

Mathematical Modelling of Stoneley Wave in Rotating Orthotropic Micropolar Elastic Solid Media

Rahul Hooda¹, Shankar Gulia², Sikander³

¹Assistant Professor, Department of Mathematics, A.I.J.H.M. College, Rohtak-124001, Haryana, India

²Research Scholar, Department of Mathematics, I.G.University, Meerpur (Rewari)-123401, Haryana, India

³Assistant Professor, Department of Mathematics, A.I.J.H.M. College, Rohtak-124001, Haryana, India

Abstract: The present paper deals with the study of rotation effect on the propagation of Stoneley waves at the interface of two dissimilar orthotropic micropolar solid media in the framework of linear theory including Coriolis and Centrifugal forces. The basic equations are solved to obtain the general surface wave solutions of the medium in x-y plane. The medium is rotating about an axis perpendicular to its plane. Following radiation conditions in the media, the particular solutions are obtained, which satisfied the appropriate boundary conditions at an interface to obtain the secular equations of the Stoneley wave in media. Limiting case of Rayleigh wave is also deduced from the present investigation.

Keywords: Stoneley wave, Rayleigh wave, rotation, secular equation, orthotropic, micropolar.

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I. INTRODUCTION

Problem of surface waves in an orthotropic elastic medium is very important for the possibility of its extensive applications in various branches of science and technology, particularly in Optics, Earthquake science, Acoustics, Geophysics and Plasma Physics. The exact number of the layers beneath the earth's surface is not known. One has, therefore, to consider various appropriate models for the purpose of theoretical investigations. These models not only provide better information about the internal composition of the earth but also helpful in exploration of valuable materials beneath the earth surface. Mathematical modelling of surface wave propagation along with the free boundary of an elastic half-space or along the interface between two dissimilar elastic half-spaces has been subject of continued interest for many years. These waves are well known in the study of geophysics, ocean acoustics, SAW devices and non-destructive evaluation.

There is a rich literature available in surface waves in terms of classical elasticity [1-7]. To explain the fundamental departure of microcontinuum theories from the classical continuum theories, a continuum model with microstructures to describe the microscopic motion or a non local model to describe the long range material interaction is developed. This theory extends the application of the continuum model to microscopic space and short-time scales.

Surface wave propagating along the free boundary of an elastic half-space, non-attenuated in their direction of propagation and damped normal to the boundary are known as Rayleigh waves in the literature, after their discovery by Rayleigh [5]. Stoneley [7] studied the existence of waves, which are similar to surface waves and propagating along the plane interface between two distinct elastic solid half-spaces in perfect contact. Stoneley waves can also propagate on interfaces either two elastic media or a solid medium and a liquid medium. Stoneley [7] derived the dispersion equation for the propagation of Stoneley waves. Since then a number of problems concerning the propagation of Stoneley waves along the solid-solid and fluid-solid boundary have been discussed by several researchers, including Strick and Ginzburg [10], Lim and Musgrave [11], Chadwick and Currie [12], Abbudi and Barnett [13], Goda [14], Tajuddin [15] and Abd-Alla [16].

Abd-Alla and Ahmed [17] studied the Stoneley and Rayleigh waves in an inhomogeneous orthotropic elastic medium under the influence of gravity field. Tomar and Singh [18] discussed the propagation of Stoneley waves at an interface between two microstretch elastic half-spaces. Markov [19] discussed propagation of Stoneley wave at the boundary of two fluid-saturated porous media. Singh and Renu [20] studied the surface wave propagation in an initially stressed transversely isotropic thermoelastic solid. Abd-Alla et al. [21] investigated the propagation of surface waves in a rotating fibre-reinforced viscoelastic anisotropic media of a higher order and fraction orders of nth order including time rate of strain with voids. Singh and Sindhu [22] studied the propagation of Rayleigh wave in micropolar piezoelectric medium. Kumar et al. [23] discussed the

propagation of Stoneley wave in transversely isotropic thermoelastic media. Recently, Hooda et al. [24] studied propagation of Stoneley waves in transversely isotropic micropolar elastic solid media.

In the present paper, we solved the governing equations of rotating orthotropic micropolar medium analytically for surface wave solutions, where the particular solutions in the half-spaces are applied at required boundary conditions at the interface to obtain the secular equation for Stoneley wave in the present model. The secular equation for the Rayleigh wave is obtained as a limiting case.

II. BASIC EQUATIONS

Following Iesan [8] and Schoenberg and Censor [9], the equations of motion for a homogeneous rotating orthotropic micropolar solid in the absence of body forces and body couples consists are

$$\sigma_{ji,j} = \rho \left[\ddot{u}_i + \left\{ \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) \right\}_i + (2\vec{\Omega} \times \vec{u})_i \right], \quad (1)$$

$$m_{i,k,i} + \varepsilon_{ijk} \sigma_{ij} = \rho j \ddot{\phi}_k \quad (i, j, k = 1, 2, 3) \quad (2)$$

The constitutive equations are written as

$$\sigma_{ij} = A_{ijkl} e_{kl} + G_{ijkl} \psi_{kl} \quad (3)$$

$$m_{ij} = G_{klij} e_{kl} + B_{ijkl} \psi_{kl} \quad (4)$$

The geometrical equations are written as

$$e_{ij} = u_{j,i} + \varepsilon_{ijk} \phi_k, \quad \psi_{ij} = \phi_{j,i} \quad (5)$$

where σ_{ij} is the stress tensor, ρ is the mass density, \vec{u} is the displacement vector, $\vec{\phi}$ is the microrotation vector, j is the micro-inertia, m_{ij} is the couple stress tensor, ε_{ijk} is the alternating symbol, e_{ij} and ψ_{ij} are kinematic strain measures and A_{ijkl} , B_{ijkl} and G_{ijkl} are constitutive coefficients. The solid is rotating uniformly with an angular velocity $\vec{\Omega} = \Omega \hat{n}$, where \hat{n} is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in rotating frame has two additional terms: Centripetal acceleration is $\vec{\Omega} \times (\vec{\Omega} \times \vec{u})$ due to time varying motion only and $(2\vec{\Omega} \times \vec{u})$ is the coriolis acceleration, where $\vec{u} = (u, v, w)$ is the dynamic displacement vector. These terms do not appear in non-rotating media. Superposed dot denotes partial differentiation with respect to the time t . Subscripts preceded by a comma denote partial differentiation with respect to the corresponding cartesian coordinate and the repeated index in the subscript implies summation.

III. FORMULATION OF THE PROBLEM

Let M and M' be two dissimilar rotating orthotropic micropolar solid media. They are perfectly welded in contact as shown in Fig. 1. These two media extend to an infinite great distance from the origin and are separated by a plane horizontal boundary and M' is to be taken above M . The origin of the Cartesian coordinate system (x, y, z) is taken at any point on the plane interface and y -axis pointing vertically downwards into M is taken which is designated as $y \geq 0$.

We consider the possibility of a type of wave travelling in the x -direction in such a manner that the disturbance is largely confined to the neighbourhood of the boundary which implies that the wave is a surface wave. The present study is restricted to the plane strain parallel to x - y plane. It is assumed that the entire half-space is rotating with constant angular rate Ω about z -axis. For two-dimensional problem, the displacement vector \vec{u}' and microrotation vector $\vec{\phi}'$ in medium M' and displacement vector \vec{u} and microrotation vector $\vec{\phi}$ in medium M are taken as

$$\vec{u}' = (u'_1, u'_2, 0), \quad \vec{\phi}' = (0, 0, \phi'_3), \quad \vec{u} = (u_1, u_2, 0) \quad \text{and} \quad \vec{\phi} = (0, 0, \phi_3) \quad (6)$$

Using eqs. (1) to (6), the equations of motion for rotating orthotropic micropolar medium M are expressed as

$$A_{11} \frac{\partial^2 u_1}{\partial x^2} + (A_{12} + A_{78}) \frac{\partial^2 u_2}{\partial x \partial y} + A_{88} \frac{\partial^2 u_1}{\partial y^2} - K_1 \frac{\partial \phi_3}{\partial y} = \rho \left(\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 - 2\Omega \frac{\partial u_2}{\partial t} \right) \quad (7)$$

$$A_{77} \frac{\partial^2 u_2}{\partial x^2} + (A_{12} + A_{78}) \frac{\partial^2 u_1}{\partial x \partial y} + A_{22} \frac{\partial^2 u_2}{\partial y^2} - K_2 \frac{\partial \phi_3}{\partial x} = \rho \left(\frac{\partial^2 u_2}{\partial t^2} - \Omega^2 u_2 + 2\Omega \frac{\partial u_1}{\partial t} \right) \quad (8)$$

$$B_{66} \frac{\partial^2 \phi_3}{\partial x^2} + B_{44} \frac{\partial^2 \phi_3}{\partial y^2} - \chi \phi_3 + K_1 \frac{\partial u_1}{\partial y} + K_2 \frac{\partial u_2}{\partial x} = \rho j \frac{\partial^2 \phi_3}{\partial t^2} \quad (9)$$

where

$$K_1 = A_{78} - A_{88}, K_2 = A_{77} - A_{78}, \chi = K_2 - K_1 \tag{10}$$

Similarly, we can get equations of motion for medium M' with all the parameters in medium M' are denoted by super script "dash".

$$A_{11}' \frac{\partial^2 u_1'}{\partial x^2} + (A_{12}' + A_{78}') \frac{\partial^2 u_2'}{\partial x \partial y} + A_{88}' \frac{\partial^2 u_1'}{\partial y^2} - K_1' \frac{\partial \phi_3'}{\partial y} = \rho' \left(\frac{\partial^2 u_1'}{\partial t^2} - \Omega'^2 u_1' - 2\Omega' \frac{\partial u_2'}{\partial t} \right) \tag{11}$$

$$A_{77}' \frac{\partial^2 u_2'}{\partial x^2} + (A_{12}' + A_{78}') \frac{\partial^2 u_1'}{\partial x \partial y} + A_{22}' \frac{\partial^2 u_2'}{\partial y^2} - K_2' \frac{\partial \phi_3'}{\partial x} = \rho' \left(\frac{\partial^2 u_2'}{\partial t^2} - \Omega'^2 u_2' + 2\Omega' \frac{\partial u_1'}{\partial t} \right) \tag{12}$$

$$B_{66}' \frac{\partial^2 \phi_3'}{\partial x^2} + B_{44}' \frac{\partial^2 \phi_3'}{\partial y^2} - \chi' \phi_3' + K_1' \frac{\partial u_1'}{\partial y} + K_2' \frac{\partial u_2'}{\partial x} = \rho' j' \frac{\partial^2 \phi_3'}{\partial t^2} \tag{13}$$

$$\text{where } K_1' = A_{78}' - A_{88}', K_2' = A_{77}' - A_{78}', \chi' = K_2' - K_1' \tag{14}$$

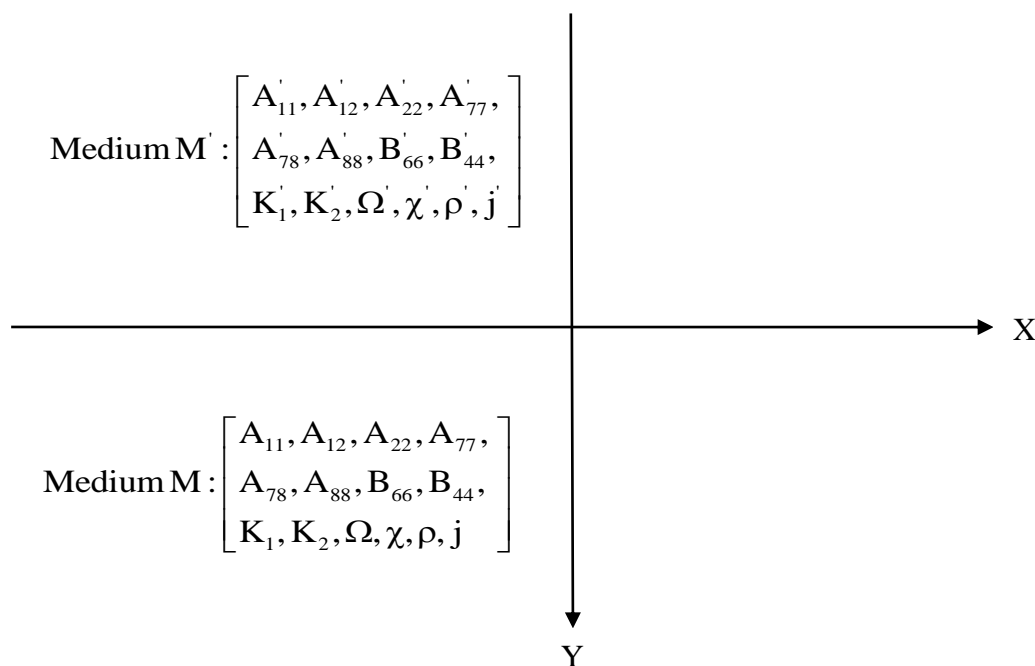


Fig. 1 Geometry of the Problem

IV. SOLUTION OF THE PROBLEM

We seek the surface wave solution of the eqs. (7) to (9) in x-y plane in the following form

$$\{u_1, u_2, \phi_3\} = \{\bar{u}_1(y), \bar{u}_2(y), \bar{\phi}_3(y)\} \exp\{i\xi(x - ct)\} \tag{15}$$

where ξ is the wave number and c is phase velocity of the wave,

Making use of eq. (15) in eqs. (7-9) and applying the radiation conditions $u_1 \rightarrow 0, u_2 \rightarrow 0, \phi_3 \rightarrow 0$ as $y \rightarrow \infty$, we obtain the following particular solutions for medium M

$$u_1 = \left(\sum_{j=1}^3 A_j e^{-m_j y} \right) e^{i\xi(x-ct)} \tag{16}$$

$$u_2 = \left(\sum_{j=1}^3 \zeta_j A_j e^{-m_j y} \right) e^{i\xi(x-ct)} \tag{17}$$

$$\phi_3 = \left(\sum_{j=1}^3 \eta_j A_j e^{-m_j y} \right) e^{i\xi(x-ct)} \tag{18}$$

where the expressions for coupling coefficients ζ_j, η_j ($j=1,2,3$) and the relations between m_j ($j=1,2,3$) are given as follow:

$$m_1^2 + m_2^2 + m_3^2 = \frac{S_1}{S_0}$$

$$m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2 = \frac{S_2}{S_0}$$

$$m_1^2 m_2^2 m_3^2 = \frac{S_3}{S_0}$$

where

$$S_0 = A_{22} A_{88} B_{44},$$

$$S_1 = (A_{22} A_{88} P + A_{88} B_{44} N + A_{22} B_{44} L - B_{44} M^2) \xi^2 - A_{22} K_1^2,$$

$$S_2 = \left[A_{22} LP + A_{88} NP + B_{44} LN - PM^2 - 4(\rho c^2)^2 \left(\frac{\Omega}{\omega} \right)^2 B_{44} \right] \xi^4 + (2K_1 K_2 M - K_1^2 N - K_2^2 A_{88}) \xi^2,$$

$$S_3 = \left[LNP - 4(\rho c^2)^2 \left(\frac{\Omega}{\omega} \right)^2 P \right] \xi^6 - LK_2^2 \xi^4,$$

$$L = (A_{11} - \rho c^2 \Omega^*), \quad M = (A_{12} + A_{78}), \quad N = (A_{77} - \rho c^2 \Omega^*), \quad P = \left(B_{66} - \rho j c^2 + \frac{\chi}{\xi^2} \right), \quad \Omega^* = 1 + \frac{\Omega^2}{\omega^2}.$$

The expressions for ζ_j, η_j ($j=1,2,3$) are given as

$$\zeta_j = i \frac{\left[\frac{m_j}{\xi} \left(\frac{m_j}{\xi} \frac{M}{A_{88}} - 2 \frac{\rho c^2}{A_{88}} \frac{\Omega}{\omega} \right) - \frac{K_2}{K_1} \left(\frac{m_j^2}{\xi^2} - \frac{L}{A_{88}} \right) \right]}{\left[\frac{K_2}{K_1} \left(\frac{m_j}{\xi} \frac{M}{A_{88}} + 2 \frac{\rho c^2}{A_{88}} \frac{\Omega}{\omega} \right) + \frac{m_j}{\xi} \frac{A_{22}}{A_{88}} \left(\frac{m_j^2}{\xi^2} - \frac{N}{A_{22}} \right) \right]}, \quad (j=1,2,3)$$

$$\frac{\eta_j}{\xi} = \frac{\left[\frac{m_j^2}{\xi^2} \frac{M^2}{A_{22} A_{88}} - 4 \frac{(\rho c^2)^2}{A_{22} A_{88}} \left(\frac{\Omega}{\omega} \right)^2 - \left(\frac{m_j^2}{\xi^2} - \frac{L}{A_{88}} \right) \left(\frac{m_j^2}{\xi^2} - \frac{N}{A_{22}} \right) \right]}{\left[\frac{K_2}{A_{88}} \left(\frac{m_j}{\xi} \frac{M}{A_{22}} + 2 \frac{\rho c^2}{A_{22}} \frac{\Omega}{\omega} \right) + \frac{m_j}{\xi} \frac{K_1}{A_{88}} \left(\frac{m_j^2}{\xi^2} - \frac{N}{A_{22}} \right) \right]}, \quad (j=1,2,3)$$

and similarly, for medium M'

$$u_1' = \left(\sum_{j=1}^3 A_j' e^{m_j y} \right) e^{i\xi(x-ct)} \tag{19}$$

$$u_2' = \left(\sum_{j=1}^3 \zeta_j' A_j' e^{m_j y} \right) e^{i\xi(x-ct)} \tag{20}$$

$$\phi_3' = \left(\sum_{j=1}^3 \eta_j' A_j' e^{m_j y} \right) e^{i\xi(x-ct)} \tag{21}$$

The quantities ζ_j', η_j', m_j' ($j=1,2,3$) are defined in the same way as their counterparts without superscript dash.

V. BOUNDARY CONDITIONS

The appropriate boundary conditions at an interface $y = 0$ are continuity of components of displacement, continuity of component of microrotation, the continuity of normal and tangential force stress components and continuity of tangential couple stress component i.e.,

$$u_1 = u_1', \quad u_2 = u_2', \quad \phi_3 = \phi_3', \quad \sigma_{22} = \sigma_{22}', \quad \sigma_{21} = \sigma_{21}', \quad m_{23} = m_{23}', \tag{22}$$

where

$$\sigma_{22} = A_{12} u_{1,1}' + A_{22} u_{2,2}', \quad \sigma_{21} = A_{12} u_{1,1}' + A_{22} u_{2,2}'$$

$$\begin{aligned} \sigma'_{21} &= A'_{78}u'_{2,1} + A'_{88}u'_{1,2} + (A'_{88} - A'_{78})\phi'_3, \\ \sigma_{21} &= A_{78}u_{2,1} + A_{88}u_{1,2} + (A_{88} - A_{78})\phi_3, \\ m'_{23} &= B'_{44}\phi'_{3,2}, \quad m_{23} = B_{44}\phi_{3,2}. \end{aligned}$$

Here, the symbols with superscript dash correspond to medium M'. Boundary conditions imply the following equations

$$\sum_{j=1}^3 A_j = \sum_{j=1}^3 A'_j \tag{23}$$

$$\sum_{j=1}^3 \zeta_j A_j = \sum_{j=1}^3 \zeta'_j A'_j \tag{24}$$

$$\sum_{j=1}^3 \eta_j A_j = \sum_{j=1}^3 \eta'_j A'_j \tag{25}$$

$$\sum_{j=1}^3 (i\xi A_{12} - m_j \zeta_j A_{22}) A_j = \sum_{j=1}^3 (i\xi A'_{12} + m'_j \zeta'_j A'_{22}) A'_j \tag{26}$$

$$\sum_{j=1}^3 \{i\xi \zeta_j A_{78} - m_j A_{88} - K_1 \eta_j\} A_j = \sum_{j=1}^3 \{i\xi \zeta'_j A'_{78} + m'_j A'_{88} - K_1 \eta'_j\} A'_j \tag{27}$$

$$\sum_{j=1}^3 (m_j \eta_j B_{44}) A_j = \sum_{j=1}^3 (-m'_j \eta'_j B'_{44}) A'_j \tag{28}$$

VI. STONELEY WAVE

Elimination of constants A_j and A'_j ($j=1,2,3$) from eqs.(23-28) gives the following secular equation for the surface wave in rotating orthotropic micropolar elastic solid media i.e

$$\det(a_{ij}) = 0; \quad (i, j = 1, 2, \dots, 6) \tag{29}$$

where

$$\begin{aligned} a_{11} &= 1, a_{12} = 1, a_{13} = 1, a_{14} = -1, a_{15} = -1, a_{16} = -1, \\ a_{21} &= \zeta_1, a_{22} = \zeta_2, a_{23} = \zeta_3, a_{24} = -\zeta'_1, a_{25} = -\zeta'_2, a_{26} = -\zeta'_3, \\ a_{31} &= \eta_1, a_{32} = \eta_2, a_{33} = \eta_3, a_{34} = -\eta'_1, a_{35} = -\eta'_2, a_{36} = -\eta'_3, \\ a_{4j} &= \begin{cases} (i\xi A_{12} - m_j \zeta_j A_{22}), & (j = 1, 2, 3) \\ -(i\xi A'_{12} + m'_{j-3} \zeta'_{j-3} A'_{22}), & (j = 4, 5, 6) \end{cases} \\ a_{5j} &= \begin{cases} \{i\xi \zeta_j A_{78} - m_j A_{88} - K_1 \eta_j\}, & (j = 1, 2, 3) \\ -\{i\xi \zeta'_{j-3} A'_{78} + m'_{j-3} A'_{88} - K_1 \eta'_{j-3}\}, & (j = 4, 5, 6) \end{cases} \\ a_{6j} &= \begin{cases} m_j \eta_j B_{44}, & (j = 1, 2, 3) \\ m'_{j-3} \eta'_{j-3} B'_{44}, & (j = 4, 5, 6) \end{cases} \end{aligned}$$

The eq. (29) is the secular equation for Stoneley wave rotating orthotropic micropolar solid media.

VII. RAYLEIGH WAVE AS LIMITING CASE

Rayleigh wave is a special case of the above general surface wave. In this case, we consider a model where the medium M' is replaced by vacuum. Since the boundary $y=0$ is adjacent to vacuum. It is free from traction. So the boundary conditions in this case may be expressed as

$$\sigma_{22} = 0, \sigma_{21} = 0, m_{23} = 0,$$

Thus the set of eqs. (23-28) reduces to

$$\sum_{j=1}^3 (i\xi A_{12} - m_j \zeta_j A_{22}) A_j = 0 \tag{30}$$

$$\sum_{j=1}^3 \{i\xi \zeta_j A_{78} - m_j A_{88} - K_1 \eta_j\} A_j = 0 \tag{31}$$

$$\sum_{j=1}^3 (m_j \eta_j B_{44}) A_j = 0 \quad (32)$$

Eliminating the constants A_j ($j=1,2,3$), we get the wave velocity equation for Rayleigh waves in the rotating orthotropic micropolar elastic case as follow:

$$\det(b_{ij}) = 0; \quad (i, j = 1, 2, 3) \quad (33)$$

where

$$b_{1j} = (i\xi A_{12} - m_j \zeta_j A_{22}), \quad (j=1,2,3)$$

$$b_{2j} = i\xi \zeta_j A_{78} - m_j A_{88} - K_1 \eta_j, \quad (j=1,2,3)$$

$$b_{3j} = m_j \eta_j B_{44}, \quad (j=1,2,3)$$

The eq. (33) is the secular equation for Rayleigh waves in rotating orthotropic micropolar solid half-space.

VIII. CONCLUSION

Assuming the components of the displacement and microrotation vectors in the form $\vec{u} = (u_1, u_2, 0)$ and $\vec{\phi} = (0, 0, \phi_3)$, the governing equations given in Iesan [8] and Schoenberg and Censor [9] are derived as a special case for rotating orthotropic micropolar elastic medium in x-y plane. Stoneley and Rayleigh type surface waves are studied in this medium. The secular equation of Stoneley and Rayleigh waves in rotating orthotropic micropolar elastic media is obtained. The theoretical results indicate that the speed of the surface wave depends on various material parameters. Present analytical solutions can be used to find numerically the speed of the Stoneley and Rayleigh waves for a particular material modelled as a rotating orthotropic micropolar elastic material. The results in this paper should prove useful in the field of material science, designers of new materials as well as for those working on the development of theory of elasticity.

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