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An Application of Dynamic Programming Problem in Multi Stage Transportation Problem with Fuzzy Random Parameters

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Abstract: In general, transportation from origin to final destination can't be made in a single transportation. In this paper, a multi stage transportation problem (MSTP) is formed where the unit transportation costs are fuzzy random in nature. Using dynamic programming approach, the problem first converted into a single stage problem. Such single stage problem optimized with minimized the expected total cost and expected total variance. The problem is solved for both with and without limitation of the intermediate depots. Here a numerical example is solved to check the validity of the proposed method.
Key Words: Multi stage transportation problem, Fuzzy random co-efficient, Dynamic programming problem.

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I. INTRODUCTION

The transportation problem (TP) means shipping of commodities from different sources to destinations under the objective to determine the shipping schedule that minimizes that total shipping cost while satisfying supply and demand limits. The TP finds application in communication network, scheduling, industry, planning, transportation and allotment system etc. In general transportation processes may not be performed directly between the suppliers and customers. There may exist different warehouses in different stages. Such type of transportation problems is known as multi stage transportation problems (MSTP). Geoffrion and Graves [1] were the pioneers who studied the two-stage distribution problem. Brezina and Istvanikova [9] presented a way of solving two-stage transportation problem. Das [4] describes cost varying multistage transportation problem. Malhotra and Malhotra [15] proposed a polynomial bound algorithm for a two-stage time minimization problem to obtain optimal schedules for stage I and stage II. The main objective of the multi-stage transportation problem is similar to single stage transportation problem under the satisfaction of all intermediate warehouse's limit.

Now, the concept of fuzzy-random-variable (FRV) was introduced by Kwakernaak [7], where all the calculations and theories are done under the consideration of independency nature of the variables. In his next article [8] several algorithms about conversion of fuzzy random variable to deterministic variable is performed with a lot of examples for the discrete case. Puri and Ralescu [13] developed the new idea to generate the fuzziness and they stated that the expected value could be fuzzy but the variance should be scalar. The occurrence of fuzzy-random variable/parameter makes the combination of randomness and fuzziness more persuasive. These are some decision making problems formuled and solved with fuzzy random parameter/ variables. Very few TPs have been formulated and solved with fuzzy-random costs/resources. A system involving both randomness and fuzziness [18] which can be characterized by random variables and fuzzy variables separately. Gani and Razak [2] solved a two-stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers, where a parametric approach has been used for the imprecise nature. Pandian et al. [14] proposed a method namely, zero-point method, for finding a fuzzy optimal solution for a fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers. Ritha and Vinotha [17] proposed a method for finding a best compromise solution to a multi-objective two-stage fuzzy transportation problem using geometric programming approach. Ojha et al. [3] considered mean chance in same transportation problems with fuzzy stochastic cost. These works illustrate the great significance of mean chance measure in the area of fuzzy random optimizations. A comparison of the existing literature is presented here to represent the new contribution of this paper.

Table 1: Summary of related literature

Author(s)	Type of the TP	Stage of the TP	Nature of cost	Solution Procedure
Ojha. et.al.	TP	Single stage	fuzzy-stochastic cost	Genetic algorithm
W. Ritha, J. M. Inotha	Fuzzy TP	Two stage	transportation cost	Fuzzy geometric programming approach
I. Berzina, A. Stranikova	TP	Two stage	transportation cost	Parametric approach
C. B. Das	TP	Multi-stage	varying cost	Parametric approach
S. Malhotra, R. Malhotra	Time minimizing TP	Two stage	crisps	Polynomial time algorithm
A. N. Gani, K.A. Razak	Fuzzy TP	Two stage	transportation cost	Parametric approach
P. Pandian, G. Natarajan	Fuzzy TP	Single	fuzzy cost	Parametric approach
This paper	TP	Multi stage	Fuzzy random cost	Dynamic programming

II. PRELIMINARY CONCEPTS

2.1 Fuzzy Number: A fuzzy number refer to a set of possible values characterize by its membership function $\boldsymbol{\mu_{\tilde{a}}}: \mathbb{R} \to [\mathbf{0}, \mathbf{1}]$, which satisfies the following conditions;

(i) $\mu_{\tilde{a}}$ is normal. i.e., $\exists\, x_0 \in \mathbb{R}, \mu_{\tilde{a}}(x_0) = 1$.

(ii) $\mu_{\tilde{a}}$ is convex. i.e., $\mu_{\tilde{a}}(tx + (1-t)y) \geq \min\{\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)\}\, \forall t \in [0,1], x, y \in \mathbb{R}$.

(iii) $\mu_{\tilde{a}}$ is upper semi-continuous on $\mathbb{R}$. i.e.,$\forall\, \varepsilon > 0, \exists\, \delta > 0$ such that $\mu_{\tilde{a}}(x) - \mu_{\tilde{a}}(x_0) < \varepsilon$, $|x - x_0| < \delta$.

(iv) $\mu_{\tilde{a}}$ is compactly supported. i.e., $cl\{\, x \in \mathbb{R}, \mu_{\tilde{a}} > 0\}$ is compact, where $cl(\tilde{A})$ denotes the closer of the set $\tilde{A}$.

$\alpha - cuts$: Let $\tilde{A}$ be the set of all fuzzy numbers. The set(crisp) of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α-cut or α-level set and defined by $A_\alpha = \{x \in X: \tilde{A}(x) \geq \alpha, 0 \leq \alpha \leq 1 = [A_\alpha^L, A_\alpha^R]$, where $A_\alpha^L = \min\{x \in \mathbb{R}: \tilde{A}(x) \geq \alpha\}$ and $A_\alpha^R = \max\{x \in \mathbb{R}: \tilde{A}(x) \geq \alpha\}$. The addition and scalar multiplication on A_α be defined by $[a+b]_\alpha = a_\alpha + b_\alpha$ and $\lambda a_\alpha = \lambda a_\alpha$ for all $\tilde{a}, \tilde{b} \in A$, $\tilde{\lambda} \in \mathbb{R}, \alpha \in [0,1]$. A matric on $\tilde{A}$ is defined by $d(\tilde{a}, \tilde{b}) = \frac{1}{2}\int_0^1 (|a_\alpha^L - b_\alpha^L|^2 + |a_\alpha^R - b_\alpha^R|^2) d\alpha$,$\forall \tilde{a}, \tilde{b} \in A$ where a_α^L, a_α^R are the left and right end points of A_α, then the ordered pair $(\tilde{A}, d)$ is a complete matric space.

2.2 Fuzzy random variable (FRV)

Let (Ω, F, P) be a complete probability space and **A** denote the set of all fuzzy real number. A fuzzy random variable is a function $\hat{\tilde{X}}: \Omega \to \boldsymbol{A}$ such that for any Borel set $\boldsymbol{B}$ of $\mathbb{R}$, $Cr\{\hat{\tilde{X}}(w) \in \boldsymbol{B}\}$ is a measurable function of w. Let **A** be a fuzzy real number system, then $\hat{\tilde{X}}$ is a f.r.v if and only if X_α^L and X_α^U are ordinary random variable $\forall \alpha \in [0,1]$.

The Kwakernaak FRV: Let a FRV is a mapping $\hat{\tilde{X}}: \Omega \to \boldsymbol{A}$ such that for any a $\alpha \in [0,1]$. X and all $w \in \Omega$, the real valued mapping.

$inf X_\alpha: \Omega \to \mathbb{R}$, satisfying $inf\{X_\alpha(w)\} = \ inf\{(X(w))_\alpha\}$ and

$Sup X_\alpha: \Omega \to \mathbb{R}$, satisfying $Sup\{X_\alpha(\mathrm{w})\} = \ inf\{(X(w))_\alpha\}$ are real valued random variables

Given w the unique characteristic of the Kwakernaak FRV is captured by its $\alpha - cut$, which is shown below

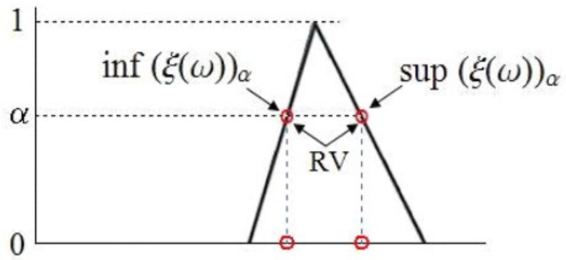

Figure 1: $\alpha - cut$ of a Kwakernaak FRV

Definition: Let (Ω, F, Pos) be a possibility space and $\hat{\tilde{X}}$ be a fuzzy random variable with membership function μ and B a set of real numbers. Then the credibility of a fuzzy event $\{w \in \Omega: \hat{\tilde{X}}\}(w) \in B\}$ is defined by

$$Cr(\hat{\tilde{X}} \in B) = \frac{1}{2}\left(Pos(\hat{\tilde{X}} \in B) + Nes(\hat{\tilde{X}} \in B)\right)$$

where *Pos* and *Nes* represent the possibility measure and the necessity measure [11, 10], respectively.
Let $\hat{\tilde{X}}$ be a fuzzy random variable defined on the probability space (Ω, *F*, *P*). Then The upper expected value of the fuzzy random variable $\hat{\tilde{X}}$ is defined as

$$\bar{E}\left(\hat{\tilde{X}}\right) = \int_{\Omega}\left[\int_{0}^{\infty} Pos\left\{\hat{\tilde{X}}(w) \geq r\right\} dr - \int_{-\infty}^{0} Nes\left\{\hat{\tilde{X}}(w) \leq r\right\} dr\right] P(r)dw$$

And the lower expected value of the fuzzy random variable is defined by

$$\underline{E}\left(\hat{\tilde{X}}\right) = \int_{\Omega}\left[\int_{0}^{\infty} Nes\left\{\hat{\tilde{X}}(w) \geq r\right\} dr - \int_{-\infty}^{0} Pos\left\{\hat{\tilde{X}}(w) \leq r\right\} dr\right] P(r)dw$$

The expected value of the fuzzy random variable is defined as

$$E\left(\hat{\tilde{X}}\right) = \int_{\Omega}\left[\int_{0}^{\infty} Cr\left\{\hat{\tilde{X}}(w) \geq r\right\} dr - \int_{-\infty}^{0} Cr\left\{\hat{\tilde{X}}(w) \leq r\right\} dr\right] P(r)dw$$

Provided at least of the two integrals is finite. If $\hat{\tilde{X}}(w)$ is a non-negative fuzzy random variable, its expected value will be

$$E\left(\hat{\tilde{X}}\right) = \int_{\Omega}\int_{0}^{\infty} Cr\left\{\hat{\tilde{X}}(w) \geq r\right\} dr\, P(r)dw$$

If $\hat{\tilde{X}}$ be a fuzzy random variable with the probability distribution $P\left(\hat{\tilde{X}} = w_i\right) = \tilde{p}_i; i = 1,2, \ldots$, then its expectation is defined by

$$\begin{aligned} E\left(\hat{\tilde{X}}\right) &= \sum_{i=1}^{n} p_i \int_{0}^{\infty} Cr\left\{\hat{\tilde{X}}(w_i) \geq r\right\} dr - \sum_{i=1}^{n} p_i \int_{-\infty}^{0} Cr\left\{\hat{\tilde{X}}(w_i) \leq r\right\} dr \\ &= \sum_{i=1}^{n} p_i \left[\int_{0}^{\infty} Cr\left\{\hat{\tilde{X}}(w_i) \geq r\right\} dr - \int_{-\infty}^{0} Cr\left\{\hat{\tilde{X}}(w_i) \leq r\right\} dr\right] \\ &= \sum_{i=1}^{n} p_i\, E\left(\hat{\tilde{X}}(w_i)\right) \end{aligned}$$

where $[E(\hat{\tilde{X}}(w_i)]$ is the expected value of fuzzy variable $\hat{\tilde{X}}(w_i)$.

Theorem: Let $\hat{\tilde{X}}$ be a fuzzy random variable, then the expected value $E[\hat{\tilde{X}}(w)]$ of fuzzy $\hat{\tilde{X}}(w)$ is a random variable.

Proof: In order to prove that the expected value of $\hat{\tilde{X}}(w)$ is an r.v., it suffices to prove that $E[\hat{\tilde{X}}(w)]$ is a measurable function of w.
In fact, by

$$\begin{aligned} E\left[\hat{\tilde{X}}(w)\right] &= \int_{0}^{\infty} Cr\left\{\hat{\tilde{X}}(w) \geq r\right\} dr - \int_{-\infty}^{0} Cr\left\{\hat{\tilde{X}}(w) \leq r\right\} dr \\ &= \lim_{N\to\infty}\lim_{N\to\infty}\left(\sum_{k=1}^{n}\frac{N}{n} Cr\left\{\hat{\tilde{X}}(w) \geq \frac{kN}{n}\right\} - \sum_{k=1}^{n}\frac{N}{n} Cr\left\{\hat{\tilde{X}}(w) \leq \frac{-kN}{n}\right\}\right) \end{aligned}$$

and the measurability $Cr\left\{\hat{\tilde{X}}(w) \geq \frac{kN}{n}\right\}$ of and $Cr\left\{\hat{\tilde{X}}(w) \leq \frac{-kN}{n}\right\}$, we deduce that $E[\hat{\tilde{X}}(w)]$ an r.v.. The proof of the theorem is complete.

Definition: Let $\hat{\tilde{X}} = x_1, x_2, \ldots, x_n$ be a fuzzy random vector and $f_j: \mathbb{R}^n \to \mathbb{R}$ continuous function for *j* = 1,2,...,*m*. Then the mean chance, denoted by Ch^c, of fuzzy random event in additive measure is defined as

$$Ch^c\left\{f_j(\hat{\tilde{X}}) \leq 0\right\} = \int_{\Omega} Cr\left\{f_j(\hat{\tilde{X}}(w)) \leq 0\right\} Pdw\,, \qquad j = 1,2, \ldots, m$$

Similarly

$$Ch^p\left\{f_j(\hat{\tilde{X}}) \leq 0\right\} = \int_{\Omega} Pos\left\{f_j(\hat{\tilde{X}}(w)) \leq 0\right\} Pdw\,, \qquad j = 1,2, \ldots, m$$

$$Ch^p\left\{f_j(\hat{\tilde{X}}) \leq 0\right\} = \int_{\Omega} Nes\left\{f_j(\hat{\tilde{X}}(w)) \leq 0\right\} Pdw\,, \qquad j = 1,2, \ldots, m$$

Criptization of fuzzy random parameter: Let $\hat{\tilde{C}}_{ij} = (C_{ij}^1, C_{ij}^2, C_{ij}^3)$ be the fuzzy random parameter. Then the membership function $\mu_{\hat{\tilde{C}}_{ij}}(y): \mathbb{R} \to [0,1]$

$$\mu_{\hat{\tilde{C}}_{ij}}(y) = \begin{cases} \dfrac{y - C_{ij}^1}{C_{ij}^2 - C_{ij}^1} & for\ C_{ij}^1 < y \le C_{ij}^2 \\ \dfrac{C_{ij}^3 - y}{C_{ij}^3 - C_{ij}^2} & for\ C_{ij}^2 < y \le C_{ij}^3 \\ 0 & otherwise \end{cases}$$

The mean of this membership function is $m\left(\hat{\tilde{C}}_{ij}\right) = \frac{C_{ij}^1 + 4C_{ij}^2 + C_{ij}^3}{6}$

*Definition :*Let $\hat{\tilde{X}}$ be an f.r.v. with finite expected value $E[\hat{\tilde{X}}]$. The variance $Var\left(\hat{\tilde{X}}\right)$of $\hat{\tilde{X}}$ is defined as the expected value of f.r.v $(\hat{\tilde{X}} - E[\hat{\tilde{X}}])^2$. i.e.,

$$Var\left(\hat{\tilde{X}}\right) = E[(\hat{\tilde{X}} - E[\hat{\tilde{X}}])^2]$$

Definition: Assume that$f: \mathbb{R}^n \to \mathbb{R}$ is a measurable function, and ξ_i is a f.r.vs on the probability spaces (Ω_i, F_i, P_i), *i=1,2,...,n* respectively. Then $\hat{\tilde{X}} = f(\xi_1, \xi_2, \ldots, \xi_n)$ is a f.r.v on the product probability space of (Ω_i, F_i, P_i), *i=1,2,...,n*, defined by $\hat{\tilde{X}}(w_1, w_2, \ldots, w_n) = f(\xi_1(w_1), \xi_2(w_2), \ldots, \xi_n(w_n)), \forall\ (w_1, w_2, \ldots, w_n) \in \prod_{i=1}^n \Omega_i$

Definition:$D: F(\mathbb{R}) \times F(\mathbb{R}) \to [0, \infty)$be the equation

$$D_{p,q}(\tilde{A}, \tilde{B}) = \begin{cases} \left(\displaystyle\int_0^1 |q(A_\alpha^R - B_\alpha^R) + (1-q)(A_\alpha^L - B_\alpha^L)|^p dq\right)^{1/p} & if\ 1 \le p < \infty \\ sup_{\alpha \in [0,1]} |q(A_\alpha^R - B_\alpha^R) + (1-q)(A_\alpha^L - B_\alpha^L)| & if\ p = \infty \end{cases}$$

Where $D_{p,q}$ is the distance defined on the set of fuzzy numbers.

Definition: Let $\left\{\hat{\tilde{X}}_n, \hat{\tilde{X}}: n \ge 1\right\}$ be a sequence of fuzzy random variables of real value. If $\hat{\tilde{X}}_n \xrightarrow{a.\ s.\ d} \hat{\tilde{X}},\ then$ $\hat{\tilde{X}}_n \xrightarrow{i.\ P.\ D} \hat{\tilde{X}}$, where a.s.D means almost surly based on $D_{p,q}$.

Definition: Let $\left\{\hat{\tilde{X}}_n, \hat{\tilde{X}}: n \ge 1\right\}$ be a sequence of fuzzy random variables of real value. If $\hat{\tilde{X}}_n \to \hat{\tilde{X}}$, then $E\left[\hat{\tilde{X}}_n\right] = E[X]$

Theorem: Let $\left\{\hat{\tilde{X}}_n, \hat{\tilde{X}}: n \ge 1\right\}$ be a sequence of fuzzy random variables and *Y* be independent random variable such that $Y \in L_1, \hat{\tilde{X}} \in L_1(F), \hat{\tilde{X}}_n \in L_1(F)$. If $\hat{\tilde{X}}_n \xrightarrow{a.\ s.\ d} \hat{\tilde{X}}$, then $E[\hat{\tilde{X}}_n Y] \xrightarrow{D} E\left[\hat{\tilde{X}}\right] E[Y]$, Where L_1 is the fuzzy integrable space.

Proof: By the above two definition, $E[\hat{\tilde{X}}_n] \xrightarrow{D} E[X]$, since $\hat{\tilde{X}}_n \xrightarrow{a.\ s.\ d} \hat{\tilde{X}}$.

Therefore, $E\left[\hat{\tilde{X}}_n\right] E[Y] \xrightarrow{D} E[\hat{\tilde{X}}]\, E[Y]$.

Now, to complete the proof we show that $E[\hat{\tilde{X}}_n]E[Y] \xrightarrow{D} E[\hat{\tilde{X}}_n Y]$. For all fuzzy random variables, we have,

$$\hat{\tilde{X}}_n(w) = \left\{[\hat{\tilde{X}}_{n\alpha}^L(w), \hat{\tilde{X}}_{n\alpha}^R(w): \alpha \in [0,1]]\right\} \forall w \in \Omega.$$

Thus, $\sigma\left(\hat{\tilde{X}}_n\right) = \sigma\left(\{\hat{\tilde{X}}_{n\alpha}: 0 \le \alpha \le 1\}\right)$. We also know that a fuzzy random variable $\hat{\tilde{X}}_n$ and areal-valued random variable Y are independent if and only if $\sigma(\hat{\tilde{X}}_n)$ and $\sigma(Y)$ are independent for $n \ge 1$, i.e, for any $A \in \sigma(\hat{\tilde{X}}_n)$ and $B \in \sigma(Y)$, $P(A \cap B) = P(A)P(B)$.

Therefore, it is enough to show that $E\left[\hat{\tilde{X}}_{n\alpha}\right] E[Y] = E\left[\hat{\tilde{X}}_{n\alpha} Y\right], \forall \alpha \in [0,1]$

Now, let $Y = I_A$ for $A \in \sigma(Y)$. Since I_A is a random set thus $I_A \hat{\tilde{X}}_{n\alpha}$ is a random set. By Aumann integral we have, $E[\hat{\tilde{X}}_{n\alpha}] = \{E(Z): Z(w) \in \hat{\tilde{X}}_{n\alpha}(w)\}$ and

$$\begin{aligned} E\left[\hat{\tilde{X}}_{n\alpha} I_A\right] &= \left\{E(Z I_A): Z(w) \in \hat{\tilde{X}}_{n\alpha}(w)\right\} \\ &= \left\{E(Z)P(A): Z(w) \in \hat{\tilde{X}}_{n\alpha}(w)\right\} \\ &= P(A)\left\{E(Z): Z(w) \in \hat{\tilde{X}}_{n\alpha}(w)\right\} \\ &= E[Y]E[\hat{\tilde{X}}_{n\alpha}] \end{aligned}$$

Now, if $Y = \sum_{i=1}^n a_i I_A$, then we have

$$E\left[Y\hat{\hat{X}}_{n\alpha}\right]=E\left[\sum_{i=1}^{n}a_i I_{A_i}\hat{\hat{X}}_{n\alpha}\right]$$
$$=\sum_{i=1}^{n}a_i E[I_{A_i}\hat{\hat{X}}_{n\alpha}]$$
$$=\sum_{i=1}^{n}a_i P(A_i)E[\hat{\hat{X}}_{n\alpha}]$$
$$=\sum_{i=1}^{n}a_i E[I_{A_i}]E[\hat{\hat{X}}_{n\alpha}]$$
$$=E[\sum_{i=1}^{n}a_i I_{A_i}]E[\hat{\hat{X}}_{n\alpha}]$$

Hence the theorem.

Addition: The addition of the two fuzzy random parameters $\tilde{a}_{ij}^1$ and $\tilde{a}_{ij}^2$, each of which can be represented by the following triangular fuzzy number and their associated probabilities,

$$\tilde{a}_{ij}^1=\begin{cases}(a_{11},a_{12},a_{13}) & with\, probability\, p_1\\ (a_{21},a_{22},a_{23}) & with\, probability\, p_2\end{cases}$$
$$\tilde{a}_{ij}^2=\begin{cases}(a_{31},a_{32},a_{33}) & with\, probability\, p_3\\ (a_{41},a_{42},a_{43}) & with\, probability\, p_4\end{cases}$$

Adding these two fuzzy random parameters we get

$$\tilde{a}_{ij}^1+\tilde{a}_{ij}^2=\begin{cases}(a_{11}+a_{31}\,,\ a_{12}+a_{32}\,,\ a_{13}+a_{33}) & with\, probability\, p_1p_3\\ (a_{11}+a_{41}\,,\ a_{12}+a_{42}\,,\ a_{13}+a_{43}) & with\, probability\, p_1p_4\\ (a_{21}+a_{31}\,,\ a_{22}+a_{32}\,,\ a_{23}+a_{33}) & with\, probability\, p_2p_3\\ (a_{21}+a_{41}\,,\ a_{22}+a_{42}\,,\ a_{23}+a_{43}) & with\, probability\, p_2p_4\end{cases}$$

Scalar multiplication: Let $\tilde{a}_{ij}$ is a f.r.v represented as a triangular fuzzy number its probability(p_i)

$$\lambda\tilde{a}_{ij}^1=\begin{cases}(\lambda a_{11},\lambda a_{12},\lambda a_{13}) & with\, probability\, p_1\\ (\lambda a_{21},\lambda a_{22},\lambda a_{23}) & with\, probability\, p_2\end{cases}$$

Then, adding two f.r.vs we get $\tilde{a}_{ij}=\{(X_1,P_1),(X_2,P_2),(X_3,P_3),(X_4,P_4)\}$. Then expectation of $\tilde{a}_{ij}$ is

$$E(\tilde{a}_{ij})=X_1P_1+X_2P_2+X_3P_3+X_4P_4$$

Example 1: Let $\hat{\hat{C}}_{ij}$ be the f r.v whose fuzzy parameters represented by TFN and individual probabilities are discreate as given below:

$$\hat{\hat{C}}_{ij}=\begin{cases}(C_{11}^1,C_{12}^2,C_{13}^3) & with\, probability\ \ 0.6\\ (C_{21}^1,C_{22}^2,C_{23}^3) & with\, probability\ \ 0.4\end{cases}$$

Then the expected value is

$$E\left(\hat{\hat{C}}_{ij}\right)=\frac{C_{11}^1+4C_{12}^2+C_{13}^3}{6}0.6+\frac{C_{21}^1+4C_{22}^2+C_{23}^3}{6}0.4$$

Example 2: Same as Example-1, where individual probabilistic are continuous. The mean chance of these fuzzy number are:

For $0\le y\le C_{ij}^1$

$$Ch^c\left(\hat{\hat{C}}_{ij}\ge y\right)=Cr\left(\hat{\hat{C}}_{ij}\ge y\right)Pr+Cr\left(\hat{\hat{C}}_{ij}\ge y\right)Pr;\ C_{ij}^1\le\hat{\hat{C}}_{ij}\le C_{ij}^3, C_{ij}^3\le\hat{\hat{C}}_{ij}\le C_{ij}^5$$

Similarly, for $C_{ij}^1\le y\le C_{ij}^3$

$$Ch^c\left(\hat{\hat{C}}_{ij}\ge y\right)=Cr\left(\hat{\hat{C}}_{ij}\ge y\right)Pr+Cr\left(\hat{\hat{C}}_{ij}\ge y\right)Pr;\ C_{ij}^1\le\hat{\hat{C}}_{ij}\le C_{ij}^3, C_{ij}^3\le\hat{\hat{C}}_{ij}\le C_{ij}^5$$

Similarly, for $C_{ij}^3\le y\le C_{ij}^5$

$$Ch^c\left(\hat{\hat{C}}_{ij}\ge y\right)=Cr\left(\hat{\hat{C}}_{ij}\ge y\right)Pr+Cr\left(\hat{\hat{C}}_{ij}\ge y\right)Pr;\ C_{ij}^1\le\hat{\hat{C}}_{ij}\le C_{ij}^3, C_{ij}^3\le\hat{\hat{C}}_{ij}\le C_{ij}^5$$

Similarly, for $C_{ij}^5\le y\le C_{ij}^6$

$$Ch^c\left(\hat{\hat{C}}_{ij}\ge y\right)=Cr\left(\hat{\hat{C}}_{ij}\ge y\right)Pr+Cr\left(\hat{\hat{C}}_{ij}\ge y\right)Pr;\ C_{ij}^1\le\hat{\hat{C}}_{ij}\le C_{ij}^3, C_{ij}^3\le\hat{\hat{C}}_{ij}\le C_{ij}^5$$

Then according to the mean chance the expected value is

$$E\left(\hat{\hat{C}}_{ij}\right)=\int_0^{C_{ij}^1}dy+\int_{C_{ij}^1}^{C_{ij}^3}\mu c_{ij}dy+\int_{C_{ij}^3}^{C_{ij}^5}\mu c_{ij}dy$$

2.3 Dynamic programming

This method or technique can be used to solve different types of optimization problems. D.P. obtains solutions by working forward and/or backward from the beginning and/or end of a problem towards the end and/or beginning. Thus, it breaking up a large unwieldy problem into a series of smaller, more tractable and inter related problems, such sub-problems are called stages. Where each stage is connected together by state variables. After solving every sub-problem, the solution of original problem can be achieved by combining them using the state variables. It is a general strategy for optimization rather than a specific set of rules. Consequently, the particular equation must be developed to fit each problem. Abdelwali H. A. [6] introduced parametric multi-objective dynamic programming to solve some of automotive problems.

In this paper, the D.P. technique is applied to find the shortest route between each source of the first stage to each destinations of the last stage of a multistage transportation problem. Here, the shortest route means having the minimum transportation cost from any source of the first stage to any destinations of the last stage of the transportation network.

Regarding to find the fuzzy random costs of the shortest routes, the algebraic operation proposed by H. Kwakernak is used here. Then find out the expected cost of each route and corresponding variance. After that with respect to total expected cost and total expected variance, the model is optimized.

III. Mathematical Formulation

Let us consider a K-stage transportation problem. Also m_k be the number of depots (if k = 1, then if is terms as origin/ source) of the k–th level and $n_k$be the number of depots (if $k = K$, then it is called as destination) of the (k + 1)-th level. The associate cost is $\hat{\tilde{c}}_{j_{k+1}}^{i_k}$ required to transported the unit commodity from $i_k - th$ depot ($i_k = 1,2, \dots , m_k$) to $j_k - th$ depot ($j_k = 1,2, \dots , n_k$), which is fuzzy random in nature. The K stage fuzzy random transportation problem can be formulated as

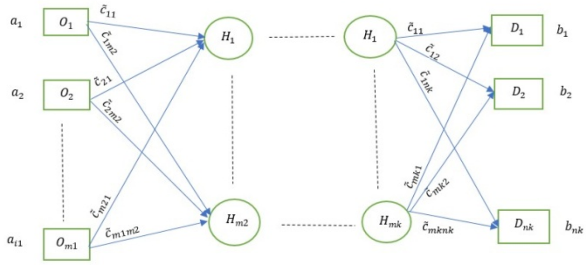

Figure 2: Multi stage transportation problem

$$\min_{X_1 \in S} \left[\hat{\tilde{T}}_1(X_1) + \min_{X_2 \in S_1} \left(\hat{\tilde{T}}_2(X_2) \right) + \cdots + \min_{X_n \in S_{n-1}} \left(\hat{\tilde{T}}_K(X_K) \right) \right]$$

As each $T_k(X_k)$is linear homogeneous of this form, $\sum \sum \hat{\tilde{c}}_{j_{k+1}}^{i_k} x_{j_{k+1}}^{i_k}$ then this problem is equivalent to

$$Minimize\ \hat{\tilde{T}} = \sum_{k=1}^{K} \sum_{i_k=1}^{m_k} \sum_{j_{k+1}=1}^{m_{k+1}} \hat{\tilde{c}}_{j_{k+1}}^{i_k} \left(1 - \sum_{p_{k-1}=1}^{m_{k-1}} \lambda_{i_k}^{p_{k-1}} \right) x_{j_{k+1}}^{i_k} \tag{1}$$

Depending on the availabilities of the origins/ sources and demand of the destinations the constrains of such transportation system is given by:

$$\sum_{j_2=1}^{m_2} x_{j_2}^{i_1} = a_{i_1};\ i_1 = 1,2, \dots , m_1 \tag{2}$$

$$\sum_{i_{K-1}=1}^{m_K} x_{j_K}^{i_{K-1}} = b_{j_k}; j_k = 1,2, \dots , m_K \tag{3}$$

$$x_{j_k}^{i_k} \geq 0, \forall i, j \tag{4}$$

If $\sum_{i_1=1}^{m_1} a_{i_1} = \sum_{j_{k+1}=1}^{m_{k+1}} b_{j_k}$, then the problem is said to be balance and constraints (4) is due to the feasibility condition of the MSTP.

Here, the fuzzy random unit transportation cost denoted as $\hat{\tilde{c}}_{j_{k+1}}^{i_k}$ is of the form $\hat{\tilde{c}}_{j_{k+1}}^{i_k} = \left\{ \left(c_{j_k}^{(1)i_k}, c_{j_k}^{(2)i_k}, c_{j_k}^{(3)i_k} \right), p_{i_k} p_{j_k} \right\}$ is the set of triangular fuzzy numbers occurred probability $p_{i_k} p_{j_k}$.

Other notation represents, a_{i_1}= availabilities of the commodity present for the first state.

j_k= demand of commodity for the first state.

$x_{j_{k+1}}^{i_k}$ = amounts transported from i_k-th to j_{k+1}-th destinations.
$\lambda_{i_k}^{p_{k-1}}$= rate of defective to transport the quantity from p_{k-1} interpot to depot i_k.

3.1 Recursive Equation for solving dynamic programming problem:
As mentioned above, the dynamic programming technique will be used to convert the multistage problem into a single-stage problem. The recursive equations for the backward dynamic program for the shortest route problem are illustrated in equations (5) and (6). For minimum transportation cost to a certain destination from all the sources at the last stage, is given by the following formula.
Let, for the last stage K,

$$\hat{\tilde{F}}^K(i_{K-1}) = \hat{\tilde{c}}_{j_K}^{i_{K-1}}, i_K = 1,2,\dots,m_K \; for \; all \; j_K = 2,3,\dots,n_K \tag{5}$$

$$\text{Then calculate } \hat{\tilde{F}}^k(i_k) = \min\left\{\hat{\tilde{c}}_{j_{k+1}}^{i_k} + \hat{\tilde{F}}^{k+1}(i_k)\right\} \tag{6}$$

$$for \; i_k = 1,2,\dots,m_k, j_k = 2,\dots,n_k; k = K-1, K-2,\dots,1$$

Where
K: number of stages in general, k=1,2,3,...,K
i_k: the source number at any stage.
j_k: the destination number at any stage.
m_k: the total number of sources at stage k.
n_k: the total number of destinations at stage k.
$\hat{\tilde{c}}_{j_{k+1}}^{i_k}$: fuzzy random transportation cost of stage k from its sources i_k to its destination j_{k+1}.
$\hat{\tilde{F}}^k(i_k)$: fuzzy random optimum value of the studied objective for each source at any stage k.
Therefore, the reduced single-stage transportation problem with fuzzy random cost coefficient are

$$Minimize \; \hat{\tilde{Z}} = \sum_{i_1=1}^{m_1}\sum_{j_{K=1}}^{n_K} \hat{\tilde{d}}_{j_K}^{i_1} Y_{j_K}^{i_1} \tag{7}$$

Where $Y_{j_K}^{i_1} = \min\{x_{j_2}^{i_1}, x_{j_3}^{i_2}, \dots, x_{j_K}^{i_{k-1}}\}$ and $\hat{\tilde{d}}_{j_K}^{i_1}$ are calculated through equations (5) and (6) is represented by

$$\left(d_{j_K}^{(1)i_1}, p_i^{(l)}\right), l = 1,2,..,L$$

Model-1: Now, the fuzzy Expectation of (7) is given by

$$E\left(\hat{\tilde{Z}}\right) = \sum_{i=1}^{n}\left(\frac{Z^l + 4Z^M + Z^R}{6}\right)p_i \tag{8}$$

Following the expression of expected value of fuzzy random variable. So, the problem become minimization if $E\left(\hat{\tilde{Z}}\right)$ subject to (2) and (3).
Model-2: The fuzzy variance of (7) is given by

$$Var\left(\hat{\tilde{Z}}\right) = \sum_{i=1}^{n}[E(\hat{\tilde{Z}}^2) - \left\{E\left(\hat{\tilde{Z}}\right)\right\}^2] \, (Y_{j_K}^{i_1})^2 \tag{9}$$

The expression of (9) is the variance of fuzzy random variable. So, the problem become minimization if $V(\hat{\tilde{Z}})$ subject to (2) and (3).

IV. NUMERICAL EXAMPLE

Example-1: Let us consider a 3-stage transportation problem. The availabilities of sources at stage-1 equal are those values of ($a_1 = 120, a_2 = 70$ and $a_3 = 60$). The requirement of the destinations at the last stage are those values of ($b_1 = 95, b_2 = 103$ and $b_3 = 52$). There are no transportation restrictions on the middle stages availabilities or requirement.

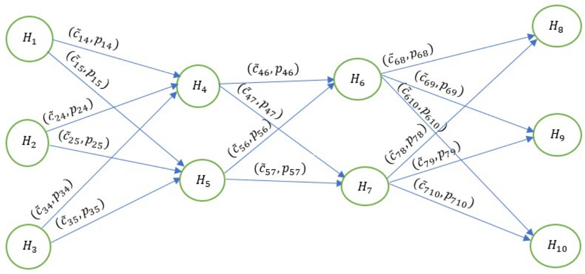

Figure 3: Multi stage transportation problem

Table-2: fuzzy random unit transportation cost

$\tilde{c}_{14}$	(8,9,12) (4,7,10)	with probability 0.2 with probability 0.8	$\tilde{c}_{56}$	(6,7,8) (10,11,12)	with probability 0.9 with probability 0.1
$\tilde{c}_{15}$	(3,5,7) (4,7,9)	with probability 0.7 with probability 0.3	$\tilde{c}_{57}$	(9,12,15) (13,15,17)	with probability 0.6 with probability 0.4
$\tilde{c}_{24}$	(5,8,11) (6,9,12)	with probability 0.8 with probability 0.2	$\tilde{c}_{68}$	(8,11,14) (7,10,13)	with probability 0.4 with probability 0.6
$\tilde{c}_{25}$	(7,11,13) (5,6,7)	with probability 0.4 with probability 0.6	$\tilde{c}_{69}$	(9,11,13) (6,8,10)	with probability 0.6 with probability 0.4
$\tilde{c}_{34}$	(11,12,13) 10,13,16)	with probability 0.6 with probability 0.4	$\tilde{c}_{610}$	(7,9,11) (6,8,10)	with probability 0.4 with probability 0.6
$\tilde{c}_{35}$	(12,15,20) (11,12,13)	with probability 0.9 with probability 0.1	$\tilde{c}_{78}$	(5,9,12) (12,14,16)	with probability 0.7 with probability 0.3
$\tilde{c}_{46}$	(9,12,15) (11,14,17)	with probability 0.7 with probability 0.3	$\tilde{c}_{79}$	(4,6,8) (5,7,9)	with probability 0.7 with probability 0.3
$\tilde{c}_{47}$	(13,17,18) (11,14,17)	with probability 0.4 with probability 0.6	$\tilde{c}_{710}$	(3,5,7) (2,4,6)	with probability 0.2 with probability 0.8

Now we using the dynamic programming problem to convert 3-stage transportation problem to single stage transportation problem and we get the shortest route of the given 3- stage transportation problem.

Table 3: DPP and fuzzy random variable

shortest route	fuzzy random cost with probability
$H_1 - H_5 - H_7 - H_8$	{(17,26,34),0.294},{(24,31,38),0.196},{(21,29,36),0.126}, {(28,34,40),0.084}, {(18,28,36),0.126},{(25,33,40),0.084},{(22,31,38),0.054},{(29,36,42),0.036}
$H_2 - H_4 - H_6 - H_8$	{(22,31,40),0.224}, {(24,33,42),0.096}, {(21,30,39),0.336}, {(23,32,41),0.144}, {(23,32,41),0.056},{(25,34,43),0.024},{(22,31,40),0.084},{(24,33,42),0.036}
$H_3 - H_4 - H_6 - H_8$	{(28,35,42),0.168},{(30,37,44),0.072},{(27,34,41),0.252},{(29,36,43),0.108}, {(27,36,45),0.112}, {(29,38,47),0.048},{(26,35,44),0.168},{(28,37,46),0.072}
$H_1 - H_5 - H_6 - H_9$	{(25,32,38),0.056},{(26,33,39),0.024},{(23,29,37),0.084},{(24,30,39),0.036}, {(21,30,36),0.224}, {(22,31,37),0.096},{(19,27,35),0.336}, {(20,28,37),0.144}
$H_1 - H_5 - H_7 - H_9$	{(22,29,34),0.216},{(19,26,31),0.144},{(26,33,38),0.024},{(23,30,35),0.016}, {(20,24,28),0.324}, {(17,21,25),0.216},{(24,28,32),0.036}, {(21,25,29),0.024}
$H_1 - H_5 - H_7 - H_9$	{(24,32,38),0.168},{(25,33,39),0.072},{(22,29,37),0.252},{(23,30,39),0.108}, {(25,34,40),0.112}, {(26,35,41),0.048},{(23,31,39),0.168}, {(24,32,41),0.072}
$H_1 - H_5 - H_7 - H_8$	{(14,19,24),0.252},{(18,23,28),0.028},{(15,20,25),0.378},{(19,24,29),0.042}, {(15,21,24),0.108},{(19,25,30),0.012)},{(16,22,27),0.162},{(20,26,31),0.018}
$H_1 - H_5 - H_7 - H_8$	{(18,25,30),0.144},{22,29,34),0.016},{(19,26,31),0.216},{(23,30,35),0.024}, {(16,20,24),0.216}, {(20,24,28),0.024},{(17,21,25),0.324},{(21,25,29),0.036}
$H_1 - H_5 - H_7 - H_8$	{(23,31,37),0.048},{(21,28,36),0.072},{(22,30,36),0.192},{(20,27,35),0.288}, {(24,33,39),0.032},{(22,30,38),0.048},{(23,32,38),0.128},{(21,29,37),0.192}

All the shortest routes according to their unit transportation cost(which is also a fuzzy random parameters) are shown in the figure

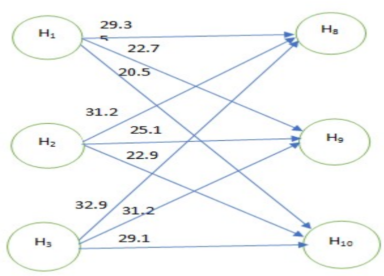

Figure 4: single stage transportation problem

To find optimal transportation unit at first, we find out the expected cost of each route then the converted transportation problem minimized through Lingo-14.0 subject to the constraints (2) and (3). The optimal solution is

Table 4: Optimum transportation for Model-1

Route	Trans. Unit (min)	Exp. Value(min)
$H_1 - H_5 - H_7 - H_8$	0	6554.1
$H_2 - H_4 - H_6 - H_8$	35	
$H_3 - H_4 - H_6 - H_8$	60	
$H_1 - H_5 - H_6 - H_9$	103	
$H_2 - H_5 - H_6 - H_9$	0	
$H_3 - H_4 - H_7 - H_9$	0	
$H_1 - H_5 - H_6 - H_{10}$	17	
$H_2 - H_5 - H_6 - H_{10}$	35	
$H_3 - H_4 - H_7 - H_{10}$	0	

And for model-2:

Table 5: Optimum transportation for Model-2

Route	Trans. Unit (min)	Var.(min)
$H_1 - H_5 - H_7 - H_8$	0	4675.05
$H_2 - H_4 - H_6 - H_8$	35	
$H_3 - H_4 - H_6 - H_8$	60	
$H_1 - H_5 - H_6 - H_9$	68	
$H_2 - H_5 - H_6 - H_9$	35	
$H_3 - H_4 - H_7 - H_9$	0	
$H_1 - H_5 - H_6 - H_{10}$	52	
$H_2 - H_5 - H_6 - H_{10}$	0	
$H_3 - H_4 - H_7 - H_{10}$	0	

Example-2: Example-2 is similar as example-1, where the internal depots are not in infinite capacity. Instead of that, internal depots have finite capacity of storing. Let the capacity of the depots are

Depots	H_4	H_5	H_5	H_7
Capacity	55	65	50	60

Then the optimal solution is

Table 6: optimum result with capacity of depots

Route	Transportation Unit	min cost
$H_1 - H_5 - H_7 - H_8$	20	5933.6
$H_2 - H_4 - H_6 - H_8$	25	
$H_3 - H_4 - H_6 - H_8$	50	
$H_1 - H_5 - H_6 - H_9$	50	
$H_2 - H_5 - H_6 - H_9$	43	
$H_3 - H_4 - H_7 - H_9$	10	
$H_1 - H_5 - H_6 - H_{10}$	0	
$H_2 - H_5 - H_6 - H_{10}$	2	
$H_3 - H_4 - H_7 - H_{10}$	0	

Example-3: Let us consider an example, same as example-1 with the positive defective rate, and corresponding results are

Table 7: Optimum results with defectiveness of the item

Defective	Shortest Route	transportation unit of stage-1	Transportation unit of stage-2	transportation unit of stage-3	min cost
$\lambda_{15}=0.3$, $\lambda_{57}=0.3$	$H_1-H_5-H_7-H_8$	35	24.5	14	4475.32
$\lambda_{24}=0.3$, $\lambda_{46}=0.3$	$H_2-H_4-H_6-H_8$	0	0	0	
$\lambda_{34}=0.4$, $\lambda_{46}=0.3$	$H_3-H_4-H_6-H_8$	60	36	18	
$\lambda_{15}=0.3$, $\lambda_{56}=0.2$	$H_1-H_5-H_6-H_9$	33	23.1	16.5	
$\lambda_{25}=0.4$, $\lambda_{56}=0.2$	$H_2-H_5-H_6-H_9$	70	42	28	
$\lambda_{34}=0.4$, $\lambda_{56}=0.2$	$H_3-H_4-H_7-H_9$	0	0	0	
$\lambda_{15}=0.3$, $\lambda_{56}=0.2$	$H_1-H_5-H_6-H_{10}$	52	36.4	26	
$\lambda_{25}=0.4$, $\lambda_{56}=0.2$	$H_2-H_5-H_6-H_{10}$	0	0	0	
$\lambda_{34}=0.4$, $\lambda_{47}=0.3$	$H_3-H_4-H_7-H_{10}$	0	0	0	

V. DISCUSSION

The mathematical form and its numerical illustration help the decision maker to take the following decisions: (i) As these are several paths from an origin H_1, H_2, H_3 to final destination H_8, H_9, H_{10}, the dynamic programming problem help us to find the path with minimum cost, such paths with their fuzzy random costs are display in table-3. (ii) The optimum results to minimize the expected total cost and variance of total of total cost are shown in Table-4 and Table-5. (iii) Here the optimum paths are non-degenerate in nature and solution are balanced. (iv) It is also seen that in the middle stage H_7 depots are not used, since to transport the quantity to H_9 and H_{10}through H_7 gives the more expected cost (31.23, 29.1 respectively) than that through H_6((22.75.25.06), (20.51,22.86) respectively). Similarly, when the amount is transport from H_1 gives the larger value than H_6. (v) But, if we assign the capacity of depots then through H_7 , quantity also transport. In this case the number of allocations is more than without that of without capacity. (vi) From table-7, it is also seen that if defectiveness of the items is introduced then as expected total transportation amount reduce.

VI. CONCLUSION AND FUTURE RESEARCH WORK

In reality, the last user i.e., retailer shop or seller can't get the commodity directly from the origin. There may present one or more intermediate depots. So, in general a single stage transportation problem is not realistic in nature. Such a multi stage transportation problem is constructed here. The multi stage problem create multi paths from one origin to one destination. From these multiple available paths unique path is determined here through dynamic programming problem. Here the unit cost terms are taken with vagueness presented by fuzzy random variable, which has been transformed into a deterministic one using expected value of credibility measure of the fuzzy random variable. The model can be extended with solid transportation problem, problem with deteriorating item, model with fixed charge, model with more vehicle cost, etc. Not only that, the similar problem can be formulate and examine with another type of vagueness, like- rough measures, fuzzy rough measures, type-2 fuzzy parameters, etc.

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