1,2,3)\\ -\left\{i\xi\zeta_{j-3}'A_{56}'+m_{j-3}'A_{55}'+\left(A_{56}'-A_{55}'\right)\eta_{j-3}'\right\}, & (j=4,5,6)\end{cases}$$

$$a_{6j}=\begin{cases}m_j\eta_j B_{66}, & (j=1,2,3)\\ m_{j-3}'n_{j-3}'B_{66}', & (j=4,5,6)\end{cases}$$

The eq. (25) is the secular equation for Stoneley waves in transversely isotropic micropolar solid media.

VII. RAYLEIGH WAVE AS LIMITING CASE

Rayleigh wave is a special case of the above general surface wave. In this case, we consider a model where the medium M' is replaced by vacuum. Since the boundary $z=0$ is adjacent to vacuum. It is free from traction. So the boundary conditions in this case may be expressed as

$\sigma_{33}=0,\ \sigma_{31}=0,\ m_{32}=0,$

Thus the set of eqs. (19-24) reduces to

$$\sum_{j=1}^{3}\left(i\xi A_{13}-m_j\zeta_j A_{33}\right)A_j=0 \tag{26}$$

$$\sum_{j=1}^{3}\left\{i\xi\zeta_j A_{56}-m_j A_{55}+\left(A_{56}-A_{55}\right)\eta_j\right\}A_j=0 \tag{27}$$

$$\sum_{j=1}^{3}\left(m_j\eta_j B_{66}\right)A_j=0 \tag{28}$$

Eliminating the constants $A_j\ (j=1,2,3)$, we get the wave velocity equation for Rayleigh waves in the transversely isotropic micropolar case as follow:

$$\det(b_{ij})=0; \quad (i,j=1,2,3) \tag{29}$$

where

$b_{1j}=\left(i\xi A_{13}-m_j\zeta_j A_{33}\right), \quad (j=1,2,3)$

$b_{2j}=i\xi\zeta_j A_{56}-m_j A_{55}+\left(A_{56}-A_{55}\right)\eta_j, \quad (j=1,2,3)$

$b_{3j}=m_j\eta_j B_{66}, \quad (j=1,2,3)$

The eq. (29) is the secular equation for Rayleigh waves in transversely isotropic micropolar solid half-space.

VIII. CONCLUSION

Assuming the components of the displacement and microrotation vectors in the form $\vec{u}=\left(u_1,0,u_3\right)$ and $\vec{\phi}=\left(0,\phi_2,0\right)$, the governing equations given in Iesan [8] are derived as a special case for transversely isotropic micropolar medium in x-z plane. Stoneley and Rayleigh type surface waves are studied in this medium. The secular equation of Stoneley and Rayleigh waves in transversely isotropic micropolar media is obtained. The theoretical results indicate that the speed of the surface wave depends on various material parameters. Present analytical solutions can be used to find numerically the speed of the Stoneley and Rayleigh waves for a particular material modelled as a transversely isotropic micropolar material. The results in this paper should prove useful in the field of material science, designers of new materials as well as for those working on the development of theory of elasticity.

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