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A Different Approach to Solve Fuzzy Quadratic Equation $AX^2 = B$

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Abstract: Recently fuzzy equations are used in various fields such as Physics, Economics, Finance and Statistics. Solving fuzzy quadratic equations has long been a problem in fuzzy set theory. Fuzzy quadratic equations were solved by using the method of $\alpha - \text{cut}$. This article proposed a new and simple solution method to solve a fuzzy quadratic equation $\text{AX}^2 = \text{B}$, without using the $\alpha - \text{cut}$. We use distribution function, complementary distribution function and density function to solve fuzzy quadratic equation $\text{AX}^2 = \text{B}$. The result obtained is found to be in good agreement. We also compare our new solution with classical solution of $\alpha - \text{cut}$ by plotting them into graphical representation using MATLAB.

Keywords: Fuzzy Number, Fuzzy Membership Function (fmf), Probability Distribution, Density Function.

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I. INTRODUCTION

Fuzzy set theory largely recognizes that it offers more applications than ordinary set theory. Lotfi A. Zadeh [19] presented the concept of fuzzy set and its application in 1965. One of the domains of fuzzy set theory, in which fuzzy numbers and arithmetic operations on fuzzy numbers play a decisive role are the fuzzy equations. The problem of finding the roots of equations like the quadratic equation has many applications in applied sciences such as finance, economics, and mechanics. Fuzzy equations have been studied by Dubois and Prades [10]. Sanchez [17] proposes a solution to the fuzzy equation with extended operations. Accordingly various researchers Buckley [2], [3], Jiang [12],Wasowski [18], Biacino and Lettieri [1] offered various methods to solve fuzzy equations. After that there have been many research articles that have suggested solutions to different types of fuzzy equations by using $\alpha - cut$. Buckley and Qu in [4], [5], [6], [7] defined the concept of solving fuzzy quadratic equations $AX^2 = B$, where A, B and X are fuzzy numbers, by using the method of $\alpha - cut$ and their work has been influential in the study of fuzzy equations.

Mazarbhuiya [16], Baruah [15], Chou [9], have defined arithmetic operations of fuzzy numbers without using the method $\alpha - cut$. In this article, we present a new approach to solve fuzzy quadratic equations $AX^2 = B$, without using the usual method. Our method is based on fuzzy arithmetic without $\alpha - cut$. We used the distribution function and the complementary distribution function to solve the equation with numerical examples. We denote our solution by $X_{W.C.}$ and compare with old solution.

The paper is arranged as follows. In Section 2, we discuss about the definitions and notations used in this article. In Section 3, we have reviewed the classical methods of solving fuzzy quadratic equation $AX^2 = B$ with numerical examples. In Section 4, we introduce a new solution technique of $AX^2 = B$, without using $\alpha - cut$ where we get the solution $X_{W.C.}$ and compare the new solution with the old solution by graphical representation. All the graphs are plotted by using the computer programming MATLAB function. In section 5, we give a brief conclusion of our task and lines for future work.

II. DEFINITIONS AND NOTATIONS

A function $A: X \longrightarrow [0,1]$ is called a fuzzy set on X, where X a nonempty set of objects is called referential set and $[0, 1]$ is called valuation set and $\forall x \in X;\ A(x)$ represents the grade of membership of x.

A $\alpha - cut, A^\alpha$ of a fuzzy set A is an ordinary set of elements with membership not less than α for $0 \leq \alpha \leq 1$. This means $A^\alpha = \{x \in X:\ A(x) \geq \alpha\}$. [13]

The height of A is denoted by $h(A)$ and is defined as $h(A) = \sup\{A(x): x \in X\}$. A is called normal fuzzy set iff $h(A) = \sup\{A(x)\} = 1$. [13]

A fuzzy set is said to be convex if all it $\alpha - cuts$ are convex sets (see e. g. [8]).

A fuzzy number is a convex normal fuzzy set A define on the real line such that $A(x)$ is piecewise continuous.

The support of a fuzzy set $A$is denoted by $\text{supp}(A)$ and is defined as the set of elements with membership nonzero *i.e.*, $\sup p(A) = \{x \in X: A(x) > 0\}$. [13]

A fuzzy number A, denoted by a triad $[a, b, c]$ such that $A(a) = 0 = A(c)$ and $A(b) = 1$, where $A(x)$ for $x \in [a, b]$is called the left reference function and for $x \in [b, c]$is called right reference function. The left reference function is right continuous monotone and non-decreasing where right reference function is left continuous, monotone and non-increasing. The above definition of a fuzzy number is called L-R fuzzy number. [11]

If Z is a continuous random variable with density functionn $f(z)$, then distribution of Z, denoted by $F(z)$, is defined by $F(z) = Prob[Z \leq z] = \int_{-\infty}^{z} f(z)\, dz$ and the complementary distribution of Z, denoted by $\bar{F}(z)$, is defined by $\bar{F}(z) = 1 - Prob[Z \leq z] = 1 - \int_{-\infty}^{z} f(z)\, dz$. [14]

III. CLASSICAL SOLUTION OF $AX^2 = B$, USING THE METHOD OF $\alpha - cut$

In this section, we methodically reviewed the classical solution of $AX^2 = B$ founded by Buckley in [6], [7]. We consider the simplest form of the fuzzy quadratic equation,

$$AX^2 = B \text{ ... ... } (3.1)$$

where A and B are triangular fuzzy numbers. Let $A = [a_1 / a_2 / a_3]$; $B = [b_1 / b_2 / b_3]$ and also let $A^{\alpha} = [a_1(\alpha), a_2(\alpha)]andB^{\alpha} = [b_1(\alpha), b_2(\alpha)]$ be $\alpha - cuts$ of A and B respectively. We say that this method defines the solutionX_c [6], so that $X_c^{\alpha} = [x_1(\alpha), x_2(\alpha)]$ be $\alpha - cut$ of the solution X_c. Substituting these into equation (3.1) we get,

$$[a_1(\alpha), a_2(\alpha)][x_1(\alpha), x_2(\alpha)]^2 = [b_1(\alpha), b_2(\alpha)] \text{ ... ... } (3.2)$$

which gives $$x_1(\alpha) = \pm\sqrt{\frac{b_1(\alpha)}{a_1(\alpha)}} \quad \text{and} \quad x_2(\alpha) = \pm\sqrt{\frac{b_2(\alpha)}{a_2(\alpha)}}$$

Therefore, the solutions are $X_{c_1}^{\alpha} = \left[\sqrt{\frac{b_1(\alpha)}{a_1(\alpha)}}, \sqrt{\frac{b_2(\alpha)}{a_2(\alpha)}}\right]$ and $X_{c_2}^{\alpha} = \left[-\sqrt{\frac{b_2(\alpha)}{a_2(\alpha)}}, -\sqrt{\frac{b_1(\alpha)}{a_1(\alpha)}}\right]$.(e.g. [6])

Example: Consider the fuzzy quadratic equation$[3/4/5]X^2 = [5/7/9]$. Here $A = [3/4/5]$ and $B = [5/7/9]$. Also $A^{\alpha} = [a_1(\alpha), a_2(\alpha)] = [3 + \alpha, 5 - \alpha]$ and $B^{\alpha} = [b_1(\alpha), b_2(\alpha)] = [5 + 2\alpha, 9 - 2\alpha]$.

Substitution these into equation (3.2) we get, $x_1(\alpha) = \pm\sqrt{\frac{5+2\alpha}{3+\alpha}}$ and $x_2(\alpha) = \pm\sqrt{\frac{9-2\alpha}{5-\alpha}}$.

Then there are two possible classical solutions of the equation whose $\alpha - cuts$ are given by,

$$X_{c_1}^{\alpha} = \left[\sqrt{\frac{5+2\alpha}{3+\alpha}}, \sqrt{\frac{9-2\alpha}{5-\alpha}}\right] \quad \text{and} \quad X_{c_2}^{\alpha} = \left[-\sqrt{\frac{9-2\alpha}{5-\alpha}}, -\sqrt{\frac{5+2\alpha}{3+\alpha}}\right].$$

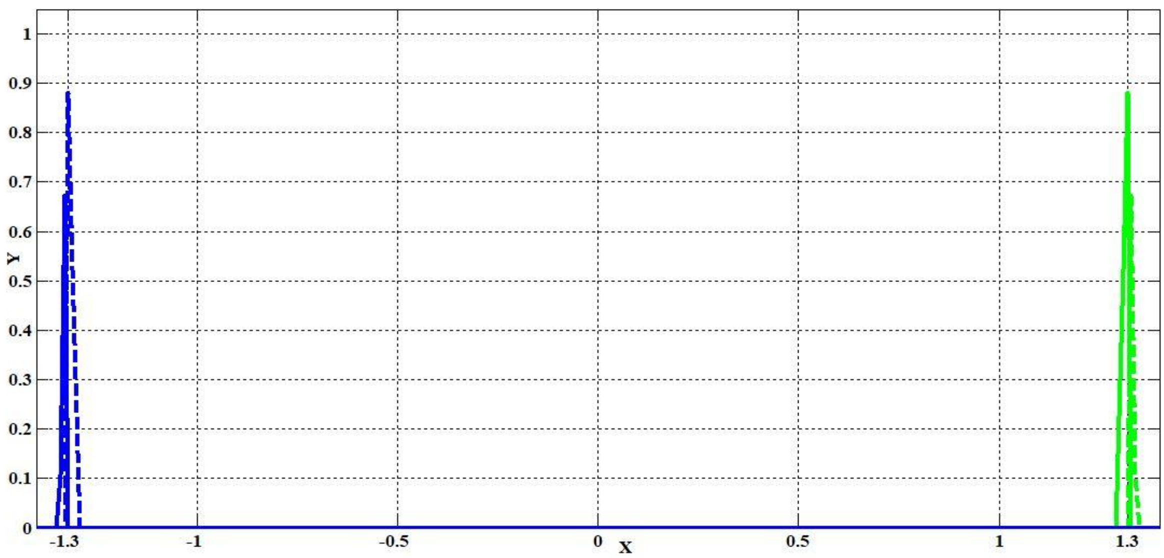

Fig. 3.1. Solution X_C

IV. SOLUTION OF $AX^2 = B$, WITHOUT USING THE METHOD OF $\alpha - cut$

In this section we are going to solve the simple fuzzy quadratic equation $AX^2 = B$, without using $\alpha - cut$. A new way to evaluate equation (3.1) is to use distribution function and complementary distribution function and we get the solution which we denoted by $X_{W.C.}$. Here we have followed [16], [15], [9]. Chou [9] has forwarded a method of finding the fuzzy membership function for a triangular fuzzy numbers X without using $\alpha - cut$.

4.1. Arithmetic of fuzzy number without using $\alpha - cut$

We now define a triangular fuzzy number $X = [a/b/c]$ with membership function

$$X(x) = \begin{cases} L(x)\,, & a \le x \le b \\ R(x)\,, & b \le x \le c \\ 0\,, & otherwise \end{cases}$$

Where $L(x)$ being continuous non-decreasing function in the interval $[a, b]$ and $R(x)$ being a continuous non-increasing function in the interval $[b, c]$ with $L(a) = R(c) = 0$ and $L(b) = R(b) = 1$. Where $L(x)$ is left reference function and $R(x)$ is right reference function of the concerned fuzzy number. In this discussion, we are going to demonstrate the easiness of applying this method in evaluating the arithmetic of fuzzy numbers if start from the simple assumption that the left reference function is a distribution function, and similarly the right reference function is a complementary distribution function. Accordingly, the functions $L(x)$ and $(1 - R(x))$ would have to be associated with densities

$$\frac{d}{dx} L(x) \text{ in } [a, b] \text{ and } \frac{d}{dx}\big(1 - R(x)\big) \text{ in } [b, c].$$

Now consider $X = [a/b/c]$ and $Y = [p/q/r]$ be two triangular fuzzy numbers.

Suppose $Z = X * Y = [a * p\,, b * q\,, c * r]$ be the fuzzy number of $X * Y$, where '$*$'denotes addition$(+)$ and multiplication$(\cdot)$.

Let $X(x) = \begin{cases} L(x)\,, & a \le x \le b \\ R(x)\,, & b \le x \le c \\ 0\,, & otherwise \end{cases}$; $Y(y) = \begin{cases} L(y)\,, & p \le y \le q \\ R(y)\,, & q \le y \le r \\ 0\,, & otherwise \end{cases}$

Where $L(x)$ and $L(y)$ are the left reference functions and $R(x)$ and $R(y)$ are the right reference functions respectively. We assume that $L(x)$ and $L(y)$ are distribution functions and $R(x)$ and $R(y)$ are complementary distribution functions. Accordingly, there would exist some density functions for the distribution functions $L(x)$ and $(1 - R(x))$.

Let $f(x) = \frac{d}{dx}\big(L(x)\big),\ a \le x \le b$ and $g(x) = \frac{d}{dx}\big(1 - R(x)\big),\ b \le x \le c$.

We start with equating $L(x)$ with $L(y)$, and $R(x)$ with $R(y)$. And so we obtain

$y = \varphi_1(x)$ and $y = \varphi_2(x)$ respectively.

Let $z = x * y$, so we have $z = x * \varphi_1(x)$ and $z = x * \varphi_2(x)$

So that $x = \psi_1(z)$ and $x = \psi_2(z)$ (say).

Replacing x by $\psi_1(z)$ in $f(x)$, and $\psi_2(z)$ in $g(x)$, we get

$f(x) = \eta_1(z)$ and $g(x) = \eta_2(z)$ (say).

Now $\frac{dx}{dz} = \frac{d}{dz}\big(\psi_1(z)\big) = m_1(z)$ and $\frac{dx}{dz} = \frac{d}{dz}\big(\psi_2(z)\big) = m_2(z)$

Therefore, the fuzzy arithmetic without $\alpha - cut$ would exist as follows:

4.1.1. Addition:

$$(X + Y)(x) = \begin{cases} \displaystyle\int_{a+p}^{x} \eta_1(z) m_1(z) dz\,, & a + p \le x \le b + q \\ 1 - \displaystyle\int_{b+q}^{x} \eta_2(z) m_2(z) dz\,, & b + q \le x \le c + r \\ 0 & , \ otherwise \end{cases} \quad \ldots\ldots(4.1)$$

4.1.2. Multiplication:

$$(X\cdot Y)(x)=\begin{cases}\int_{ap}^{x}\eta_1(z)m_1(z)dz\,, & ap\le x\le bq\\ 1-\int_{bq}^{x}\eta_2(z)m_2(z)dz\,, & bq\le x\le cr\\ 0\,, & otherwise\end{cases} \quad \ldots\ldots(4.2)$$

4.1.3. Subtraction:

Suppose $Z = X - Y$. Then the fmf of $Z = X - Y$ would be given by $Z = X + (-Y)$. At first, we have to find the fmf of $(-Y)$. Let $(-Y) = [-r/-q/-p]$ be the fuzzy number of $(-Y)$.

Therefore $f(y) = \frac{d}{dy}\big(L(y)\big)\,,\ p \le y \le q$ and $g(y) = \frac{d}{dy}\big(1 - R(y)\big)\,,\ q \le y \le r.$

Now let $t = -y$ so that $\frac{dy}{dt} = -1 = m(t)$, (say).

Replacing $y = -t$ in $f(y)$ and $g(y)$, we obtain $f(y) = \eta_1(t)$ and $g(y) = \eta_2(t)$ (say). Then the fmf of $(-Y)$ would be given by

$$(-Y)(y)=\begin{cases}\int_{-r}^{y}\eta_2(t)m(t)dt\,, & -r\le y\le -q\\ 1-\int_{-q}^{y}\eta_1(t)m(t)dt\,, & -q\le y\le -p\\ 0\,, & otherwise\end{cases} \quad \ldots\ldots(4.3)$$

Then we can easily find the fmf of $X - Y$ by addition of fuzzy numbers X and $(-Y)$ as described in the earlier section.

4.1.4. Division:

Suppose $Z = \frac{X}{Y}$. Then the fmf of $Z = \frac{X}{Y}$ would be given by $Z = X.Y^{-1}$. First we have to find the fmf of Y^{-1}. Suppose $Y^{-1} = (r^{-1}/q^{-1}/p^{-1})$ be the fuzzy number of Y^{-1}.

Therefore, $f(y) = \frac{d}{dy}\big(L(y)\big)\,,\ p \le y \le q$ and $g(y) = \frac{d}{dy}\big(1 - R(y)\big)\,,\ q \le y \le r.$

Now let $t = y^{-1}$ so that $\frac{dy}{dt} = -\frac{1}{t^2} = m(t)$, (say).

Replacing $t = y^{-1}$ in $f(y)$ and $g(y)$ we obtain $f(y) = \eta_1(t)$ and $g(y) = \eta_2(t)$ (say).
Then the fmf of (Y^{-1}) would be given by

$$(Y^{-1})(y)=\begin{cases}\int_{r^{-1}}^{y}\eta_2(t)m(t)dt\,, & r^{-1}\le y\le q^{-1}\\ 1-\int_{q^{-1}}^{y}\eta_1(t)m(t)dt\,, & q^{-1}\le y\le p^{-1}\\ 0\,, & otherwise\end{cases} \quad \ldots\ldots(4.4)$$

Next we can easily find the fmf of $\frac{X}{Y}$ by multiplication of fuzzy numbers X and Y^{-1} as described in the earlier section.

4.1.5. The fmf of $\sqrt{X}$ for a triangular fuzzy number X:

Consider $X = [a/b/c]$, with fmf , $X(x) = \begin{cases} L(x)\,, & a\le x\le b\\ R(x)\,, & b\le x\le c\\ 0\,, & otherwise\end{cases}$

Let $f(x) = \frac{d}{dx}\big(L(x)\big)$, $a \le x \le b$ and $g(x) = \frac{d}{dx}\big(1 - R(x)\big)$, $b \le x \le c$.

Put $t = \sqrt{x}$ so that $\frac{dx}{dt} = 2t = m(t)$, (say).

Replacing $t = \sqrt{x}$ in $f(x)$ and $g(x)$ we obtain $f(x) = \eta_1(t)$ and $g(x) = \eta_2(t)$ (say).

Therefore fmf of $\sqrt{X}$ would be given by

$$(\sqrt{X})(x) = \begin{cases} \int_{\sqrt{a}}^{x} \eta_1(t)m(t)dt\,, & \sqrt{a} \le x \le \sqrt{b} \\ 1 - \int_{\sqrt{b}}^{x} \eta_2(t)m(t)dt\,, & \sqrt{b} \le x \le \sqrt{c} \\ 0 & , \quad otherwise \end{cases} \quad \dots\dots (4.5)$$

Now applying these fuzzy arithmetic without using the method of $\alpha - cut$ we can easily solve the fuzzy quadratic equation $AX^2 = B$.

Example: We are going to solve same problem discussed in section 3, by our new method. Consider the fuzzy quadratic equation $[3/4/5]X^2 = [5/7/9]$. Where $A = [3/4/5]$ and $B = [5/7/9]$.

$$\therefore\ B(x) = \begin{cases} \frac{x-5}{2}\,, & 5 \le x \le 7 \\ \frac{9-x}{2}\,, & 7 \le x \le 9 \\ 0\,, & x < 5 \ or\ x > 9 \end{cases} \quad \text{and} \quad A(y) = \begin{cases} y-3\,, & 3 \le x \le 4 \\ 5-y\,, & 4 \le x \le 5 \\ 0\,, & x < 3 \ or\ x > 5 \end{cases}$$

We have to find the fmf of $\sqrt{\frac{B}{A}}$ that is

$$X_{W.C.} = \sqrt{\frac{B}{A}}(x) \dots\dots (4.6)$$

By using (4.4) the fmf of $\frac{1}{A} = [\frac{1}{5}/\frac{1}{4}/\frac{1}{3}]$ is given by

$$\frac{1}{A}(y) = \begin{cases} 5 - \frac{1}{y}, & \frac{1}{5} \le y \le \frac{1}{4} \\ \frac{1}{y} - 3, & \frac{1}{4} \le y \le \frac{1}{3} \\ 0, & y < \frac{1}{5} \ or\ y > \frac{1}{3} \end{cases}$$

Here $L(x) = \frac{x-5}{2}$ and $L(y) = 5 - \frac{1}{y}$ are distributions function and $R(x) = \frac{9-x}{2}$ and $R(y) = \frac{1}{y} - 3$ are complementary distribution functions. Then the density functions would be given by,

$$f(x) = \frac{d}{dx}\left(\frac{x-5}{2}\right) = \frac{1}{2},\ 5 \le x \le 7 \ \text{ and } \ g(x) = \frac{d}{dx}\left(1 - \frac{9-x}{2}\right) = \frac{1}{2},\ 7 \le x \le 9$$

Suppose $X^2 = \frac{B}{A} = [1/\frac{7}{4}/3]$. Equating distribution function and complementary distribution function we get,

$$L(x) = L(y) \quad \text{and} \quad R(x) = R(y)$$

$$\frac{x-5}{2} = 5 - \frac{1}{y} \quad \text{and} \quad \frac{9-x}{2} = \frac{1}{y} - 3$$

$$y = \frac{2}{15-x} = \varphi_1(x) \quad \text{and} \quad y = \frac{2}{15-x} = \varphi_2(x)$$

Let $z = x \cdot y$ so we shall have $z = x \cdot \varphi_1(x) = \frac{2}{15-x}$ and $z = x \cdot \varphi_2(x) = \frac{2}{15-x}$

So that $x = \frac{15z}{z+2} = \psi_1(z)$ and $x = \frac{15z}{z+2} = \psi_2(z)$.

Replacing x by $\psi_1(z)$ in $f(x)$ and $\psi_2(z)$ in $g(x)$, we obtain

$$f(x) = \frac{1}{2} = \eta_1(z) \qquad \text{and} \qquad g(x) = \frac{1}{2} = \eta_2(z)$$

Now let

$$m_1(z) = \frac{d}{dz}\left(\psi_1(z)\right) = \frac{30}{(z+2)^2} \ \text{ and } \ m_2(z) = \frac{d}{dz}\left(\psi_2(z)\right) = \frac{30}{(z+2)^2}.$$

Putting this values into (4.2) we get the fmf of $\frac{B}{A}$, which is given by

$$X^2 = \frac{B}{A}(x) = \begin{cases} \frac{5x-5}{x+2}\,, & 1 \le x \le \frac{7}{4} \\ \frac{9-3x}{x+2}\,, & \frac{7}{4} \le x \le 3 \\ 0 & , x < 1 \ or\ x > 3 \end{cases} \quad \dots\dots (4.7)$$

Now, to get the solution $X_{W.C.} = \sqrt{X^2}$, we have to find the square root of equation (4.7). Again let $f(x) = \frac{d}{dx}\left(\frac{5x-5}{x+2}\right) = \frac{15}{(x+2)^2}$, $1 \le x \le \frac{7}{4}$ and $g(x) = \frac{d}{dx}\left(1 - \frac{9-3x}{x+2}\right) = \frac{15}{(x+2)^2}, \frac{7}{4} \le x \le 3$.
Let $t = \sqrt{x}$ so that $m(t) = \frac{dx}{dt} = 2t$. Replacing $t = \sqrt{x}$ in $f(x)$ and $g(x)$ we obtain

$$f\,(x) = \frac{15}{(t^2+2)^2} = \eta_1(t) \quad \text{and} \quad g(x) = \frac{15}{(t^2+2)^2} = \eta_2(t)\,.$$

Finally, using eqn.(4.5) we get the desired solution and the fmf of $X_{W.C.}$ would be given by,

$$X_{W.C.} = \sqrt{\frac{B}{A}}(x) = \begin{cases} \dfrac{5x^2 - 5}{x^2 + 2}\,, & 1 \le x \le \sqrt{\dfrac{7}{4}} \\ \dfrac{9 - 3x^2}{x^2 + 2}\,, & \sqrt{\dfrac{7}{4}} \le x \le \sqrt{3} \\ 0 & , x < 1 \; or \; x > \sqrt{3} \end{cases}$$

$X_{W.C}$ is shown in the following figure 4.1.

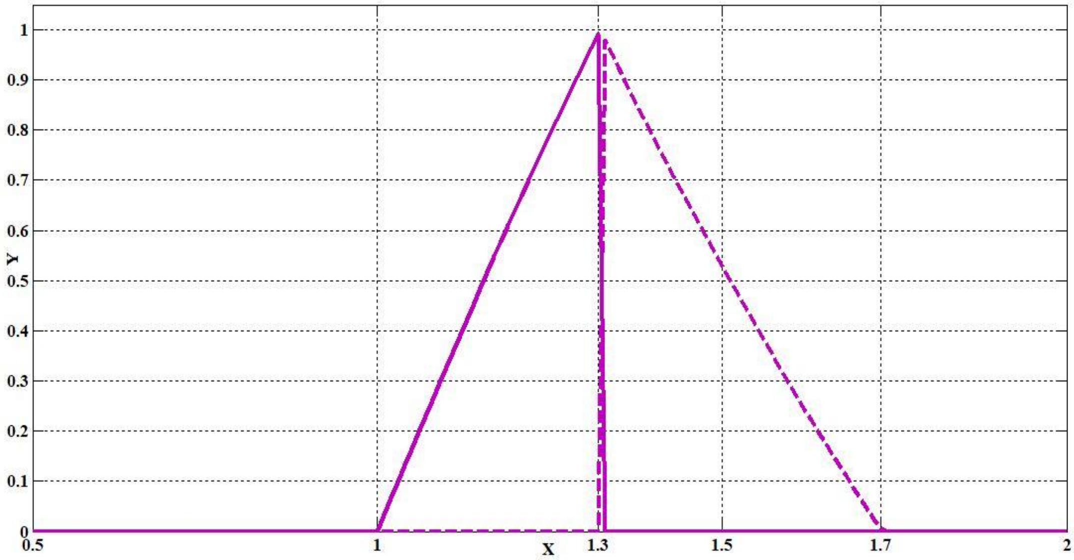

Fig 4.1. New Solution $X_{W.C.}$

4.2. The comparison of new solution and classical solution are given in the following figure:

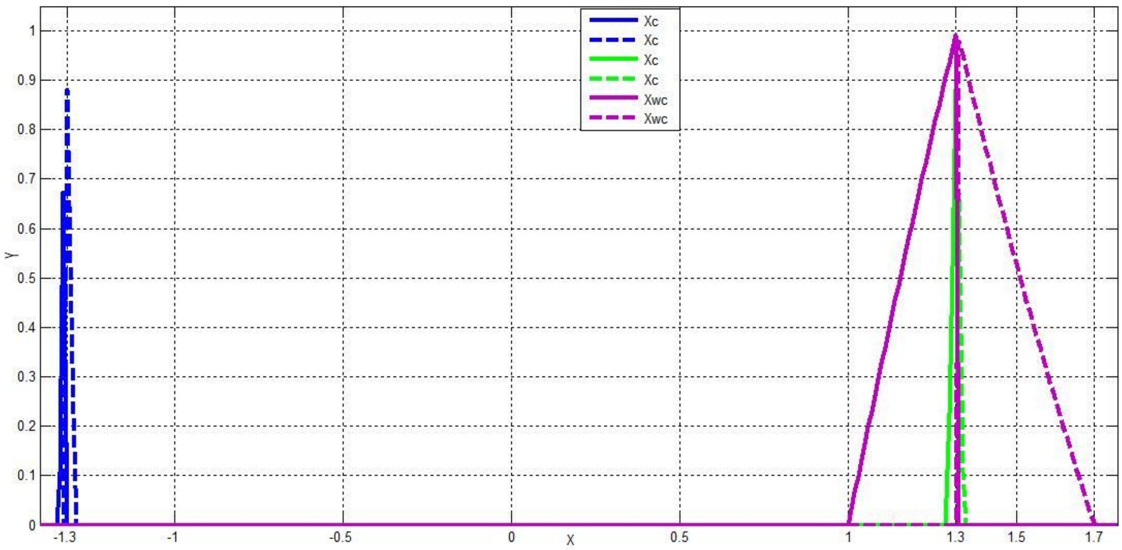

Fig 4.2. Comparison between solutions $X_{W.C.}, X_c$.

V. CONCLUSION

The classical method of $\alpha - cut$ is very limited. This is because there is no solution very often or there is a solution by applying very strong conditions to the equation. This has prevented the application of fuzzy set theory to algebra, economics, decision theory, and physics. This fact motivated us to develop new solutions technique. In this article we have acquainted with a new technique to solve fuzzy quadratic equation $AX^2 = B$, and our new solution is more effective than classical solution. In our new solution the referential set is larger than the classical solution. Our future aim is to solve the fully fuzzy quadratic equation $AX^2 + Bx + C = D$ by using this new technique.

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