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Poisson Ishita Distribution: A New Compounding Probability Model

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Abstract: In this article, we propose a new probability distribution by compounding Poisson distribution with Ishita distribution. Important mathematical and statistical properties of the distribution have been derived and discussed.The expressions for coefficient of variation, skewness , kurtosis ,reliability analysis and order statistics has been obtained. Then, parameter estimation is discussed using maximum likelihood method of estimation. Finally, real data set is analyzed to investigate the suitability of the proposed distribution in modeling count dataset representing epileptic seizure counts**.**

Keywords: Poisson distribution, Ishita distribution, compound distribution, Count data, Maximum likelihood estimation.

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I. INTRODUCTION

Compounding of probability distributions is a sound and innovative technique to obtain new probability distributions to fit data sets not adequately fit by common parametric distributions. Compound distributions serve well to describe various phenomena in biology, epidemiology, genetics and so on. Mathematically, compound distribution arises when all or some parameters of a distribution known as parent distribution vary according to some probability distribution called the compounding distribution, for instance negative binomial distribution can be obtained from Poisson distribution when its rate parameter follows gamma distribution. If the parent distribution is discrete/continuous then resultant compound distribution will also be discrete/continuous respectively i.e., the support of the parent distribution determines the support of compound distributions. The work has been done in this particular area since 1920. It is well known that Greenwood and Yule(1920) established a relationship between Poisson distribution and a negative binomial distribution through compounding mechanism by treating the rate parameter in Poisson distribution as gamma variate. Skellam(1948) derived a probability distribution from the binomial distribution by regarding the probability of success as a beta variable between sets of trials. Lindely (1958) suggested a one parameter distribution to illustrate the difference between fiducial distribution and posterior distribution. Dubey (1970) derived a compound gamma, beta and F distribution by compounding a gamma distribution with another gamma distribution and reduced it to the beta Ist and beta 2nd kind and to the F distribution by suitable transformations. Gerstenkorn(1993,1996) proposed several compound distributions, he obtained compound of gamma distribution with exponential distribution by treating the parameter of gamma distribution as an exponential variate and also obtained compound of polya with beta distribution. Mahmoudi et al. (2010) generalized the Poisson-Lindely distribution of Sankaran (1970) and showed that their generalized distribution has more flexibility in analyzing count data. Zamani and Ismail (2010) constructed a new compound distribution by compounding negative binomial with one parameter Lindley distribution that provides good fit for count data where the probability at zero has a large value. A new generalized negative binomial distribution was proposed by Gupta and Ong (2004), this distribution arises from Poisson distribution if the rate parameter follows generalized gamma distribution; the resulting distribution so obtained was applied to various data sets and can be used as better alternative to negative binomial distribution. Rashid, Ahmad and Jan (2016) proposed a new competitive count data model, by compounding negative binomial distribution with Kumaraswamy distribution that finds its application in biological sciences. Para and Jan (2018) introduced two compounding models with applications to handle count data in medical sciences.

In this paper we propose a new compounding distribution by compounding Poisson distribution with Ishita distribution, as there is a need to find more flexible model for analyzing statistical data.

II. DEFINITION OF PROPOSED MODEL (POISSON ISHITA DISTRIBUTION)

If $X|\lambda \sim \text{SBP}(\lambda)$, where λ is itself a random variable following Ishita distribution with parameter θ, then determining the distribution that results from marginalizing over λ will be known as a compound of Poisson distribution with that of Ishita distribution, which is denoted by $PID(X;\theta)$. It may be noted that proposed model will be a discrete since the parent distribution is discrete.

Theorem 2.1: The probability mass function of a Poisson Ishita Distribution i.e., $PID(X;\theta)$ is given by

$$P(X=x)=\frac{\theta^3}{\theta^3+2}\left[\frac{\theta(1+\theta)^2+(x+1)(x+2)}{(1+\theta)^{x+3}}\right];x=0,1,2,3,\ldots;\theta>0$$

Proof: Using the definition (2), the pmf of a Poisson Ishita Distribution i.e., $PID(X;\theta)$ can be obtained as

$$g(x|\lambda)=\frac{e^{-\lambda}\lambda^x}{(x)!}\quad ;\ x=0,1,2,3,\ldots;\lambda>0$$

When its parameter λ follows Ishita distribution (ID) with pdf

$$h(\lambda;\theta)=\frac{\theta^3}{\theta^3+2}(\theta+\lambda^2)e^{-\theta\lambda}\ ;\lambda>0,\theta>0$$

We have

$$P(X=x)=\int_0^\infty g(x|\lambda).h(\lambda;\theta)d\lambda$$

$$P(X=x)=\int_0^\infty \frac{e^{-\lambda}\lambda^x}{(x)!}\left(\frac{\theta^3}{\theta^3+2}\right)(\theta+\lambda^2)e^{-\theta\lambda}d\lambda$$

$$P(X=x)=\frac{\theta^3}{x!(\theta^3+2)}\left(\int_0^\infty \theta e^{-(1+\theta)\lambda}\lambda^x\,d\lambda+\int_0^\infty e^{-(1+\theta)\lambda}\lambda^{x+2}d\lambda\right)$$

$$P(X=x)=\frac{\theta^3}{(\theta^3+2)}\left[\frac{\theta(1+\theta)^2+(x+1)(x+2)}{(1+\theta)^{x+3}}\right];x=0,1,2,3,\ldots;\theta>0 \qquad (2.1)$$

which is the p.m.f. of PID

The corresponding c.d.f of Poisson Ishita distribution is obtained as:

$$F_X(x)=\sum_{n=0}^{x}\frac{\theta^3}{(\theta^3+2)}\left[\frac{\theta(1+\theta)^2+(x+1)(x+2)}{(1+\theta)^{x+3}}\right]$$

$$1-\frac{\theta^5+2\theta^4+\theta^3+x^2\theta^2+5x\theta^2+6\theta^2+2x\theta+6\theta+2}{(1+\theta)^{x+3}(2+\theta^3)}\ ;x>0,\theta>0 \qquad (2.2)$$

III. STATISTICAL PROPERTIES

In this section, structural properties of the Poisson Ishita model have been evaluated. These include moment, moment generating function and probability generating function.

3.1 Moments

3.1.1 Factorial Moments

Using (2.1), the rth factorial moment about origin of the PID (2.1) can be obtained as

$$\mu_{(r)}'=E[E(X^{(r)}|\lambda)],\ where\ X^{(r)}=X(X-1)(X-2)\ldots(X-r+1)$$

$$\mu_{(r)}'=\int_0^\infty\left[\sum_{x=0}^\infty x^{(r)}\frac{e^{-\lambda}\lambda^x}{(x)!}\right].\frac{\theta^3}{\theta^3+2}(\theta+\lambda^2)e^{-\theta\lambda}d\lambda$$

$$\mu_{(r)}' = \frac{\theta^3}{\theta^3+2}\int_0^\infty \left[\lambda^r\left(\sum_{x=r}^{\infty}\frac{e^{-\lambda}\lambda^{x-r}}{(x-r)!}\right)\right](\theta+\lambda^2)e^{-\theta\lambda}d\lambda$$

$Taking\,u = x-r, we\,get$

$$\mu_{(r)}' = \frac{\theta^3}{\theta^3+2}\int_0^\infty \left[\lambda^r\left(\sum_{u=0}^{\infty}\frac{e^{-\lambda}\lambda^{u}}{u!}\right)\right](\theta+\lambda^2)e^{-\theta\lambda}d\lambda$$

$$\mu_{(r)}' = \frac{\theta^3}{\theta^3+2}\left(\int_0^\infty \theta\lambda^r e^{-\theta\lambda}d\lambda + \int \lambda^{r+2}e^{-\theta\lambda}d\lambda\right)$$

$$\mu_{(r)}' = \frac{r!}{\theta^3+2}\left[\frac{\theta^3+(r+1)(r+2)}{\theta^r}\right] \qquad \textit{(3.1.1.1)}$$

Taking r=1,2,3,4 in (3.1.1.1), the first four factorial moments about origin of Poisson Ishita Distribution can be obtained as

$$\mu_{(1)}' = \frac{\theta^3+6}{\theta(\theta^3+2)}$$

$$\mu_{(2)}' = \frac{2\theta^3+24}{\theta^2(\theta^3+2)}$$

$$\mu_{(3)}' = \frac{6\theta^3+120}{\theta^3(\theta^3+2)}$$

$$\mu_{(4)}' = \frac{24\theta^4+1440}{\theta^4(\theta^3+2)}$$

3.1.2 Moments about origin (Raw moments)

The first four moments about origin , using the relationship between factorial moments about origin and the moments about origin, of PID (2.1)can be obtained as

$$\mu_1' = \frac{\theta^3+6}{\theta(\theta^3+2)}$$

$$\mu_2' = \frac{\theta^4+2\theta^3+6\theta+24}{\theta^2(\theta^3+2)}$$

$$\mu_3' = \frac{\theta^5+6\theta^4+6\theta^3+6\ \theta^2+72\theta+120}{\theta^3(\theta^3+2)}$$

$$\mu_4' = \frac{\theta^6+14\theta^5+60\theta^4+6\theta^3+168\theta^2+72\theta+1440}{\theta^4(\theta^3+2)}$$

3.1.3 Moments about the Mean (Central moments)

Using the relationship $\mu_r = E(Y-\mu_1')^r = \sum_{K=0}^{r}\binom{r}{k}\mu_k'(-\mu_1')^{r-k}$ between moments about the mean and the moments about origin, the moments about the mean of the PID (2.1) can be obtained as

$$\mu_2 = \frac{\theta^7+\theta^6+8\theta^4+16\theta^3+12\theta+12}{\theta^2(\theta^3+2)^2}$$

$$\mu_3 = \frac{\theta^{11}+3\theta^{10}+10\theta^8+56\theta^7+36\theta^6+280\theta^5+156\theta^4+24\theta^2+144\theta+50}{\theta^3(\theta^3+2)^3}$$

$$\mu_4 = \frac{\theta^{15} + 10\theta^{14} + 42\theta^{13} - 3\theta^{12} + 188\theta^{11} + 24\theta^{10} + 1440\theta^{9} + 814\theta^{8} - 2194\theta^{7} + 6200\theta^{6} + 1360\theta^{5} - 6010\theta^{4} + 7632\theta^{3} + 768\theta^{2} - 3744\theta + 6480}{\theta^4(\theta^3+2)^4}$$

3.2 Coefficient of variation, skewness, kurtosis and Index of Dispersion

The coefficient of variation (C.V), coefficient of skewness $(\sqrt{\beta_1})$, coefficient of kurtosis (β_2) ,and index of dispersion (γ) of the SBPAD are thus obtained as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\theta^7 + \theta^6 + 8\theta^4 + 16\theta^3 + 12\theta + 12}{\theta^3 + 6}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\theta^{11} + 3\theta^{10} + 10\theta^8 + 56\theta^7 + 36\theta^6 + 280\theta^5 + 156\theta^4 + 24\theta^4 + 144\theta + 50}{(\theta^7 + \theta^6 + 8\theta^4 + 16\theta^3 12\theta + 12)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^{2}} = \frac{\theta^{15} + 10\theta^{14} + 42\theta^{13} - 3\theta^{12} + 188\theta^{11} + 24\theta^{10} + 1440\theta^{9} + 814\theta^{8} - 2194\theta^{7} + 6200\theta^{6} + 1360\theta^{5} - 6010\theta^{4} + 7632\theta^{3} + 768\theta^{2} - 3744\theta + 6480}{(\theta^7 + \theta^6 + 8\theta^4 + 16\theta^3 + 12\theta + 12)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\theta^{7+}\theta^6 + 8\theta^4 + 16\theta^3 + 12\theta + 12}{\theta(\theta^3+2)(\theta^6+6)}$$

3.3 Moment generating function and Probability generating function of Poisson Ishita Distribution

We will derive moment generating function and Probability generating function of PID in this section.

Theorem 3.3.1: If X has the PID(θ), then the Probability generating function $P_X(t)$ has the following form

$$P_X(t) = \frac{\theta^3}{(\theta^3+2)(1+\theta)^2}\left[\frac{\theta(1+\theta)^2+2}{(\theta+1-t)} + \frac{4t\theta - 2t^2 + 4t}{(\theta+1-t)^3}\right]$$

Proof: We begin with the well known definition of the probability generating function given by

$$P_X(t) = \sum_{x=0}^{\infty} t^x\left[\frac{\theta^3}{(\theta^3+2)}\left(\frac{\theta(1+\theta)^2 + (x+1)(x+2)}{(1+\theta)^{x+3}}\right)\right]$$

$$P_X(t) = \frac{\theta^3}{(\theta^3+2)(1+\theta)^3}\left[\theta(1+\theta)^2\sum_{x=0}^{\infty}\left(\frac{t}{1+\theta}\right)^x + \sum_{x=0}^{\infty} x^2\left(\frac{t}{1+\theta}\right)^x + 3\sum_{x=0}^{\infty} x\left(\frac{t}{1+\theta}\right)^x + 2\sum_{x=0}^{\infty}\left(\frac{t}{1+\theta}\right)^x\right]$$

$$P_X(t) = \frac{\theta^3}{(\theta^3+2)(1+\theta)^3}\left[\frac{\theta(1+\theta)^2(1+\theta)}{(\theta+1-t)} + \frac{(t(\theta+1-t)+2t^2)(1+\theta)}{(\theta+1-t)^3} + \frac{3t(1+\theta)}{(\theta+1-t)^2} + \frac{2(1+\theta)}{(\theta+1-t)}\right]$$

$$P_X(t) = \frac{\theta^3}{(\theta^3+2)(1+\theta)^2}\left[\frac{\theta(1+\theta)^2+2}{(\theta+1-t)} + \frac{t\theta + t^2 + t}{(\theta+1-t)^3} + \frac{3t}{(\theta+1-t)^2}\right]$$

$$P_X(t) = \frac{\theta^3}{(\theta^3+2)(1+\theta)^2}\left[\frac{\theta(1+\theta)^2+2}{(\theta+1-t)} + \frac{4t\theta - 2t^2 + 4t}{(\theta+1-t)^3}\right]$$

Theorem 3.3.2 : If X has the PID (θ), then the moment generating function $M_X(t)$ has the following form

$$M_X(t) = \frac{\theta^3}{(\theta^3+2)(1+\theta)^2}\left[\frac{\theta(1+\theta)^2+2}{(\theta+1-e^t)} + \frac{4e^t\theta - 2e^{2t} + 4e^t}{(\theta+1-e^t)^3}\right]$$

Proof: We begin with the well known definition of the moment generating function given by

$$M_X(t)=\sum_{x=0}^{\infty}t^x\left[\frac{\theta^3}{(\theta^3+2)}\left(\frac{\theta(1+\theta)^2+(x+1)(x+2)}{(1+\theta)^{x+3}}\right)\right]$$

$$M_X(t)=\frac{\theta^3}{(\theta^3+2)(1+\theta)^3}\left[\theta(1+\theta)^2\sum_{x=0}^{\infty}\left(\frac{e^t}{1+\theta}\right)^x+\sum_{x=0}^{\infty}x^2\left(\frac{e^t}{1+\theta}\right)^x+3\sum_{x=0}^{\infty}x\left(\frac{e^t}{1+\theta}\right)^x+2\sum_{x=0}^{\infty}\left(\frac{e^t}{1+\theta}\right)^x\right]$$

$$M_X(t)=\frac{\theta^3}{(\theta^3+2)(1+\theta)^3}\left[\frac{\theta(1+\theta)^2(1+\theta)}{(\theta+1-e^t)}+\frac{(e^t(\theta+1-e^t)+2e^{2t})(1+\theta)}{(\theta+1-e^t)^3}+\frac{3e^t(1+\theta)}{(\theta+1-e^t)^2}+\frac{2(1+\theta)}{(\theta+1-e^t)}\right]$$

$$M_X(t)=\frac{\theta^3}{(\theta^3+2)(1+\theta)^2}\left[\frac{\theta(1+\theta)^2+2}{(\theta+1-e^t)}+\frac{4e^t\theta-2e^{2t}+4e^t}{(\theta+1-e^t)^3}\right]$$

IV. RELIABILITY ANALYSIS

In this section, we have obtained the reliability, hazard rate, reverse hazard rate and Mills ratio of the proposed Poisson Ishita model.

4.1 Reliability Function R(x)

The reliability function is defined as the probability that a system survives beyond a specified time. It is also referred to as survival or survivor function of the distribution. It can be computed as complement of the cumulative distribution function of the model. The reliability function or the survival function of Poisson Ishita distribution is calculated as:

$$R(x,\theta)=\left(\frac{\theta^5+2\theta^4+\theta^3+x^2\theta^2+5x\theta^2+6\theta^2+2x\theta+6\theta+2}{(1+\theta)^{x+3}(2+\theta^3)}\right)$$

4.2 Hazard Function

The hazard function is also known as hazard rate , instantaneous failure rate or force of mortality is given as:

$$\text{H.R}=h(x,\theta)=\frac{f(x,\theta)}{R(x,\theta)}=\frac{\theta^3[\theta(1+\theta)^2+(x+1)(x+2)]}{\theta^5+2\theta^4+\theta^3+x^2\theta^2+5x\theta^2+6\theta^2+2x\theta+6\theta+2}$$

4.3 Reverse Hazard Rate and Mills Ratio

The reverse hazard rate and the mills ratio of Poisson Ishita distribution are respectively given as:

$$\text{R.H.R}=h_r(x,\theta)=\frac{\theta^3[\theta(1+\theta)^2+(x+1)(x+2)]}{(1+\theta)^{x+3}(\theta^3+2)-(\theta^5+2\theta^4+\theta^3+x^2\theta^2+5x\theta^2+6\theta^2+2x\theta+6\theta+2)}$$

$$\text{Mills ratio}=\frac{(1+\theta)^{x+3}(\theta^3+2)-(\theta^5+2\theta^4+\theta^3+x^2\theta^2+5x\theta^2+6\theta^2+2x\theta+6\theta+2}{\theta^3[\theta(1+\theta)^2+(x+1)(x+2)]}$$

V. ORDER STATISTICS

Let $X_{(1)}, X_{(2)}, X_{(3)}...., X_{(n)}$ be the ordered statistics of the random sample $X_1, X_2, X_3,.... X_n$ drawn from the discrete distribution with cumulative distribution function $F_X(x)$ and probability mass function $P_X(x)$, then the probability mass function of rth order statistics $X_{(r)}$ is given by:

$$f_{x(r)}(x,c,\theta)=\frac{n!}{(r-1)!(n-r)!}P(x)[F(x)]^{r-1}[1-F(x)]^{n-r}.\ r=1, 2, 3,\ldots,n$$

Using the equations (2.1) and (2.2), the probability density function of rth order statistics of Poisson Ishita distribution is given by:

$$f_{(r)}(x,,\theta)=\frac{n!}{(r-1)!(n-r)!}\frac{\theta^3(\theta(1+\theta)^2+(x+1)(x+2))}{(\theta^3+2)(1+\theta)^{x+3}}\left[1-\frac{\theta^5+2\theta^4+\theta^3+x^2\theta^2+5x\theta^2+6\theta^2+2x\theta+6\theta+2}{(1+\theta)^{x+3}(\theta^3+2)}\right]^{r-1}$$
$$\left[\frac{\theta^5+2\theta^4+\theta^3+x^2\theta^2+5x\theta^2+6\theta^2+2x\theta+6\theta+2}{(1+\theta)^{x+3}(\theta^3+2)}\right]^{n-r}$$

Then, the pmf of first order $X_{(1)}$ Poisson Ishita distribution is given by:

$$f_1(x,\theta)=n\frac{\theta^3(\theta(1+\theta)^2+(x+1)(x+2))}{(\theta^3+2)(1+\theta)^{x+3}}\left[\frac{\theta^5+2\theta^4+\theta^3+x^2\theta^2+5x\theta^2+6\theta^2+2x\theta+6\theta+2}{(1+\theta)^{x+3}(\theta^3+2)}\right]^{n-1}.$$

and the pmf of nth order $X_{(n)}$ Poisson Ishita model is given as:

$$f_{w(n)}(x,\theta)=n\frac{\theta^3(\theta(1+\theta)^2+(x+1)(x+2))}{(\theta^3+2)(1+\theta)^{x+3}}\left[1-\frac{\theta^5+2\theta^4+\theta^3+x^2\theta^2+5x\theta^2+6\theta^2+2x\theta+6\theta+2}{(1+\theta)^{x+3}(\theta^3+2)}\right]^{n-1}$$

VI. ESTIMATION OF PARAMETERS

In this section, we estimate the parameters of the Poisson Ishita Distribution using methods of maximum likelihood estimation.

6.1 Method of Maximum Likelihood Estimation

This is one of the most useful method for estimating the different parameters of the distribution. Let $X_1, X_2, X_3,..., X_n$ be the random size of sample n draw from Poisson Ishita Distribution(PID), then the likelihood function of PID is given as

$$L(x\mid\theta)=\frac{\theta^{3n}}{(\theta^3+2)^n}\prod_{i=1}^{n}\left(\frac{(\theta(1+\theta)^2+(x+1)(x+2))}{(1+\theta)^{x+3}}\right)$$

$$\log L=3n\log\theta+\sum_{i=1}^{n}\log(\theta(1+\theta)^2+(x+1)(x+2))-n\log(\theta^3+2)-(\sum_{i=1}^{n}x_i+3n)\log(1+\theta)$$

$$\frac{\delta}{\delta\theta}\log L=\frac{3n}{\theta}+\sum_{i=1}^{n}\frac{(3\theta^2+4\theta+1)}{(\theta(1+\theta)^2+(x+1)(x+2)}-\frac{3n\theta^2}{(\theta^3+2)}-\frac{\sum_{i=1}^{n}x_i+3n}{(1+\theta)}=0$$

VIII. APPLICATIONS OF POISSON ISHITA DISTRIBUTION

In this section, we fit our proposed distribution to a dataset representing epileptic seizure counts (see Chakraborty (2010)) so as to illustrate our claim that our proposed model fits well when compared to other competing models. The data set representing epileptic seizure counts has a long right tail and approaches to zero slowly. The data set is given in table 1 below:

Table 1: Dataset representing epileptic seizure counts (see Chakraborty (2010))

epileptic seizure (X)	0	1	2	3	4	5	6	7	8
Observed Counts	126	80	59	42	24	8	5	4	3

In each of these distributions, the parameters are estimated by using the maximum likelihood method. We have analyzed the data using R software. Parameter estimates along with standard errors in braces and model function of the fitted distributions are given in table 2.

Table 2:.Estimated Parameters by ML method for fitted distributions for dataset representing epileptic seizure counts.

Distribution	Parameter Estimates (Standard Error)	Model function
Poisson Ishita Distribution	$\theta = 1.27(0.04)$	$P(X = x) = \frac{\theta^3}{\theta^3 + 2}\left[\frac{\theta(1+\theta)^2 + (x+1)(x+2)}{(1+\theta)^{x+3}}\right]$ $x = 0,1,2,3,\ldots; \theta > 0$
Poisson Distribution	$\lambda = 1.54\ (0.06)$	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}\ \lambda > 0;\ x = 0,1,2,\ldots..$
Poisson Lindley Distribution	$\theta = 0.973\ (0.053)$	$p(x) = \frac{\theta^2(x+\theta+2)}{(\theta+1)^{x+3}}\ x = 0,1,2,\ldots.\theta > 0$
Geometric Distribution	$p = 0.393\ (0.016)$	$p(x) = q^x p\ \ 0 < q < 1; q = 1\text{-}p; x = 0,1,2,\ldots..$
Negative Binomial Distribution	$r = 1.55, p = 0.501$ $(0.27, 0.047)$	$p(x) = \binom{x+r-1}{x} p^r q^x,\quad x = 0,1,2,\ldots$ $r > 0\ and\ 0 < p < 1$
Zero Inflated Poisson	$\alpha = 2.11, \lambda = 0.27$ $(0.10,\ 0.031)$	$p(x) = \begin{cases} \alpha + (1-\alpha)\frac{e^{-\lambda}\lambda^x}{x!}, \lambda > 0; x = 0 \\ (1-\alpha)\frac{e^{-\lambda}\lambda^x}{x!}, \lambda > 0; x = 0,1,2,.. \end{cases}$ $0 < \alpha < 1;\ \lambda > 0$

We compute the expected frequencies for fitting Poisson Ishita, Zero Inflated Poisson, Geometric, Poisson Lindley, Negative Binomial and Poisson distributions with the help of R studio statistical software and Pearson's chi-square test is applied to check the goodness of fit of the models discussed. The calculated figures are given in the table 3. Based on the chi-square, we observe that Poisson Ishita distribution provides a satisfactorily better fit for the data set representing epileptic seizure counts (see Chakraborty (2010))compared to other distributions.

Table 3: Fitted proposed distribution and other competing models to a dataset representing epileptic seizure counts

epileptic seizure (X)	Observed Counts	Poisson Distribution	Zero Inflated Poisson	Geometric Distribution	Poisson Lindley Distribution	Negative Binomial Distribution	Poisson Ishita Distribution
0	126	74.935	126.000	137.963	128.681	120.201	129.839
1	80	115.712	65.080	83.736	87.136	93.009	83.920
2	59	89.339	68.974	50.823	55.267	59.184	54.622
3	42	45.985	48.733	30.847	33.636	34.949	34.427
4	24	17.752	25.824	18.722	19.898	19.837	20.871
5	8	5.482	10.948	11.363	11.529	10.987	12.208
6	5	1.411	3.868	6.897	6.575	5.984	6.927
7	4	0.311	1.171	4.186	3.703	3.221	3.831
8	3	0.072	0.402	6.464	4.577	3.627	4.357
Degrees of freedom		4	4	6	6	5	6
Chi-Statistic Value		80.913	14.621	10.808	5.473	5.383	4.941

p-value	0.0000 1	0.0121	0.094	0.484	0.372	0.551

Furthermore, in order to compare our proposed distribution and other competing models, we consider the criteria like AIC (Akaike information criterion), AICC (corrected Akaike information criterion) and BIC (Bayesian information criterion). The better distribution corresponds to lesser AIC, AICC and BIC values.

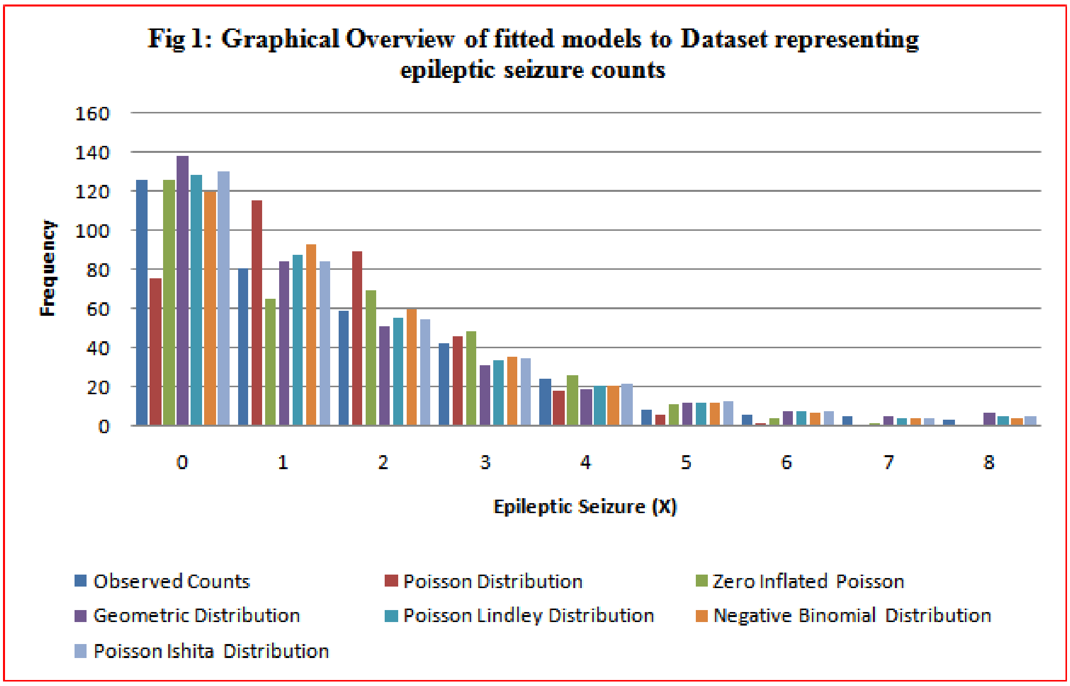
$$\text{AIC} = 2k-2\log L \qquad \text{AICC} = \text{AIC}+\frac{2k(k+1)}{n-k-1} \text{ and BIC} = k\log n-2\log L$$

where k is the number of parameters in the statistical model, n is the sample size and -2log*L* is the maximized value of the log-likelihood function under the considered model. From Table 4, it has been observed that the Poisson Ishita distribution have the lesser AIC, AICC and BIC values as compared toother competing models. Hence we can concluded that the Poisson Ishita distribution leads to a better fit than the other competing models for analyzing the data set given in table 1.

Table 4: Model comparison criterion for fitted models to a dataset representing epileptic seizure counts

Criterio n	**Poisson Distributio n**	**Zero Inflated Poisson**	**Geometric Distribution**	**Poisson Lindley Distribution**	**Negative Binomial Distribution**	**Poisson Ishita Distribution**
-logl	636.045	599.637	598.396	595.181	594.942	594.764
AIC	1274.091	1203.274	1198.791	1192.362	1193.884	1191.527
BIC	1277.952	1210.996	1202.652	1196.223	1201.605	1195.388

Figure 1 gives a graphical overview of the fitted distributions to a data set given in table 1.



IX. CONCLUSION

A new probability distribution is introduced using compounding technique. Statistical properties of the proposed model are studied and application in handling count dataset representing epileptic seizure counts is analyzed.

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