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Observer Design for an Harvesting System

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Abstract: This paper presents the design of an asymptotic converging observer for an energy harvesting system. In an energy harvesting system, an observer is used to estimate mechanical quantities from the known electrical variables. Most estimation in the past has remained a challenge owing to rigorous computational techniques. In this work, we show that if there exists mappings which are left invertible and the manifolds of these mappings are positive invariant, then, using the small gain theorem and the solution of a modified algebraic Riccati equation, an asymptotic converging observer can be designed.

Keywords: Asymptotic, Observer, Energy harvesting system, Ricatti, Converging

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I. INTRODUCTION

In most nonlinear systems, not all the states are available for measurement. In such instances, an observer is utilized to estimate the unavailable states. Designing observers for nonlinear systems has received widespread attention owing to its utmost significance and intensive mathematical computations (Karagiannis and Astolfi, 2005). In (Krener and Isidori, 1983) and (Krener and Respondek, 1985), an approach to nonlinear design of observers is presented. It consists of linearizing the observed plant and then applying basic observer design techniques. However, not all nonlinear system can be linearized. A typical example is the unicycle whose nonlinearity is so severe that it cannot be approximated around operating points. Gauthier *et.al* (1992) and Gauthier and Kupka (1994) presents another approach to observer design by using the Lipschitz condition and the gain of the plant. (Yu, 2004) presents an observer design for a class of uncertain nonlinear multiple-input-multiple-output mechanical systems whose dynamics are first-order differentiable. The observer is immune to noise and parameter variations. However, the system has to be first order differentiable. To obtain the unmeasured states of the control system used in Dou *et. al.*, (2017), an observer design without a model base was utilized. In all these instances, certain assumptions need to be drawn which in practice is hard to satisfy. To relax these assumptions, a general framework for designing observers which are globally convergent was presented in (Karagiannis and Astolfi, 2005). This paper is based on this approach. We show that if there exists mappings which are left invertible and the manifolds of these mappings are positive invariant then, using the small gain theorem and the solution of a modified algebraic Riccati equation, an asymptotic converging observer can be designed.

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II. METHODOLOGY

Preliminaries

Karagiannis and Astolfi (2005) described a novel means of designing observers for nonlinear systems.

Consider the dynamic system

$$\dot{x}_1 = f_1(x_1, x_2, t) \quad (1)$$

$$\dot{x}_2 = f_2(x_1, x_2, t) \quad (2)$$

$x_1 \in \mathbb{R}^n$ is the unmeasured part of the state and $x_2 \in \mathbb{R}^m$ is the measurable output.

The vector fields $f_1(\cdot)$ and $f_2(\cdot)$ are assumed to be forward complete, *ie* their trajectories are defined for all times $t \geq t_0$.

Defination 1. The dynamical system

$$\dot{\widehat{x_1}} = \propto (x_2, \widehat{x_1}, t) \quad (3)$$

With $\widehat{x_1} \in \mathbb{R}^p$, p $\geq$n, is called an observer for the system (1) – (2) if there exist mappings $\mu(.) : \mathbb{R}^n \; x \; \mathbb{R}^m \; x \; \mathbb{R}^p \rightarrow \mathbb{R}^p$ and $\sigma(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^p$, with $(\cdot)$ left invertible, such that the manifold $\gamma = (x_1, x_2, \widehat{x_1}, t) \in \mathbb{R}^n x \mathbb{R}^m x \mathbb{R}^p : \mu(x_2, \widehat{x_1}, t) = \sigma(x_1)$ is positive invariant which implies that all trajectories of the extended system (1) – (2) – (3) that start on γ remain there for all future times, and is attractive, which implies that the trajectories of (1) – (2) – (3) that start in a neighborhood of ωasymptotically converge to γ.

Consider the equation

$$z = \gamma(x_2, \widehat{x_1}, t) - \sigma(x_1, x_2, t) \quad (4)$$

Using definition 1 to construct an observer implies that γ should be designed in such a way that equation (3) above asymptotically converges to zero, uniformly in x_1, x_2 and t.

2.1. Third Order Observer

Inductor current, I_L and capacitor voltage, Vc are variables associated with the boost converter presented in (Nunna *et.al*, 2014) and (Ukoima, 2016). These are measurable, therefore known. ϑ, w and w_r are assumed unknown and have to be accurately estimated. This assumption is useful for sensorless operation of the harvester.

Consider the following equations.

$$z_1 = \widehat{x_1} - x_1 + \gamma_1(x_3) \quad (5)$$

$$z_2 = \widehat{x_2} - x_2 + \gamma_2(x_3) \quad (6)$$

$$z_3 = \widehat{w_s} - w_s + \gamma_3(x_3) \quad (7)$$

The estimate is taken to be the sum of the observed output and a function of one of the known variables. Out of the two known variables (current and voltage), current (x_3) is chosen. The reason for choosing x_3 stems from the fact that x_3 contains information on the unknown variables. This is easily seen from equation (1) in (Ukoima, 2016).

$$x_{iestimate} = \widehat{x_1} + \gamma_1(x_3) \quad (8)$$

γ_i is a function which embeds the design variables.

$\Longrightarrow$ xiestimate = xi + zi

From (9), it can be observed that if $z_i \longrightarrow 0$, then $x_{iestimate} \longrightarrow x_i$. $i = 1, 2, 3$.

Taking the time derivative of (5), (6) and (7),

$$\dot{z_1} = \dot{\widehat{x_1}} - \dot{x_1} + \frac{\partial\gamma_1(x_3)}{\partial x_3}\dot{x_3}$$

$$\dot{z_2} = \dot{\widehat{x_2}} - \dot{x_2} + \frac{\partial\gamma_2(x_3)}{\partial x_3}\dot{x_3}$$

$$\dot{z_3} = \dot{\widehat{w_s}} - \dot{w_s} + \frac{\partial\gamma_3(x_3)}{\partial x_3}\dot{x_3}$$

Let $\gamma_i(x_3) = \gamma_i x_3$

$$\Longrightarrow \frac{\partial\gamma_i(x_3)}{\partial x_3} = \gamma_i$$

Substituting values of $\dot{x_1}$ and $\dot{X_3}$ from (1) in (Ukoima, 2016).

$$\dot{z_1} = \dot{\widehat{x_1}} - \widehat{x_2} + \frac{\partial\gamma_1(x_3)}{\partial x_3}\left[\frac{-(1-\delta)x4}{L} + \frac{kE(w_s - x_2)(1-\delta)^2 RL}{L\,(Ra(1-\delta)^2 + RL)}\right]$$

$$x_2 = \widehat{x_2} - z_2 + \gamma_2(x_3)$$

$$w_s = \widehat{w_s} - z_3 + \gamma_3(x_3)$$

$$x_1 = \widehat{x_1} - z_1 + \gamma_1(x_3)$$

$$\dot{z_1} = \dot{\widehat{x_1}} - [\widehat{x_2} + \gamma_2(x3) - z_2] - \frac{\gamma_1(1-\delta)x4}{L} + \frac{\gamma_1 kE(w_s + \widehat{(x3)} - z_3)(1-\delta)^2 RL}{L\,(Ra(1-\delta)^2 + RL)} - \frac{\gamma_1 kE(\widehat{x_2} + \gamma_2(x3) - z2)(1-\delta)^2 RL}{L\,(Ra(1-\delta)^2 + RL)}$$

Expanding $\dot{z_1}$

$$\dot{z_1} = \dot{\widehat{x_1}} - \widehat{x_2} - \gamma_2(x3) + z_2 - \frac{\gamma_1(1-\delta)x4}{L} + \frac{\gamma_1 kE(\widehat{w_s})(1-\delta)^2 RL}{L\,(Ra(1-\delta)^2 + RL)} + \frac{\gamma_1 kE(\gamma_3(x3) - z3)(1-\delta)^2 RL}{L\,(Ra(1-\delta)^2 + RL)} - \frac{\gamma_1 kE(\widehat{x_2})(1-\delta)^2 RL}{L\,(Ra(1-\delta)^2 + RL)} - \frac{\gamma_1 kE(\gamma_2(x3) - z2)(1-\delta)^2 RL}{L\,(Ra(1-\delta)^2 + RL)}$$

Let $\rho = \frac{kE\,(1-\delta)^2 RL}{L\,(Ra\,(1-\delta)^2 + RL)}$

Substituting and rearranging $\dot{z_1}$

$$\Rightarrow \dot{z_1} = \dot{\widehat{x_1}} - \widehat{x_2} - \gamma_2(x3) + z_2 - \frac{\gamma_1(1-\delta)x4}{L} + \rho\gamma_1\widehat{w_s} + \rho\gamma_1(\gamma_3(x3) - z_3) - \rho\gamma_1\widehat{x_2} - \rho\gamma_1(\gamma_2(x3) - z_2)$$

$\widehat{x_2}, \gamma_2(x3), \gamma_1, \gamma_3(x3)$are terms which are unknown. Therefore, $\dot{\widehat{x_1}}$ can be chosen such that these terms are eliminated.

$$\dot{\widehat{x_1}} = \widehat{x_2} + \gamma_2(x3) + \frac{\gamma_1(1-\delta)x4}{L} - \rho\gamma_1\widehat{w_s} - \rho\gamma_1\left(\gamma_3(x3)\right) + \rho\gamma_1\widehat{x_2} + \rho\gamma_1(\gamma_2(x3))$$

Let $\zeta = \frac{\gamma_1 (1-\delta)x4}{L} + \rho(\widehat{x_2} + \gamma_2(x3) - \widehat{w_s} - \gamma_3(x3))$

$$\dot{\widehat{x_1}} = \widehat{x_2} + \gamma_2(x3) + \gamma_1 \zeta \quad (9)$$

$$\Rightarrow \dot{z_1} = z_2 + \rho\gamma_1(z_2 - z_3) \quad (10)$$

Similarly, substituting values of $\dot{x_2}$ and $\dot{x_3}$

$$\dot{z_2} = \dot{\widehat{x_2}} - \left[\frac{-mglsinx_1}{J} + \frac{kEkT(w_s - x_2)}{J(Ra+(1-\delta)^2RL)}\right] + \frac{\partial\gamma_2(x_3)}{\partial x_3}\left[\frac{-(1-\delta)x4}{L} + \frac{kE(w_s - x_2)(1-\delta)^2RL}{L(Ra+(1-\delta)^2RL)}\right]$$

Substituting $v = \frac{kEkT(w_s - x_2)}{J(Ra+(1-\delta)^2RL)}$ and recalling that $\rho = \frac{kE(1-\delta)^2RL}{L(Ra(1-\delta)^2+RL)}$ and $\frac{\partial\gamma_i(x_3)}{\partial x_3} = \gamma_i$

$$\dot{z_2} = \dot{\widehat{x_2}} + \frac{-mglsinx_1}{J} - v(w_s - x_2) - \frac{\gamma_2}{L}(1-\delta)x4 + \gamma_2\rho(w_s - x_2)$$

Substituting for x_2 and w_s

$$\dot{z_2} = \dot{\widehat{x_2}} + \frac{-mglsinx_1}{J} - v(\widehat{w_s} - z_3 + \gamma_3(x_3)) + v(\widehat{x_2} - z_2 + \gamma_2(x_3)) - \frac{\gamma_2}{L}(1-\delta)x4 + \gamma_2\rho(\widehat{w_s} - z_3 + \gamma_3(x_3) - (\widehat{x_2} - z_2 + \gamma_2(x_3)))$$

Recall that $z_1 = \widehat{x_1} - x_1 + \gamma_1(x_3) \Rightarrow z_1 + x_1 = \widehat{x_1} + \gamma_1(x_3)$

$$\Rightarrow \sin(\underbrace{\widehat{x_1} + \gamma_1(x_3) - z_1}_{x_1}) = \sin(x_1 + z_1) + \sin(\underbrace{\widehat{x_1} + \gamma_1(x_3)}_{x_1+z_1}) - \sin x_1$$

Substituting these in the $\dot{z_2}$ equation and factorizing,

$$\dot{z_2} = \dot{\widehat{x_2}} + \frac{-mgl}{J}\left[\sin\left(\widehat{x_1} + \gamma_1(x_3)\right) - \sin(x_1 + z_1) + \sin x_1\right] - v(\widehat{w_s} - z_3 + \gamma_3(x_3)) + v(\widehat{x_2} - z_2 + \gamma_2(x_3)) - \frac{\gamma_2}{L}(1-\delta)x4 + \gamma_2\rho(\widehat{w_s} - z_3 + \gamma_3(x_3) - (\widehat{x_2} - z_2 + \gamma_2(x_3)))$$

Again $\hat{x}_2$, $\gamma_1(x_3)$, $\gamma 2$, $\gamma 2(x_3)$ are terms which are unknown. Therefore, $\dot{\widehat{x_2}}$ is chosen so that these terms are eliminated.

$$\dot{\widehat{x_2}} = \frac{-mgl}{J}\sin\left(\widehat{x_1} + \gamma_1(x_3)\right) - v\left(\widehat{x_2} + \gamma_2(x_3) - \widehat{w_s} - \gamma_3(x_3)\right) + \frac{\gamma_2}{L}(1-\delta)x4 - \gamma_2\rho(\widehat{w_s} + \gamma_3(x_3) - \widehat{x_2} - \gamma_2(x_3))$$

Factorizing and noting that $\zeta = \frac{\gamma_1 (1-\delta)x4}{L} + \rho(\widehat{x_2} + \gamma_2(x3) - \widehat{w_s} - \gamma_3(x3))$

$$\dot{\widehat{x_2}} = \frac{-mgl}{J}\sin(\widehat{x_1} + \gamma_1(x_3)) - v(\widehat{x_2} + \gamma_2(x_3) - \widehat{w_s} - \gamma_3(x_3)) + \gamma_2\zeta \quad (11)$$

$$\Rightarrow \dot{z_2} = \frac{-mgl}{J}[\sin(z_1 + x_1) - \sin x_1] + v(-z_2 + z_3) + \gamma_2\rho(z_2 - z_3) \quad (12)$$

For this work, it is assumed that the source rotation speed (w_s) is constant

$\Rightarrow \dot{w}_s = 0$

Substituting for $\dot{w}_s$ and x_3 in (7) and following similar procedure above,

$$\dot{\widehat{w_s}} = -\gamma_3\zeta \quad (13)$$

$$\dot{z_3} = (z_2 - z_3) \quad (14)$$

Equations (9), (11) and (13) represent the nonlinear third order observer

From (10), (12) and (14), a matrix z-dot ($\dot{z}$) system can be formed. This is given below.

$$\begin{bmatrix}\dot{z_1}\\ \dot{z_2}\\ \dot{z_3}\end{bmatrix} = \begin{bmatrix}0 & (1+\gamma_1\rho) & -\gamma_1\rho\\ \frac{-mgl}{J} & (\gamma_2\rho - v) & (\gamma_2\rho - v)\\ 0 & \gamma_3\rho & -\gamma_3\rho\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} - \begin{bmatrix}0\\ \frac{mgl}{J}\\ 0\end{bmatrix}[\sin(z_1 + x_1) - \sin x_1 - z_1] \quad (15)$$

2.2. Proof of an Asymptotically Converging Observer for the Harvester

A good observer is one in which the estimation errors asymptotically converge to zero. Therefore the design variable $\gamma \in \mathbb{R}^3$ must be designed in such a way that $z_i \longrightarrow 0$. $i = 1, 2, 3$. To begin, (15) is written in a feedback interconnection form. In this form, the stability properties can be analyzed using the small gain theorem. It also enables solving for $\gamma \in \mathbb{R}^3$.

$$\begin{bmatrix}\dot{z_1}\\ \dot{z_2}\\ \dot{z_3}\end{bmatrix} = \left(\underbrace{\begin{bmatrix}0 & 1 & 0\\ \frac{-mgl}{J} & -v & v\\ 0 & 0 & 0\end{bmatrix}}_{A} + \underbrace{\begin{bmatrix}0\\ \frac{mgl}{J}\\ 0\end{bmatrix}}_{B}\underbrace{[0 \quad \rho \quad -\rho]}_{K}\right)\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} - \underbrace{\begin{bmatrix}0\\ \frac{mgl}{J}\\ 0\end{bmatrix}}_{G}\varpi$$

$$\varpi = \eta(x_1, z_1)z_1 = \left(\frac{[\sin(z_1+x_1) - \sin x_1 - z_1]}{z_1}\right)z_1 \quad (16)$$

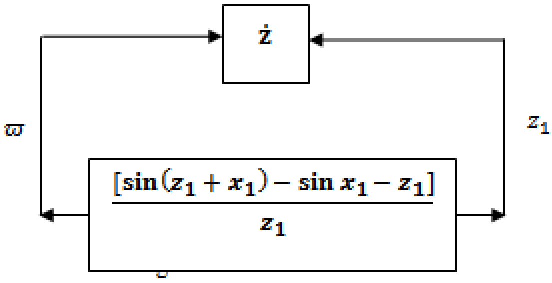

Figure 1. Feedback interconnection

Let Γ_L represent the H_∞ gain of the linear system and Γ_{NL} represent the L_2 norm of the nonlinear system. From the small gain theorem, stability occurs iff $\Gamma_L\Gamma_{NL} < 1$.

$\Gamma_L\Gamma_{NL} < 1$ implies $z_i \longrightarrow 0$.

$$\Gamma_{NL} = \underbrace{max}_{x_1 z_1} = \underbrace{\left|\frac{[\sin(z_1 + x_1) - \sin x_1 - z_1]}{z_1}\right|}_{\lambda}$$

Maxima occurs when $\frac{d\lambda}{dz_1} = 0$

$$\frac{d\lambda}{dz_1} = \frac{z_1[\cos(z_1 + x_1) - 1] - [\sin(z_1 + x_1) - \sin(x_1) - z_1}{{z_1}^2}$$

$$\Longrightarrow \frac{[\sin(z_1 + x_1) - \sin x_1 - z_1]}{z_1} = \cos(z_1 + x_1) - 1$$

Maximum value of $\cos((z_1 + x_1)) = \pm 1$

$\Rightarrow \Gamma_{NL} = |-1 - 1| = 2$

For stability, it is required that $\Gamma_L < \frac{1}{2}$

Consider the system $\dot{x} = Fx + Gu$

$z = Hx$

Where $F = A + BK$;

A, B, G and K are given in (16)

$H = [1\ 0\ 0]$

$\Gamma_L < \frac{1}{2}$ iff there exists an $x \in \mathbb{R}^{3x3} = x' > 0$:

$$F'x + xF + \frac{xGG'x}{{\Gamma_L}^2} + HH' \leq 0$$

This is the algebraic Ricatti equation Substituting for F,

$$\underbrace{(A + BK)'x + x(A + BK)}_{(A' + K'B')x + xA + xBK} + \frac{xGG'x}{{\Gamma_L}^2} + HH' < 0$$

$$[K'B'x + xBK] = \underbrace{-xBBx + xBB'x + K'K - K'K}_{=0} + K'B'x + xBK$$

$$K'K + K'B'x + xBK + BB'x = [K + B'x]'[K + B'x]$$

$$\Rightarrow A'x + xA + \frac{xGG'x}{{\Gamma_L}^2} + HH' - K'K - xBBx + xBB'x + [K + B'x]'[K + B'x] < 0$$

If $K + B'x = 0$, then the equation becomes

$$A'x + xA + \frac{xGG'x}{{\Gamma_L}^2} + HH' - K'K < 0 \qquad (17)$$

Equation (17) is the modified algebraic Riccati equation.

$K + B'x = 0 \Rightarrow B' = -Kx' \Rightarrow B = -K'x$

Or similarly, $B' = -x^{-1}K \Rightarrow B = -x^{-1}K'$

Therefore, if there exist an $x = x' > 0$, by solving (17), then the H_∞ gain of (16) with input ϖ and output z_1 is less than $\frac{1}{2}$.

It then follows that the selection $B = -x^{-1}K'$ is such that

$$\lim_{t\to\infty}(\widehat{x_1}(t) + \gamma_1(x_3) - x_1(t)) = 0, \lim_{t\to\infty}(\widehat{x_2}(t) + \gamma_2(x_3) - x_2(t)) = 0\ \&\lim_{t\to\infty}(\widehat{w_s}(t) + \gamma_3(x_3) - w_s(t)) = 0$$

That is to say the system represented by equations (9), (11) and (13) is an asymptotically converging observer for the harvester in [2] and [3]. Where $B = [\gamma_1\ \gamma_2\ \gamma_3]'$.

It is important to note that the observer presented in equations (9), (11) and (13) can be extended to the case in which w_s varies as a function of time. If $w_s = c_o + c_1 t$, then the states x_1, x_2, c_o and c_1 have to be estimated. In this case, $\dot{w}_s = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} w_s$. This thus requires a fourth order observer.

To construct this observer, consider the following equations

$$z_1 = \widehat{x_1} - x_1 + \gamma_1(x_3) \quad (18)$$

$$z_2 = \widehat{x_2} - x_2 + \gamma_2(x_3) \quad (19)$$

$$z_3 = \widehat{c_0} - c_0 + \gamma_3(x_3) \quad (20)$$

$$z_4 = \widehat{c_1} - c_1 + \gamma_4(x_3) \quad (21)$$

Following similar arguments as described previously, results in the following matrix.

$$\begin{bmatrix} \dot{z_1} \\ \dot{z_2} \\ \dot{z_3} \\ \dot{z_4} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-mgl}{J} & -v & v & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A} + \underbrace{\begin{bmatrix} 0 \\ \frac{mgl}{J} \\ 0 \\ 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} 0 & \rho & -\rho \end{bmatrix}}_{K} \right) \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} - \underbrace{\begin{bmatrix} 0 \\ \frac{mgl}{J} \\ 0 \\ 0 \end{bmatrix}}_{G} \varpi$$

$$\varpi = \eta(x_1, z_1) z_1 = \left(\frac{[\sin(z_1 + x_1) - \sin x_1 - z_1]}{z_1} \right) z_1$$

Similarly, if there exist $x \in \mathbb{R}^4 x^4 = x' > 0$ such that (17) has a solution, then $B = -x^{-1}K'$ yields an asymptotically converging observer. Where $B = [\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4]'$.

III. RESULTS AND DISCUSSION

The simulations were performed for the third order observer. It was assumed that the speed of rotation of the host is constant. The third order observer is described by equations (9), (11) and (13) with $\gamma_1 = 0.4$, $\gamma_2 = -0.4$ and $\gamma_3 = 0.35$. These values of the design parameters were obtained in matlab by solving equation (17) with a by trial and error method. Figures 2, 3 and 4 show that the estimation errors converge to zero. To confirm this, further plots were obtained in the logarithmic scale. If the errors actually converge to zero, then the log plots are expected to tend towards negative infinity. Figures 5, 6 and7 confirms this.

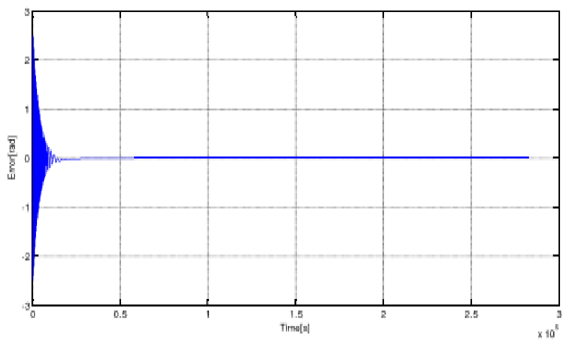

Figure 2: *Time history of the estimation error of the angular position of the mass*

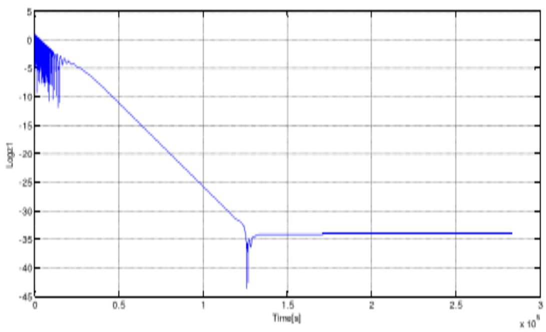

Figure 3: *Time history of the estimation error of angular position of the mass in log scale*

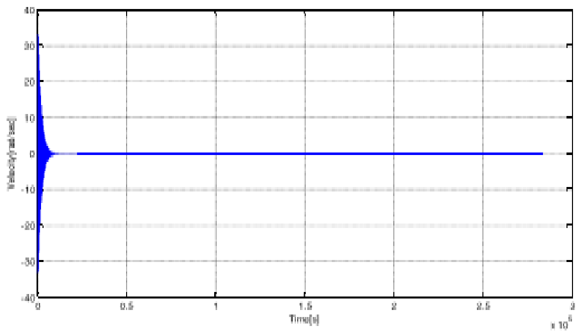

Figure 4: *Time history of the estimation error of of the angular velocity of the mass*

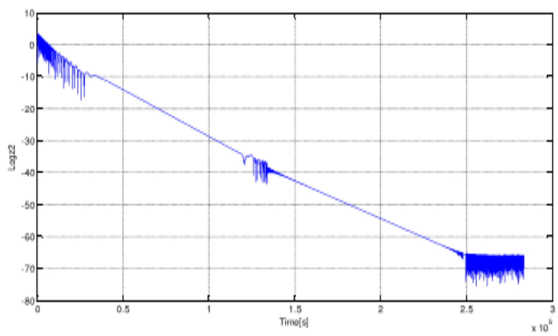

Figure 5: *Time history of the estimation error the angular velocity of the mass performed*

in log scale

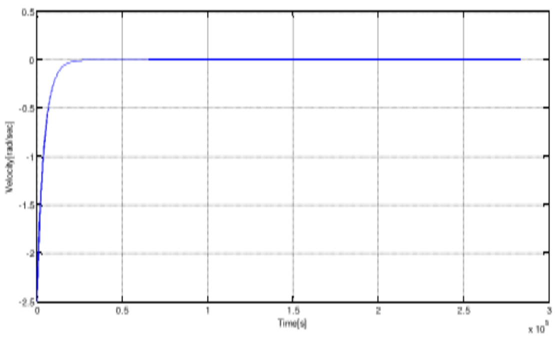

Figure 6: *Time history of the estimation error of the source rotation speed*

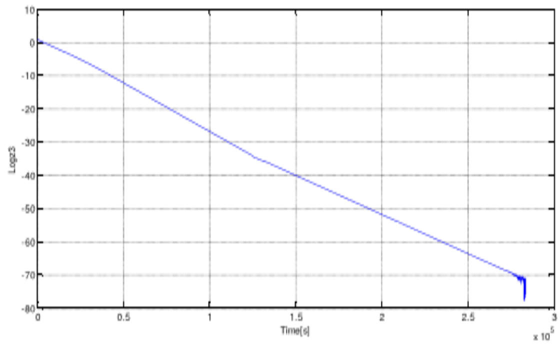

Figure 7: *Time history of the estimation error of source rotation speed performed in log scale*

IV. CONCLUSION

In conclusion, we have proved that there exists mappings $\mu(\cdot): \mathbb{R}^n \, x \, \mathbb{R}^m \, x \, \mathbb{R}^p \rightarrow \mathbb{R}^p$ and $\sigma(\cdot): \mathbb{R}^n \rightarrow \mathbb{R}^p$, with $(\cdot)$ left invertible such that the manifold $\gamma = (x_1, x_2, \hat{x}_1, t) \in \mathbb{R}^n x \mathbb{R}^m x \mathbb{R}^p : \mu(x_2, \hat{x}_1, t) = \sigma(x_1)$ is positive invariant. Therefore, the observer (3) asymptotically converges to zero uniformly in x_1, x_2 and t.

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