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Parameterized Linear Programming Based Method and its Application to Fuzzy Project Planning Problem

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Abstract: In this paper, a new method is presented to find critical path of a network project planning problem in imprecise environment using parameterized linear programming. A new parameterized defuzzification is discussed. Here network model has been developed in fuzzy environment. All activities are considered as a generalized trapezoidal fuzzy number. In this work, we develop a parameterized linear programming problem and then solve the parametric network problem. To illustrate the technique, an airport's cargo ground operation system is considered here

Keywords: Network Problem, Critical Path Method, Fuzzy Number, Trapezoidal Fuzzy Number

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I. INTRODUCTION

In today's highly competitive business environment, one of the most challenging jobs that any manager can take on is the management of a large-scale project that requires coordinating numerous activities throughout the organization to maintain competitive priorities such as on time delivery and customization. A myriad of details must be considered in planning how to coordinate all these activities, in developing a realistic schedule, and then in monitoring the progress of the project. Fortunately, two closely related operations research techniques, PERT (Program Evaluation Review Eechnique) and CPM(Critical Path Method), are available to assist the project manager in carrying out these responsibilities. These techniques make heavy use of networks to help plan and display the coordination of all the activities. They also normally use a software package to deal with all the data needed to develop schedule information and then to monitor the progress of the project. In order to solve these types of problems several methods exists in the literature. For instance, Chen[1] proposed an approach to critical path analysis for a project network with activity times being fuzzy numbers, in that membership function of fuzzy total duration time is constructed which is based on the extension principle and linear programming. Chen and Hsueh[2] presented a simple approach to solve the critical path method problem with fuzzy activity times on the basis of the linear programming formulation and the fuzzy number ranking method that are more realistic than crisp ones. They also defined that the most critical path and the relative path degree of criticality which are easy to use in practice. Sireesha and Shankar [3] presented a new method based on fuzzy theory for solving fuzzy project scheduling in fuzzy environment. Shankar et al. [4] presented an analytical method for measuring the criticality in a fuzzy project network, where the duration time of each activity is represented by a trapezoidal fuzzy number. Yakhchali et al.[5] assumed time lags which are common practice in the different projects, are imprecise and they discussed the problems of possibly critical path in the networks with interval valued activity and time lag duration. Yao and Lin [6] proposed a method for ranking fuzzy numbers without the need for any assumptions and used both positive and negative values to define ordering which then applied to critical path method. Liang and Han [7] presented an algorithm to perform fuzzy critical path analysis for project network problem. They showed that the ambiguities involved in the assessment activity times in a project network can be effectively improved and thus a more convincing and effective project management decision-making can be obtained. Zielinski [8] extended some results for interval numbers to the fuzzy case for determining the possibility distribution describing latest starting times for activity. He also proposed the time algorithm for computing the intervals of the possible values of the latest starting times of an activity in general networks with interval durations and extended the results to the networks with fuzzy duration. In this paper, we are finding an alternative way to deal with the situation,where duration can be large and in order to find a specific and optimized range, we made a concept of fuzziness in which each and every instances of duration is made a set with the given intervals by putting the function. The optimized duration is further taken by the decision makers..

II. PREREQUISITE MATHEMATICS

Fuzzy sets were first introduced by Zadeh[9] in 1965 as a mathematical way of representing impreciseness orvagueness in everyday life.

2.1. Fuzzy Set

A fuzzy set $\widetilde{A}$ in a universe of discourse X is defined as the following set of pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in X\}$.Here $\mu_{\widetilde{A}} : X \to [0,1]$ is a mapping called the membership function of the fuzzyset $\widetilde{A}$ and $\mu_{\widetilde{A}}(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set $\widetilde{A}$ .The larger $\mu_{\widetilde{A}}(x)$ is the stronger the grade of membership form in $\widetilde{A}$.

2.2. Fuzzy Number

A fuzzy number is a special case of a fuzzy set. Different definitions and properties of fuzzy numbers are encountered in the literature but they all agree on that fuzzy number represents the conception of a set of 'real numbers close to a ' where ' a ' is the number being fuzzy field. A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. A fuzzy number $\widetilde{A}$ is a fuzzy set of the real line $\Re$ whose membership function $\mu_{\widetilde{A}}(x)$ has the following characteristic with $-\infty < a_1 < a_2 < a_3 < a_4 < \infty$

$$\mu_{\widetilde{A}}(x) = \begin{cases} \mu_L(x) & for\ a_1 \le x < a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \mu_R(x) & for\ a_3 < x \le a_4 \\ 0 & for\ otherwise \end{cases}$$

where $\mu_L(x) : [a_1, a_2] \to [0,1]$ is continuous and strictly increasing ; $\mu_R(x) : [a_3, a_4] \to [0,1]$ is continuous and strictly decreasing.The general shape of a fuzzy number following the above definition is shown below

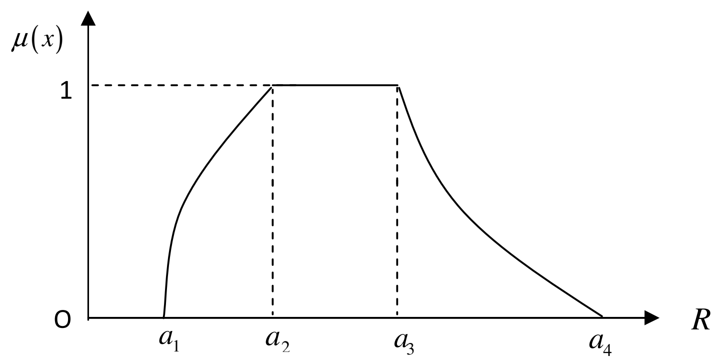

Fig-1 Fuzzy Number

2.3. α -cut of a Fuzzy Number

The α -level of a fuzzy number $\widetilde{A}$ is defined as a crisp set $A_\alpha = \{x : \mu_{\widetilde{A}}(x) \ge \alpha, x \in X, \alpha \in [0,1]\}$. A_α is non-empty bounded closed interval contained in X and it can be denoted by $A_\alpha = [A_L(\alpha), A_R(\alpha)]$, $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval , respectively. Fig-2.5 shows a fuzzy number $\widetilde{A}$ with $\alpha-$cuts $A_{\alpha_1} = [A_L(\alpha_1), A_R(\alpha_1)], A_{\alpha_2} = [A_L(\alpha_2), A_R(\alpha_2)]$. It is seen that if $\alpha_2 \ge \alpha_1$ then $A_L(\alpha_2) \ge A_L(\alpha_1)$ and $A_R(\alpha_2) \ge A_R(\alpha_1)$.

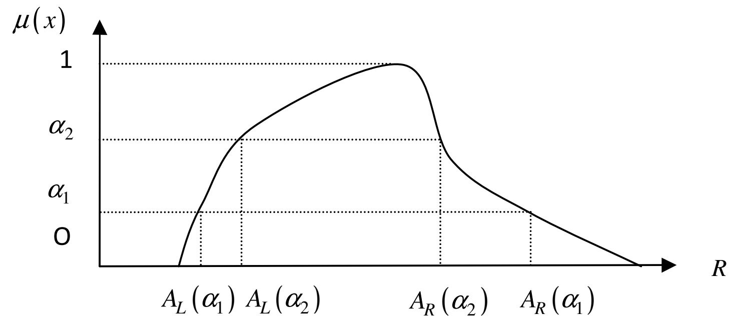

Fig-2 Fuzzy number $\widetilde{A}$ with α -cut

2.4.Generalized Fuzzy Number (GFN)

Generalized fuzzy number $\widetilde{A}$ as $\widetilde{A}=(a,b,c,d;w)$ where $0<w\le 1$ and a,b,c and d are real numbers. The generalized fuzzy number $\widetilde{A}$ is a fuzzy subset of real line R , whose membership function $\mu_{\widetilde{A}}(x)$ satisfies the following conditions:

(1) $\mu_{\widetilde{A}}(x)$ is a continuous mapping from R to the closed interval $[0,1]$.
(2) $\mu_{\widetilde{A}}(x)=0$ where $-\infty<x\le a$;
(3) $\mu_{\widetilde{A}}(x)$ is strictly increasing with constant rate on $[a,b]$
(4) $\mu_{\widetilde{A}}(x)=w$ where $b\le x\le c$;
(5) $\mu_{\widetilde{A}}(x)$ is strictly decreasing with constant rate on $[c,d]$;
(6) $\mu_{\widetilde{A}}(x)=0$ where $d\le x<\infty$.

Note: $\widetilde{A}$ is a convex fuzzy set and it is a non-normalized fuzzy number till $w\ne 1$.It will be normalized for $w=1$.

(i) If $a=b=c=d$ and $w=1$, then $\widetilde{A}$ is called a real number a .

Here $\widetilde{A}=(x,\mu_{\widetilde{A}}(x))$ with membership function $\mu_{\widetilde{A}}(x)=\begin{cases}1 \text{ if } x=a\\0 \text{ if } x\ne a\end{cases}$

(ii) If $a=b$ and $c=d$, then $\widetilde{A}$ is called crisp interval $[a,b]$.

Here $\widetilde{A}=(x,\mu_{\widetilde{A}}(x))$ with membership function $\mu_{\widetilde{A}}(x)=\begin{cases}1 \text{ if } a\le x\le d\\0 \quad otherwise\end{cases}$

(iii) If $b=c$, then $\widetilde{A}$ is called a generalized triangular fuzzy number (GTFN) as $\widetilde{A}=(a,b,d;w)$

(iv) If $b=c$, $w=1$ then it is called a triangular fuzzy number (TFN) as $\widetilde{A}=(a,b,d)$.

Here $\widetilde{A}=(x,\mu_{\widetilde{A}}(x))$ with membership function $\mu_{\widetilde{A}}(x)=\begin{cases}w\left(\dfrac{x-a}{b-a}\right) & for\ \ a\le x\le b\\ w\left(\dfrac{d-x}{d-b}\right) & for\ b\le x\le d\\ 0 & otherwise\end{cases}$

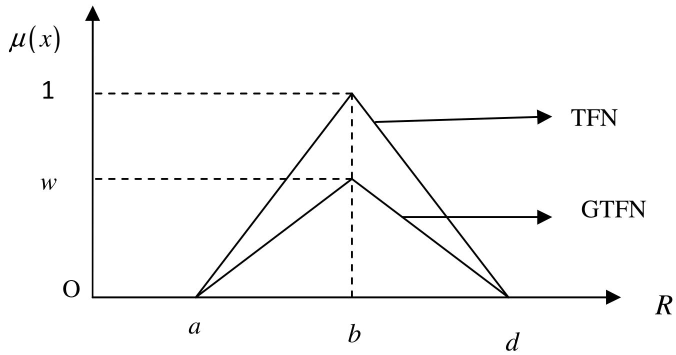

Fig-3 TFN and GTFN

(v) If $b \neq c$, then $\widetilde{A}$ is called a generalized trapezoidal fuzzy number (GTrFN) as $\widetilde{A} = (a,b,c,d;w)$

(vi) If $b \neq c$, $w=1$ then it is called a trapezoidal fuzzy number (TrFN) as $\widetilde{A} = (a,b,c,d)$.

Here $\widetilde{A} = (x, \mu_{\widetilde{A}}(x))$ with membership function $\mu_{\widetilde{A}}(x) = \begin{cases} w\left(\dfrac{x-a}{b-a}\right) & for\ \ a \le x \le b \\ w & for\ b \le x \le c \\ w\left(\dfrac{d-x}{d-c}\right) & for\ c \le x \le d \\ 0 & otherwise \end{cases}$

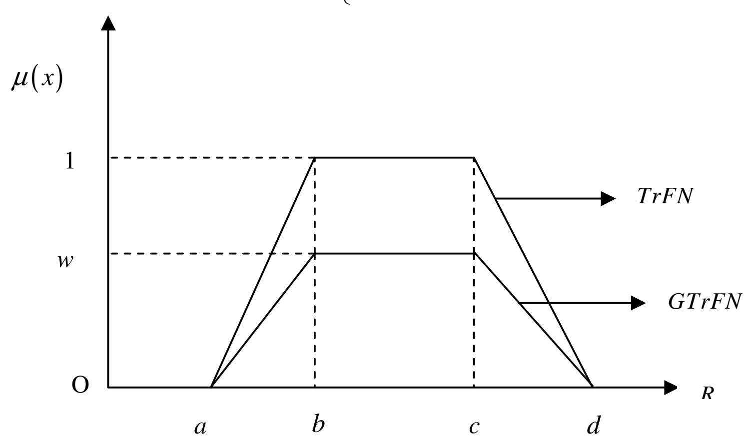

Fig-4 TrFN and GTrFN

Fig-4shows GTrFNs $\widetilde{A} \equiv (a,b,c,d;w)$ and TrFN $\widetilde{A} \equiv (a,b,c,d)$ which indicate different decision maker's opinions for different values of $w, 0 < w \le 1$.The values of w represents the degree of confidence of the opinion of the decision maker.

Because of traditional fuzzy arithmetic operations we can any deal with normalized fuzzy numbers; they not only change the type of membership function of fuzzy number after arithmetic operations, but also have a drawback of requiring troublesome and tedious arithmetic operations.

III. DEFUZZIFICATION OF FUZZY NUMBERS WITH RESPECT THEIR TOTAL INTEGRAL VALUE

Let $\lambda \in [0,1]$ be a pre-assigned parameter called the degree of optimism. The graded mean value (or, total λ-integral value) of $\widetilde{A}$ is defined as $I_\lambda(\widetilde{A}) = \lambda I_R(\widetilde{A}) + (1-\lambda) I_L(\widetilde{A})$ where $I_R(\widetilde{A})$ and $I_L(\widetilde{A})$ are the right and left interval values of $\widetilde{A}$ defined as

$$I_L^w(\widetilde{A}) = \int_0^1 \left(\mu_{L\widetilde{A}}^w\right)^{-1} \alpha d\alpha$$

$I_R^w(\widetilde{A}) = \int_0^1 \left(\mu_{R\widetilde{A}}^w\right)^{-1} \alpha d\alpha$

Now, for GTrFN $\widetilde{A} \equiv (a,b,c,d;w)$

$\left(\mu_{L\widetilde{A}}^w\right)^{-1} \alpha = a + \frac{\alpha}{w}(b-a)$ and $\left(\mu_{R\widetilde{A}}^w\right)^{-1} \alpha = d - \frac{\alpha}{w}(d-c)$

Therefore the left and right integral values are $I_L^w(\widetilde{A}) = w\left(\frac{a+b}{2}\right)$ and $I_R^w(\widetilde{A}) = w\left(\frac{c+d}{2}\right)$

Hence the total λ-integral value of $\widetilde{A}$ is $I_\lambda^w(\widetilde{A}) = \left[\lambda w\left(\frac{c+d}{2}\right) + (1-\lambda)w\left(\frac{a+b}{2}\right)\right]$

The total λ-integral value is a convex combination of the right and left integral values through the degree of optimism. The left integral value is used to reflect the pessimistic viewpoint and the right integral value is used to reflect the optimistic viewpoint of the decision-maker. A large value of λ specifies the higher degree of optimism. For instance, when $\lambda = 1$, the total integral value $I_1^w(\widetilde{A}) = w\left(\frac{c+d}{2}\right) = I_R^w(\widetilde{A})$ represents an optimistic viewpoint. On the other hand, when $\lambda = 0$, the total λ-integral value is $I_0^w(\widetilde{A}) = w\left(\frac{a+b}{2}\right) = I_L^w(\widetilde{A})$ represents a pessimistic viewpoint. When $\lambda = 0.5$ the total λ-integral is $I_{0.5}^w(\widetilde{A}) = \frac{1}{2}\left[w\left(\frac{a+b}{2}\right) + w\left(\frac{c+d}{2}\right)\right]$.

It reflects a moderately optimistic decision-makers viewpoint and is the same as the defuzzification of the fuzzy number $\widetilde{A}$.

Property 3.1

(a) If GTrFN $\widetilde{U} = (u_1,u_2,u_3,u_4;w)$ and $y = ku$ (with $k < 0$) then $\tilde{y} = k\tilde{u}$ is a GTrFN $(ku_1,ku_2,ku_3,ku_4;w)$.

(b) If $y = ku$ (with $k < 0$) then $\tilde{y} = k\tilde{u}$ is a GTrFN $(ku_1,ku_2,ku_3,ku_4;w)$.

Proof:

(a) When $k > 0$ with the transformation $y = ku$, we can find the membership function of fuzzy set $\tilde{y} = k\tilde{u}$ by α -cut method.

α -cut of $\widetilde{U}$ is $\left[\left(\mu_{L\widetilde{U}}^w\right)^{-1}\alpha, \left(\mu_{R\widetilde{U}}^w\right)^{-1}\alpha\right] = \left[u_1 + \frac{\alpha}{w}(u_2-u_1), u_4 - \frac{\alpha}{w}(u_4-u_3)\right]$ for any $\alpha \in [0,1]$

i.e. $u \in \left[u_1 + \frac{\alpha}{w}(u_2-u_1), u_4 - \frac{\alpha}{w}(u_4-u_3)\right]$

So $y(=ku) \in \left[ku_1 + \frac{\alpha}{w}(ku_2-ku_1), ku_4 - \frac{\alpha}{w}(ku_4-ku_3)\right]$.

Thus, we get the membership function of $\tilde{y} = k\tilde{u}$ as

$$\mu_{\tilde{y}}^w(x) = \begin{cases} w\left(\dfrac{y-ku_1}{ku_2-ku_1}\right) & for \;\; ku_1 \le y \le ku_2 \\ w & for \;\; ku_2 \le y \le ku_3 \\ w\left(\dfrac{ku_4-y}{ku_4-ku_3}\right) & for \;\; ku_3 \le y \le ku_4 \\ 0 & otherwise \end{cases}$$

(b) Similarly we can prove that $y = ku$, $k < 0$ then

$$\mu_{\tilde{y}}(x)=\begin{cases} w\left(\dfrac{y-ku_1}{ku_2-ku_1}\right) & for\ \ ku_2\le y\le ku_1 \\ w & for\ ku_3\le y\le ku_2 \\ w\left(\dfrac{ku_4-y}{ku_4-ku_3}\right) & for\ \ ku_4\le y\le ku_3 \\ 0 & otherwise \end{cases}$$

Property 3.2

If $\tilde{A}_1=(a_1,b_1,c_1,d_1;w_1)$ and $\tilde{A}_2=(a_2,b_2,c_2,d_2;w_2)$ then $\tilde{A}_1\oplus\tilde{A}_2$ is a fuzzy number $(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2;\min(w_1,w_2))$.

Proof:

With the transformation $y=x_1+x_2$ we can find the membership function of fuzzy set $\tilde{y}=\tilde{A}_1\oplus\tilde{A}_2$ by α -cut method.

α - cut of $\tilde{A}_1$ is $\left[\left(\mu^{w_1}_{L\tilde{A}_1}\right)^{-1}\alpha,\left(\mu^{w_1}_{R\tilde{A}_1}\right)^{-1}\alpha\right]=\left[a_1+\dfrac{\alpha}{w_1}(b_1-a_1),d_1-\dfrac{\alpha}{w_1}(d_1-c_1)\right]\ \forall\ \alpha\in[0,1]$

i.e. $x_1\in\left[a_1+\dfrac{\alpha}{w_1}(b_1-a_1),d_1-\dfrac{\alpha}{w_1}(d_1-c_1)\right]$.

α -cut of $\tilde{A}_2$ is $\left[\left(\mu^{w_1}_{L\tilde{A}_2}\right)^{-1}\alpha,\left(\mu^{w_1}_{R\tilde{A}_2}\right)^{-1}\alpha\right]=\left[a_2+\dfrac{\alpha}{w_2}(b_2-a_2),d_2-\dfrac{\alpha}{w_2}(d_2-c_2)\right]\ \forall\ \alpha\in[0,1]$

i.e. $x_2\in\left[a_2+\dfrac{\alpha}{w_2}(b_2-a_2),d_2-\dfrac{\alpha}{w_2}(d_2-c_2)\right]$.

So, $y(=x_1+x_2)\in\left[a_1+a_2+\dfrac{\alpha}{w}\left((b_1-a_1)+(b_2-a_2)\right),d_1+d_2-\dfrac{\alpha}{w}\left((d_1-c_1)+(d_2-c_2)\right)\right]$ where $w=\min(w_1,w_2)$. Therefore ,we have

$\alpha=w\left(\dfrac{y-a_1-a_2}{b_1+b_2-a_1-a_2}\right)$, $a_1+a_2\le y\le b_1+b_2$, and $\alpha=w\left(\dfrac{d_1+d_2-y}{d_1+d_2-c_1-c_2}\right)$, $c_1+c_2\le y\le d_1+d_2$

So, we have the membership function of $\tilde{y}=\tilde{A}_1\oplus\tilde{A}_2$ is

$$\mu^{w}_{\tilde{y}}(y)=\begin{cases} w\left(\dfrac{y-a_1-a_2}{b_1+b_2-a_1-a_2}\right) & for\ a_1+a_2\le y\le b_1+b_2 \\ w & for\ b_1+b_2\le y\le c_1+c_2 \\ w\left(\dfrac{d_1+d_2-y}{d_1+d_2-c_1-c_2}\right) & for c_1+c_2\le y\le d_1+d_2 \\ 0 & otherwise \end{cases}$$

Thus we have $\tilde{A}_1\oplus\tilde{A}_2=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2;\min(w_1,w_2))$.

Note: If we have the transformation $\tilde{y}=k_1\tilde{A}_1\oplus k_2\tilde{A}_2$ (k_1,k_2 are (not all zero) real numbers) then the fuzzy set $\tilde{y}=k_1\tilde{A}_1\oplus k_2\tilde{A}_2$ is the following GTrFN:

(i) $k_1>0,\ k_2\ge 0$ or $k_1\ge 0,k_2>0$,

$\tilde{y}=(k_1a_1+k_2a_2,k_1b_1+k_2b_2,k_1c_1+k_2c_2,k_1d_1+k_2d_2:w)$

(ii) $k_1>0,\ k_2\le 0$ or $k_1\ge 0,k_2<0$,

$\tilde{y}=(k_1a_1+k_2d_2,k_1b_1+k_2c_2,k_1c_1+k_2b_2,k_1d_1+k_2a_2:w)$

(iii) $k_1<0,\ k_2\ge 0$ or $k_1\le 0,k_2>0$,

$\tilde{y} = (k_1 d_1 + k_2 d_2, k_1 c_1 + k_2 c_2, k_1 b_1 + k_2 b_2, k_1 a_1 + k_2 a_2 : w)$

IV. LINEAR PROGRAMMING FORMULATION OF FUZZY CRITICAL PATH PROBLEM

Consider a project network $S = \langle V, A, t \rangle$, consisting of a finite set V of nodes (events) and a set $A \subset V \times V$ of arcs with crisp activity times, which are determined by a function $t : A \to \mathbb{R}^+$ and attached to the arcs. Denote t_{ij} as the time period of activity $(i, j) \in A$. The linear programming formulation assumes that a unit flow enters the project network at the start node and leaves at the finish node. Let x_{ij} be the decision variable denoting the amount of flow an activity $(i, j) \in A$. Since only one unit of flow could be in any arc at any one time, the variable x_{ij} must assume binary values (0 or 1) only. The CPM problem with n nodes is formulated as:

$$D = Maximize \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} x_{ij}$$

$$Subject\ to \sum_{j=1}^{n} x_{1j} = 1,$$

$$\sum_{j=1}^{n} x_{ij} = \sum_{k=1}^{n} x_{ki}, \quad i = 2, 3, \ldots, n-1. \tag{1}$$

$$\sum_{k=1}^{n} x_{kn} = 1,$$

$$x_{ij} = 0\, or\, 1, (i, j) \in A$$

The objective is to maximize the total duration time of the project network from node 1 to node n. The critical path for this network consists of a set of activities $(i, j) \in A$ from the start to the finish in which each activity in the path corresponds to the optimal decision variable $x_{ij}^* = 1$ in the optimal solution to model (1).

Intuitively, if any of the activity duration time t_{ij}, the coefficient in the objective function of the model (1), is fuzzy, the total duration time d becomes fuzzy as well.

Consequently, the fuzzy CPM problem is of the following form:

$$\tilde{D} = Maximize \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{T}_{ij} x_{ij}$$

$$Subject\ to \sum_{j=1}^{n} x_{1j} = 1,$$

$$\sum_{j=1}^{n} x_{ij} = \sum_{k=1}^{n} x_{ki}, \quad i = 2, 3, \ldots, n-1. \tag{2}$$

$$\sum_{k=1}^{n} x_{kn} = 1,$$

$$x_{ij} = 0\, or\, 1, (i, j) \in A$$

Denote fuzzy number $\tilde{T}_{ij}$ as the fuzzy duration time of activity $(i, j) \in A$, and its membership function is $\mu_{\tilde{T}_{ij}}(t_{ij})$. Then we have $\tilde{T}_{ij} = \left\{ \left(t_{ij}, \mu_{T_{ij}}(t_{ij}) \right) \middle| t_{ij} \in S(\tilde{T}_{ij}) \right\}, (i, j) \in A$, where $S(\tilde{T}_{ij})$ is the support of $\tilde{T}_{ij}$,which denotes the universe set of activity time of activity $(i, j) \in A$.

V. CRISP TRANSFORMATION USING $\lambda-$ INTEGRAL VALUE

Using the $\lambda-$ integral values of the fuzzy duration activity $\widetilde{T}_{ij}$ in the above problem (2), we get

$$\widetilde{D} = Maximize \sum_{i=1}^{n} \sum_{j=1}^{n} I_{\lambda}^{w_{ij}} \left(\widetilde{T}_{ij}\right) x_{ij}$$

$$Subject\ to \sum_{j=1}^{n} x_{1j} = 1,$$

$$\sum_{j=1}^{n} x_{ij} = \sum_{k=1}^{n} x_{ki}, \quad i = 2,3,...,n-1. \tag{3}$$

$$\sum_{k=1}^{n} x_{kn} = 1,$$

$$x_{ij} = 0\, or\, 1, (i, j) \in A$$

VI. APPLICATION OF METHOD TO AIRPORT'S CARGO GROUND OPERATION SYSTEM

As the volume of cargo traffic has grown and the demand for cargo transport continues to rise, surface congestion has become an increasing problem, within an airport's cargo terminal. If an airport terminal's internal operations and service systems are inefficient, there will be a delay in ground operations. Therefore, cargo operations time needs to be shortened and passengers' luggage must be processed before cargo goods in order to maintain customer satisfaction. Fig.5 shows an international airport cargo terminal's ground operation procedures network. With the set of nodes $\mathrm{N} = \{1,2,3,4,5\}$,the fuzzy activity time for each activity is shown in table 1.

Table 1: Fuzzy activity time for each activity

Activity A_{ij}	Fuzzy activity time (minutes)
A_{12}	(10,15,15,20;0.8)
A_{13}	(30,40,40,50;0.7)
A_{23}	(30,40,40,50;0.7)
A_{14}	(15,20,25,30;0.6)
A_{25}	(60,100,150,180;0.9)
A_{35}	(60,100,150,180;0.9)
A_{45}	(60,100,150,180;0.9)

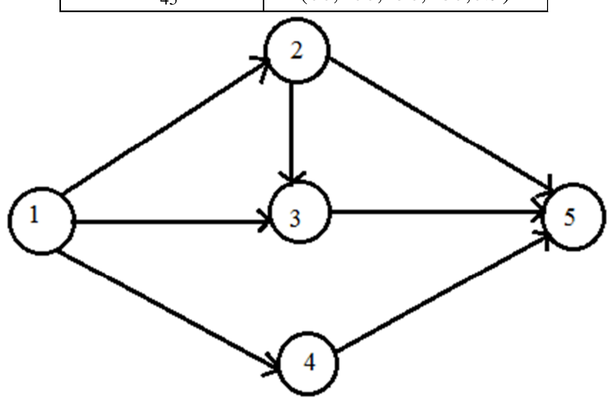

Fig-5.Airport cargo terminal's ground operation network

Solution: According to model (2), this problem can be formulated as follows:

$$\begin{aligned}
&Maximize\left(\tilde{T}_{12}x_{12}+\tilde{T}_{13}x_{13}+\tilde{T}_{14}x_{14}+\tilde{T}_{23}x_{23}+\tilde{T}_{25}x_{25}+\tilde{T}_{35}x_{35}+\tilde{T}_{45}x_{45}\right)\\
&Subject\ to\ \ x_{12}+x_{13}+x_{14}=1,\\
&x_{12}=x_{23}+x_{25},\\
&x_{13}+x_{23}=x_{35},\\
&x_{14}=x_{45},\\
&x_{25}+x_{35}+x_{45}=1,\\
&x_{12},x_{13},x_{14},x_{23},x_{25},x_{35},x_{45}\geq 0
\end{aligned} \tag{4}$$

The associated mathematical programming problem based on model (3) is:

$$\begin{aligned}
&Maximize\begin{pmatrix} I_\lambda^{w_{12}}\left(\tilde{T}_{12}\right)x_{12}+I_\lambda^{w_{13}}\left(\tilde{T}_{13}\right)x_{13}+I_\lambda^{w_{14}}\left(\tilde{T}_{14}\right)x_{14}+I_\lambda^{w_{23}}\left(\tilde{T}_{23}\right)x_{23}+I_\lambda^{w_{25}}\left(\tilde{T}_{25}\right)x_{25}+I_\lambda^{w_{35}}\left(\tilde{T}_{35}\right)x_{35}\\ +I_\lambda^{w_{45}}\left(\tilde{T}_{45}\right)x_{45} \end{pmatrix}\\
&Subject\ to\ \ x_{12}+x_{13}+x_{14}=1,\\
&x_{12}=x_{23}+x_{25},\\
&x_{13}+x_{23}=x_{35},\\
&x_{14}=x_{45},\\
&x_{25}+x_{35}+x_{45}=1,\\
&x_{12},x_{13},x_{14},x_{23},x_{25},x_{35},x_{45}\geq 0
\end{aligned} \tag{5}$$

The optimal solution of the fuzzy project network model for different values of λ are presented in table 2.

Table 2 optimal solution of the fuzzy Airport cargo terminal's ground operation network

Test	Decision variables	Critical Path	Duration
Optimistic i.e $\lambda=1$	$x_{12}=x_{23}=x_{35}=1;\ x_{13}=x_{14}=x_{25}=x_{45}=0$	1-2-3-5	194.00
About optimistic i.e $\lambda=0.7$	$x_{12}=x_{23}=x_{35}=1;\ x_{13}=x_{14}=x_{25}=x_{45}=0$	1-2-3-5	167.25
Moderate i.e $\lambda=0.5$	$x_{12}=x_{23}=x_{35}=1;\ x_{13}=x_{14}=x_{25}=x_{45}=0$	1-2-3-5	150.25
About pessimistic i.e $\lambda=0.2$	$x_{12}=x_{23}=x_{35}=1;\ x_{13}=x_{14}=x_{25}=x_{45}=0$	1-2-3-5	124.00
Pessimistic i.e $\lambda=0$	$x_{12}=x_{23}=x_{35}=1;\ x_{13}=x_{14}=x_{25}=x_{45}=0$	1-2-3-5	106.50

The critical path is 1-2-3-5.

VII. CONCLUSION

This paper presents a simple approach to solve the CPM problem with fuzzy number (generalized trapezoidal fuzzy number) activity times that are more realistic than crisp ones. On the basis of $\lambda-$integral value, the fuzzy CPM problem is transformed into a crisp CPM problem. We than solved the problem. Here decision maker may obtain the optimal project duration according to his/her expectations of optimistic/pessimistic/moderate values of project duration. This method presented is quite general and can be applied in other areas of operations research.

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