IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 09, Issue 1 (January. 2019), ||V (III) || PP 72-76

Perfect Domination Polynomial for Lollipop Graph $L_{n,1}$

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Abstract: Let $G = (V, E)$ be a connected simple undirected graph. A subset S⊆ V is a dominating set if for every vertex $v \in V/S$, there exists a vertex $w \in S$ such that $vw \in E$; that is every vertex outside S has atleast one neighbor in S. The minimum cardinality of a dominating set of G is the domination number of G, it is denoted by $\gamma(G)$. A subset S⊆ V is a perfect dominating set if for every vertex $v \in V/S$, there exists exactly one vertex $w \in S$ such that $vw \in E$; that is every vertex outside S has exactly one neighbor in S. The minimum cardinality of a perfect dominating set of G is the perfect domination number of G, it is denoted by $\gamma_p(G)$. We introduce new concept that a perfect domination polynomial of a graph G. The perfect domination polynomial of a graph G of order n is the polynomial

$$PD(G,x) = \sum_{i=\gamma_p(G)}^{n} pd(G,i)x^i$$

Where $pd(G,i)$ is the number of perfect dominating sets of G of size i and $\gamma_p(G)$ is the perfect domination number of G. we obtain some properties of D(G,x) and we compute this polynomial for Lollipop graph $L_{n,1}$.

Keywords: Domination set, Domination polynomial, Lollipop graph, Perfect domination, Perfect domination polynomial.

Date of Submission: 12-01-2019Date of acceptance: 29-01-2019

I. INTRODUCTION

In literature, the concept of domination in graphs introduced by Claude Berge in 1958 and Oystein Ore in 1962 [6] is currently receiving much attention. Following the article of Ernie Cockayne and Stephen Hedetniemi [2], the domination in graphs became an area of study by many researchers. One type of domination in graphs is the perfect domination. This was introduced by Cockayne et al. [1] in the paper Perfect domination in graphs.

Let $G = (V, E)$ be a connected simple graph and $v \in V$. The open neighbourhood of v is the set $N(v) = \{u \in V; uv \in E\}$. The closed neighbourhood of v is the set $N[v] = N(v) \cup \{v\}$.A subset S⊆ V is a dominating set if for every vertex $v \in V/S$, there exists a vertex $w \in S$ such that $vw \in E$; that is every vertex outside S has atleast one neighbor in S. The minimum cardinality of a dominating set of G is the domination number of G, it is denoted by $\gamma(G)$. A subset S⊆ V is a perfect dominating set if for every vertex $v \in V/S$, there exists exactly one vertex $w \in S$ such that $vw \in E$; that is every vertex outside S has exactly one neighbor in S. The minimum cardinality of a perfect dominating set of G is the perfect domination number of G, it is denoted by $\gamma_p(G)$. Let D(G, i) be the family of dominating sets of a graph G with cardinality i and let d(G, i) = |D(G, i)|. Then the domination polynomial D(G, x) of G is defined as $D(G,x) = \sum_{i=\gamma(G)}^{n} d(G,i)x^i$

In the second section, we introduce the domination polynomial and in the third section we derive perfect domination polynomial for the Lollipop graph.

II. PERFECT DOMINATION POLYNOMIAL

The perfect domination polynomial of a graph G of order n is the polynomial

$$PD(G,x) = \sum_{i=\gamma_p(G)}^{n} pd(G,i)x^i$$

Where $pd(G,i)$ is the number of perfect dominating sets of G of size i and $\gamma_p(G)$ is the perfect domination number of G.

Theorem 2.1.

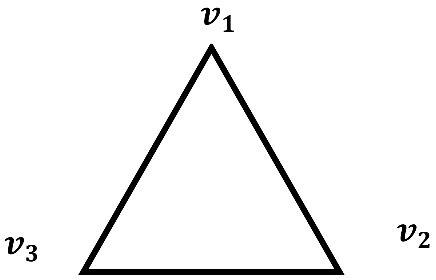
$$\mathrm{PD}(K_n, x) = x(n + x^{n-1})$$

Proof:
We prove this theorem by using induction method.
When n=2, consider $G = K_2$.

Here Perfect domination number K_2 is 1(i.e.,$\gamma_p(K_2) = 1$). Perfect domination sets of size 1 are $\{v_1\}$ and $\{v_2\}$. So $pd(G, 1) = 2$.
Perfect domination set of size 2 is $\{v_1, v_2\}$. $pd(G, 2) = 1$
Therefore the perfect domination polynomial is defined by

$$PD(G, x) = 2x^1 + 1x^2 = x(2 + x)$$

When n=3, consider $G = K_3$.

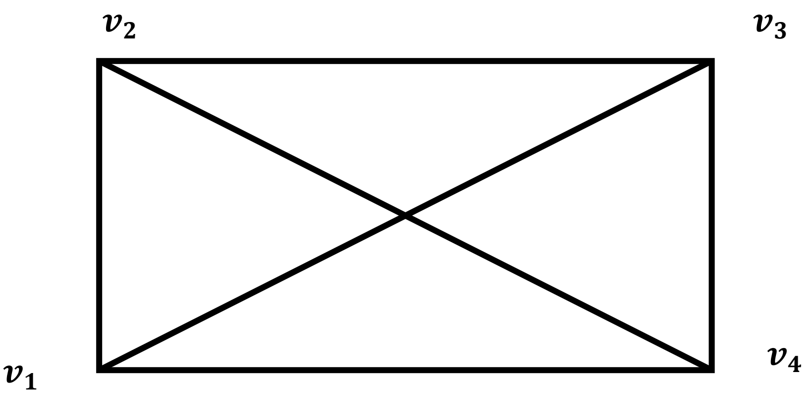


Here Perfect domination number K_3 is 1(i.e.,$\gamma_p(K_3) = 1$). Perfect domination sets of size 1 are $\{v_1\}, \{v_2\}$ and $\{v_3\}$. So $pd(G, 1) = 3$.
By the definition of Perfect domination set, there is no perfect domination set of size 2, because in the complete graph every vertex is a neighbourhood of other vertex. $pd(G, 2) = 0$. Perfect domination set of size 3 is $\{v_1, v_2, v_3\}$. $pd(G, 3) = 1$

Therefore the perfect domination polynomial is defined by

$$PD(G, x) = 3x^1 + 0x^2 + 1x^3 = x(3 + x^2)$$

When n=4, consider $G = K_4$.



Here Perfect domination number K_4 is 1(i.e.,$\gamma_p(K_4) = 1$). Perfect domination sets of size 1 are $\{v_1\}, \{v_2\}, \{v_3\}$ and $\{v_4\}$. So $pd(G, 1) = 4$.

By the definition of Perfect domination set, there is no perfect domination set of size 2 and of size 3, because in the complete graph every vertex is a neighbourhood of other vertex. $pd(G,2) = pd(G,3) = 0$. Perfect domination set of size 4 is $\{v_1, v_2, v_3, v_4\}$. $pd(G,4) = 1$

Therefore the perfect domination polynomial is defined by

$$PD(G,x) = 4x^1 + 0x^2 + 0x^3 + 1x^4 = x(4 + x^3)$$

By proceeding in this manner we get the general term for n.

$$PD(K_n, x) = x(n + x^{n-1})$$

Theorem 2.2.
Let $\overline{K_n}$ be the empty graph with n vertices. Then

$$D(\overline{K_n}, x) = x^n.$$

Proof:
Since $D(\overline{K_1}, x) = x$., we have the result by Theorem 2.1.

Theorem 2.3:
Perfect domination polynomial of any graph G has no constant term.
Proof:
Since $\gamma_p(G) \geq 1$, the perfect domination polynomial has no term of degree 0. Therefore it has no constant term.

Theorem 2.4:
Let G be a graph and H be any induced subgraph of G. Then $deg\ PD(G,x) \geq deg\ PD(H,x)$

Proof:
Let G be a graph of order n (i.e., $|V(G) = n|$), by the definition of PD(G,x), it is of order n.
Let $H \subseteq G$, then H is a subgraph of order m which is less than or equal to n$(m \leq n)$. So the highest degree of PD(H,x) is atmost n. Hence we conclude that $deg\ PD(G,x) \geq deg\ PD(H,x)$

III. PERFECT DOMINATION POLYNOMIAL FOR LOLLIPOP GRAPH $L_{n,1}$.

Definition 3.1.
The lollipop graph is the graph obtained by joining a complete graph K_n to a path graph P_1 with a bridge and it is denoted by $L_{n,1}$

Theorem 3.2.

$$PD(L_{n,1}, x) = x(n + x^{n-1})$$

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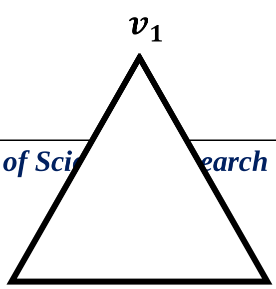
Proof:
We prove this theorem by using induction method.

When n=3, consider $G = L_{3,1}$.

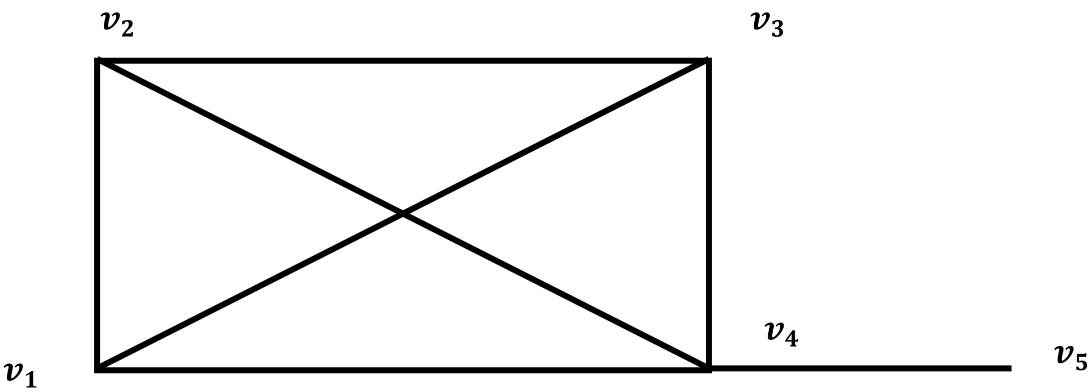


Here Perfect domination number $L_{3,1}$ is 1(i.e.,$\gamma_p(L_{3,1}) = 1$). Perfect domination set of size 1 is $\{v_3\}$. So $pd(G, 1) = 1$. Perfect domination set of size 2 is $\{v_3, v_4\}$. So $pd(G, 2) = 1$. Perfect domination set of size 3 is $\{v_1, v_2, v_3\}$. $i.e., pd(G, 3) = 1$. Perfect domination set of size 4 is $\{v_1, v_2, v_3, v_4\}$. $pd(G, 4) = 1$

Therefore the perfect domination polynomial is defined by

$$PD(G, x) = 1x^1 + 1x^2 + 1x^3 + 1x^4 = x(1 + x + x^2 + x^3)$$

When n=4, consider $G = L_{4,1}$.



Here Perfect domination number $L_{4,1}$ is 1(i.e.,$\gamma_p(L_{4,1}) = 1$). Perfect domination set of size 1 is $\{v_4\}$. So $pd(G, 1) = 1$.

Perfect domination set of size 2 is $\{v_4, v_5\}$. So $pd(G, 2) = 1$. There is no Perfect domination set of size 3, so $pd(G, 3) = 0$. Perfect domination set of size 4 is $\{v_1, v_2, v_3, v_4\}$. $pd(G, 4) = 1$. Perfect domination set of size 5 is $\{v_1, v_2, v_3, v_4, v_5\}$. $pd(G, 5) = 1$

Therefore the perfect domination polynomial is defined by

$$PD(G, x) = 1x^1 + 1x^2 + 0x^3 + 1x^4 + 1x^5 = x(1 + x + x^3 + x^4)$$

When n=5, consider $G = L_{5,1}$.

Here Perfect domination number $L_{5,1}$ is 1(i.e.,$\gamma_p(L_{5,1}) = 1$). Perfect domination set of size 1 is $\{v_5\}$. So $pd(G, 1) = 1$.

Perfect domination set of size 2 is $\{v_5, v_6\}$. So $pd(G, 2) = 1$. There is no Perfect domination set of size 3 and 4, so $pd(G, 3) = pd(G, 4) = 0$. Perfect domination set of size 5 is $\{v_1, v_2, v_3, v_4, v_5\}$. $pd(G, 5) = 1$. Perfect domination set of size 6 is $\{v_1, v_2, v_3, v_4, v_5, v_6\}$. $pd(G, 6) = 1$

Therefore the perfect domination polynomial is defined by

$$PD(G, x) = 1x^1 + 1x^2 + 0x^3 + 0x^4 + 1x^5 + 1x^6 = x(1 + x + x^4 + x^5)$$

By proceeding in this manner we get the general term for n.

$PD(L_{n,1}, x) = x(1 + x + x^{n-1} + x^n)$.

IV. CONCLUSION

In this paper we introduced a new polynomial namely, Perfect domination polynomial. And we derive some theorems and then derive perfect domination polynomial for the Lollipop graph $L_{n,1}$.

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S.Chandrasekaran “Perfect Domination Polynomial for Lollipop Graph L(n,1).” IOSR Journal