IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 09, Issue 3 (March. 2019), ||S (III) || PP 51-53

Some Expansion Formulae For The Aleph (ℵ)-Function

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Abstract: In the present paper, the author has established two expansion formula of Aleph ℵ -Function.
Keywords: ℵ -Function, Expansion Formula, Gamma Function
(2010 Mathematical Subject Classification: 33C99)

Date of Submission: 11-03-2019 Date of acceptance: 28-03-2019

I. INTRODUCTION

The ℵ - function introduced by Suland et.al. [3] defined and represented in the following form:

$$\aleph[z] = \aleph^{m,n}_{p_i,q_i;\tau_i;r}[z] = \aleph^{m,n}_{p_i,qi;\tau_i;r}\left[z \,\middle|\, \begin{matrix} (a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i} \\ (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i} \end{matrix} \right]$$

$$= \frac{1}{2\pi w}\int_L \theta(s) z^s ds \tag{1.1}$$

Where $w = \sqrt{-1}$;

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\tau_i\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s\right\}} \tag{1.2}$$

We shall use the following notations:

$$A^* = (a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}; B^* = (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}$$

II. EXPANSION FORMULA

First Formula

$$\aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[\eta\omega\middle|\begin{matrix}A^*\\B^*\end{matrix}\right] = \eta^{\frac{b_1}{\beta_1}}\sum_{r=0}^{\infty}\frac{\left[1-\eta^{1/\beta_1}\right]^r}{r!}\aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[\omega\middle|\begin{matrix}A^*\\(r+b_1,\beta_1),(b_j,\beta_j)_{2,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right] \tag{2.1}$$

Where η is written for $m=1$ and for

$m>1, |\eta^{1/\beta}-1|<1; \arg(\eta\omega) = \beta_1\arg(\eta^{1/\beta_1}) + \arg\omega$ and $|\arg(\eta^{1/\beta_1})| < \frac{\pi}{2}$.

Proof: R.H.S.= $\eta^{\frac{b_1}{\beta_1}}\sum_{r=0}^{\infty}\frac{\left[1-\eta^{1/\beta_1}\right]^r}{r!}\aleph^{m,n}_{p_i,q_i;\tau_i;r}\left[\omega\middle|\begin{matrix}A^*\\(r+b_1,\beta_1),(b_j,\beta_j)_{2,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right]$

$$= \eta^{b_1/\beta_1} \sum_{r=0}^{\infty} \frac{\left[1-\eta^{1/\beta_1}\right]^r}{r!} \frac{1}{2\pi w} \int_L \frac{\Gamma(r+b_1-\beta_1 s)\prod_{j=2}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\tau_i\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right\}} \omega^s ds$$

On changing the order of integration and summation under the integral sign, we get

$$\text{R.H.S.} \quad = \frac{1}{2\pi w} \int_L \theta(s)\omega^s \left[\eta^{b_1/\beta_1} \sum_{r=0}^{\infty} \frac{\left[1-\eta^{1/\beta_1}\right]^r}{r!} \Gamma(r+b_1-\beta_1 s) \right] ds$$

$$= \frac{1}{2\pi w} \int_L \theta(s)\omega^s \left[\eta^{b_1/\beta_1} \sum_{r=0}^{\infty} \frac{\left[1-\eta^{1/\beta_1}\right]^r}{r!} (b_1-\beta_1 s)_r \Gamma(b_1-\beta_1 s) \right] ds$$

$$= \frac{1}{2\pi w} \int_L \theta(s)\omega^s \left[\eta^{b_1/\beta_1} \left[1-(1-\eta^{1/\beta_1})\right]^{-(b_1-\beta_1 s)} \quad \Gamma(b_1-\beta_1 s) \right] ds$$

$$\left[\because \sum \frac{x^r}{r!}(a)_r = (1-x)^{-a} \right]$$

$$= \frac{1}{2\pi w} \int_L \theta(s)\omega^s \left[\eta^{b_1/\beta_1} \left[(\eta)^{1/\beta_1}\right]^{-b_1+\beta_1 s} \quad \Gamma(b_1-\beta_1 s) \right] ds$$

$$= \frac{1}{2\pi w} \int_L \theta(s)\omega^s \eta^s \Gamma(b_1-\beta_1 s) ds$$

$$= \frac{1}{2\pi w} \int_L \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\tau_i\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right\}} (\omega\eta)^s ds = \text{L.H.S.}$$

Second Formula

$$\aleph_{p_i,q_i:\tau_i:r}^{m,n}\left[\eta\omega\middle|_{B^*}^{A^*}\right] = \eta^{\frac{b_q}{\beta_q}} \sum_{r=0}^{\infty} \frac{\left[\eta^{1/\beta_q}-1\right]^r}{r!} \aleph_{p_i,q_i:\tau_i:r}^{m,n}\left[\omega\middle|_{(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-1},(r+b_q,\beta_q)}^{A^*}\right] \quad (2.2)$$

Where $q > m, |\eta^{1/\beta_q} - 1| < 1; \arg(\eta\omega) = \beta_q \arg(\eta^{1/\beta_q}) + \arg\omega$ and $|\arg(\eta^{1/\beta_q})| < \frac{\pi}{2}$.

Proof: R.H.S.$= \eta^{\frac{b_q}{\beta_q}} \sum_{r=0}^{\infty} \frac{\left[\eta^{1/\beta_q}-1\right]^r}{r!} \aleph_{p_i,q_i:\tau_i:r}^{m,n}\left[\omega\middle|_{(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-1},(r+b_q,\beta_q)}^{A^*}\right]$

$$= \eta^{b_q/\beta_q} \sum_{r=0}^{\infty} \frac{\left[\eta^{1/\beta_q}-1\right]^r}{r!} \frac{1}{2\pi w} \int_L \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\tau_i\left\{\Gamma(1-r-b_q+\beta_q s)\prod_{j=m+1}^{q_i-1}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right\}} \omega^s ds$$

On changing the order of integration and summation under the integral sign, we yield

R.H.S. $$=\frac{1}{2\pi w}\int_L \theta(s)\omega^s \left[\eta^{b_q/\beta_q}\sum_{r=0}^{\infty}\frac{\left[\eta^{1/\beta_q}-1\right]^r}{r!\Gamma(1-r-b_q+\beta_q s)}\right]ds$$

$$=\frac{1}{2\pi w}\int_L \theta(s)\omega^s \left[\eta^{b_q/\beta_q}\sum_{r=0}^{\infty}\frac{\left[\eta^{1/\beta_q}-1\right]^r}{r!}\frac{1}{(1-b_q+\beta_q s)_r\Gamma(1-b_q+\beta_q s)}\right]ds$$

$$=\frac{1}{2\pi w}\int_L \frac{\theta(s)\omega^s}{\Gamma(1-b_q+\beta_q s)}\sum_{r=0}^{\infty}\frac{\left[\eta^{1/\beta_q}-1\right]^r(-1)^r\left[1-(1-b_q+\beta_q)\right]_r}{r!}ds$$

$$=\frac{1}{2\pi w}\int_L \frac{\theta(s)\omega^s}{\Gamma(1-b_q+\beta_q s)}\sum_{r=0}^{\infty}\frac{\left[1-\eta^{1/\beta_q}\right]^r}{r!}\eta^{b_q/\beta_q}\left[b_q-\beta_q s)\right]_r ds$$

$$=\frac{1}{2\pi w}\int_L \frac{\theta(s)\omega^s}{\Gamma(1-b_q+\beta_q s)}\left[1-(1-\eta^{1/\beta_q})\right]^{-(b_q-\beta_q s)}\eta^{b_q/\beta_q}ds$$

$$\left[\because \sum\frac{x^r}{r!}(a)_r=(1-x)^{-a}\right]$$

$$=\frac{1}{2\pi w}\int_L \frac{\theta(s)\omega^s}{\Gamma(1-b_q+\beta_q s)}\left[(\eta)^{1/\beta_q}\right]^{-(b_q-\beta_q s)}\eta^{b_q/\beta_q}ds$$

$$=\frac{1}{2\pi w}\int_L \frac{\theta(s)\omega^s\eta^s}{\Gamma(1-b_q+\beta_q s)}ds$$

$$=\frac{1}{2\pi w}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\tau_i\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right\}}(\omega\eta)^s ds \text{ =L.H.S.}$$

For $\tau_i=1, r=1$ in (2.1),(2.2), we get the results in terms of Fox's H-function [1], [2].

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Poonam Kumari. “Some Expansion Formulae For The aleph(ℵ)-Function.” IOSR Journal of Engineering (IOSRJEN), vol. 09, no. 03, 2019, pp. 51-53.