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Studying Quasi-Static Void Growth Based on Strain Gradient Elasto-Plasticity

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Abstract: The void growth as the principal mechanism inductile fracturehas attracted much attention. Based on the strain gradient elasto-plasticity, the quasi-static void growth problemis solvedwith the consideration of size effect.Numerical results show thatin the early stage, the void does not grow. After the loading has increased to a certain amount, voids begin to grow rapidly. The size effectin elastic limitfurther increases the strength of materials and the critical stress, and delays the time of the quasi-static void growth, especially for the smaller voids. Furthermore, larger scale parameter makes the scale effect more significant.

Keywords: quasi-static void growth, strain gradient, elasto-plasticity, size effect, critical stress

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I. INTRODUCTION

The fracture behavior of metal materials has become a hot research topic in the fields of aviation and materials.Benzerga[1] believed that ductile fracture of metal materials usually begins with void nucleation of foreign particles or other defects.Through the elasto-plastic deformation of the surrounding medium, voids will grow sharply and coalesce, eventually leading to ductile fracture. It is well proven by the dimple-like traces at the crack interface found in the experiment.

Although the growth behavior of voids has been studied for decades, there are still some interesting things to be concerned. Wu et al.[2]pointed out that there is a critical stress in the growth process of quasi-static voids under the hydrostatic tensile loading, only when the loading exceeds the critical stress will voids begin to grow rapidly.However, the correlation between quasi-static void growth process and initial void radius has not been mentioned. In the microscale deformation, the magnitude of the geometrically necessary dislocation (GND) becomes equal to the statistical storage dislocation (SSD), resulting in thestrain gradient corresponding to GND playing a non-negligible role in the growth of micro-void.Thus, Wu el al.[3] studied the growth of voids on the basis of strain gradient and suggested that strain gradient enhances the strength of materials and increases the critical stress required for void growth.Based on strain gradient plasticity, Molinari et al.[4-6] studied the growth of voids and the effect of initial void shape.Tvergaard and Hutchinson[7]suggested that the initial void shape has less influence on the critical stress, so the deviation caused by the shape of the void is can be negligible, Void shapeis assumed to remain spherical all the time in this paper. Liu et al.[8]analyzed lengthscale effects in void growth and proposed a simple elastic–plastic decompositionmethod. In recent years, it has been found that metal materials can withstand greater elastic strain before entering plastic deformationin some micro-scale experiments[9-11]. That is to say, themicroscale metal materials exhibitthe size effect in elastic limitand are characterized by"smaller and stronger".However, the above and other papers[12-14]about void growth cannot capture the size effect in elastic limit. Based on the Taylor model, Liu and Soh[15] proposed the strain gradient elasto-plasticity with a new yield function and emphasized thecontribution of elastic part in the wholemicroscale deformation.The theoretical model can well capture the strain hardening and the size effect inelastic limit observed in the experiment. The numerical simulation results of the wire twist are also consistent with the experimental results. Thus, we will use the strain gradientelasto-plasticity to analyze the growth behavior of quasi-static void.

The remainder of this paper is organized as follows. Section II details the quasi-static void growth model, the SGEP theoretical framework isbriefly outlined, thekinematics and equilibrium equation are also given. Numerical simulations of the void growth behavior considering size effect and discussion of the results are detailed in the Section III. Finally, a brief summary closes the paper in Section IV.

II. QUASI-STATIC VOID GROWTH MODEL

2.1 Strain Gradient Elasto-plasticity

In the conventional J2plasticity,since the elastic strain is much smaller than the plastic strain, the elastic deformation is negligible in the elasto-plastic deformation. However, the magnitude of the elastic effective strain gradient is much larger than the magnitude of the plastic effective strain in the microscale deformation, which makes the contribution of the elastic part to the whole deformation particularly important. Based on Taylor plasticity, theconcept called total effective elasto-plastic strain $\varepsilon_e^{t,ep}$ is presentedin the Strain gradient elasto-plasticity (SGEP). Considering that the elasticlengthscales is much smaller than plasticlengthscales and the contribution of higher order stresses can be negligible, the total effective elasto-plastic strain $\varepsilon_e^{t,ep}$ is redefined here as:

$$\varepsilon_e^{t,ep} = \left\{ \left(\varepsilon_e^e + \varepsilon_e \right)^{2\beta} + \left[l \left(\eta_e^e + \eta_e \right) \right]^{2\beta} \right\}^{\frac{1}{2\beta}} \tag{1}$$

where ε_e^e is the effective elastic strain, ε_e is the effective plastic strain, η_e^e is the effective elastic strain gradient, η_e is the effective plastic strain gradient, and the contributions of strain and strain gradient taken into account by using plastic scale parameter l to plastic hardening are controlled by parameter β whose value is set to 1.
In order to make the theoretical framework workable,a Taylor-based yield function isdefined as:

$$\sigma^t = \begin{cases} \sigma_Y + \left(\sigma_0^t - \sigma_Y \right) \left(\dfrac{\varepsilon_e^{t,ep}}{\varepsilon_0^t} \right) & \text{if } \varepsilon_e^{t,ep} \le \varepsilon_0^t, \\ \sigma_o^t \left(\dfrac{\varepsilon_e^{t,ep}}{\varepsilon_0^t} \right)^N & \text{if } \varepsilon_e^{t,ep} > \varepsilon_0^t \end{cases} \tag{2}$$

where,

$$\varepsilon_0^t = \left(\frac{\varepsilon_e^{t,ep}}{\varepsilon_e^e + \varepsilon_e} \right)^{\gamma_\varepsilon} \varepsilon_Y , \sigma_0^t = \left(\frac{\varepsilon_e^{t,ep}}{\varepsilon_e^e + \varepsilon_e} \right)^{\gamma_\sigma} \sigma_Y \tag{3}$$

σ_t is the current yield strength, ε_0^t is the reference strain, σ_0^t is the reference stress, ε_0^t and σ_0^t are considered as the yield strain and yield stress in the current model, respectively.Comparison with traditional yield strain ε_Y and yield stress σ_Y, both ε_0^t and σ_0^t change with the deformation process.Equation(3)is proposed to incorporate the size effect in elastic limit. Therelation of material parameters γ_ε and γ_σ is assumed to be $\gamma_\sigma = 2\gamma_\varepsilon = 2\beta$ based on the flow rule[16].

2.2 Kinematics

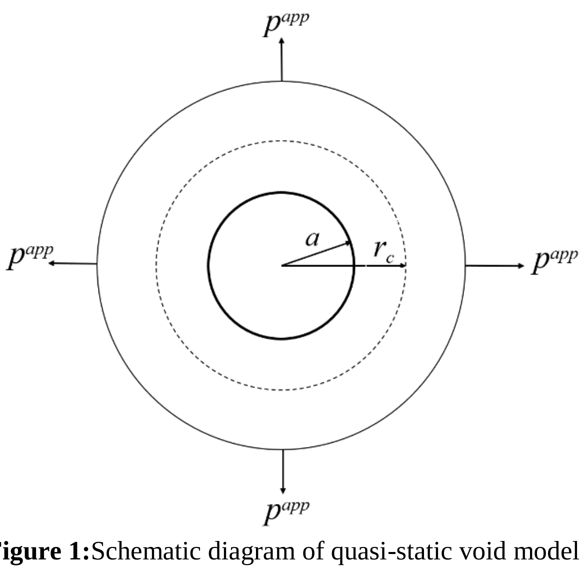

Figure 1: Schematic diagram of quasi-static void model

Assume that the infinite medium contains a single void, as shown in Fig. 1, the initial void radius is A, the initial radius of a reference point is R ,the void is deformed under the far-field hydrostatic tensile loading

p^{app}, the current void radius is a, the current radius of a reference point is r. The material is assumed to be incompressible, the existence relation is:

$$r^3\text{-R}^3 = a^3\text{-A}^3 \tag{4}$$

According to material incompressibility, the radius of void surface has the following relationship with the radius of reference point:

$$\dot{r} = a^2\dot{a}/r^2 \tag{5}$$

The strain component and the strain gradient component during the growth of the void are expressed in increments as:

$$\begin{aligned}
&\dot{\varepsilon}_{rr} = \frac{dv_r}{dr} = -\frac{2a^2\dot{a}}{r^3},\ \dot{\varepsilon}_{\theta\theta} = \dot{\varepsilon}_{\varphi\varphi} = -\frac{1}{2}\dot{\varepsilon}_{rr} = \frac{a^2\dot{a}}{r^3},\\
&\dot{\eta}_{rrr} = \frac{\partial\dot{\varepsilon}_{rr}}{\partial r} = \frac{6a^2\dot{a}}{r^4}, \dot{\eta}_{\theta\theta r} = \frac{\partial\dot{\varepsilon}_{\theta\theta}}{\partial r} = \frac{\text{-}3a^2\dot{a}}{r^4}, \dot{\eta}_{\varphi\varphi r} = \frac{\partial\dot{\varepsilon}_{\varphi\varphi}}{\partial r} = \frac{\text{-}3a^2\dot{a}}{r^4},\\
&\dot{\eta}_{r\theta\theta} = \dot{\eta}_{\theta r\theta} = \frac{\dot{\varepsilon}_{rr} - \dot{\varepsilon}_{\theta\theta}}{r} = \frac{\text{-}3a^2\dot{a}}{r^4}, \dot{\eta}_{r\varphi\varphi} = \dot{\eta}_{\varphi r\varphi} = \frac{\dot{\varepsilon}_{rr} - \dot{\varepsilon}_{\varphi\varphi}}{r} = \frac{\text{-}3a^2\dot{a}}{r^4}
\end{aligned} \tag{6}$$

The current radius increment $\dot{a}$ is decomposed into an elastic part $\dot{a}^e$ and a plastic part $\dot{a}^p$:

$$\dot{a} = \dot{a}^e_{\varepsilon} + \dot{a}^p_{\varepsilon}\, sign(\dot{a}) = \dot{a}^e_{\eta} + \dot{a}^p_{\eta}\, sign(\dot{a}) \tag{7}$$

Correspondingly, both the strain and the strain gradient are decomposed into elastic and plastic parts.The elastic constitutive law relates stresses to elastic strains in the form:

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon^e_{mm} + 2\mu\varepsilon^e_{ij} \tag{8}$$

where σ_{ij} is the Cauchy stress; λ and μ are the Lamé constants.

2.3Equilibrium Equation

The void suffers from the far-field hydrostatic tensile loading p^{app} as shown in Fig. 1, and the equilibrium equation in the spherical coordinates is:

$$\frac{d\sigma_{rr}}{dr} - \frac{2\sigma_e}{r} = 0 \tag{9}$$

The boundary conditions are:

$$\sigma(r,t) = 0 \text{ at } r = a,\ \ \sigma_r = p^{app} \text{ at } r \to \infty \tag{10}$$

Integrate the equilibrium equation to get the relationship:

$$p^{app} = \int_a^{\infty}\frac{2\sigma_e}{r}dr = \int_{r_c}^{\infty}\frac{2\sigma_e}{r}dr + \int_a^{r_c}\frac{2\sigma_e}{r}dr = \frac{2}{3}\sigma_0^t\Big|_{r_c} + \int_a^{r_c}\frac{2\sigma_e}{r}dr \tag{11}$$

where $\sigma_0^t\Big|_{r_c}$ represents the reference stress of the reference point in elastic-plastic boundary and reflectsthe size effect inelastic limit in the quasi-static void growth.

III. RESULTS AND DISCUSSION

3.1Simulation Setting

Table 1: The simulation parameters

Young's modulusE (GPa)	Yield stress σ_Y (MPa)	Posson'sratio υ	Scale parameter l (μm)	Work hardening exponentN	Controlparameter β
120	316.5	0.361	1.4	0.15	1

In this paper, the simulation parameters are shown in Table 1. It's worth noting that the material properties selected are the same as Wu et al.[2] to facilitate the observation of the size effect. In addition, all simulation paths for quasi-static void growth are shown in Fig. 2.The hydrostatic tensile loading p^{app} increases from zero at a constant rate, reaches p_s after time t_1, and then remains constant for time t_2. Under such a simulation setting, the study of quasi-static void growth considering strain gradient will be carried out next.

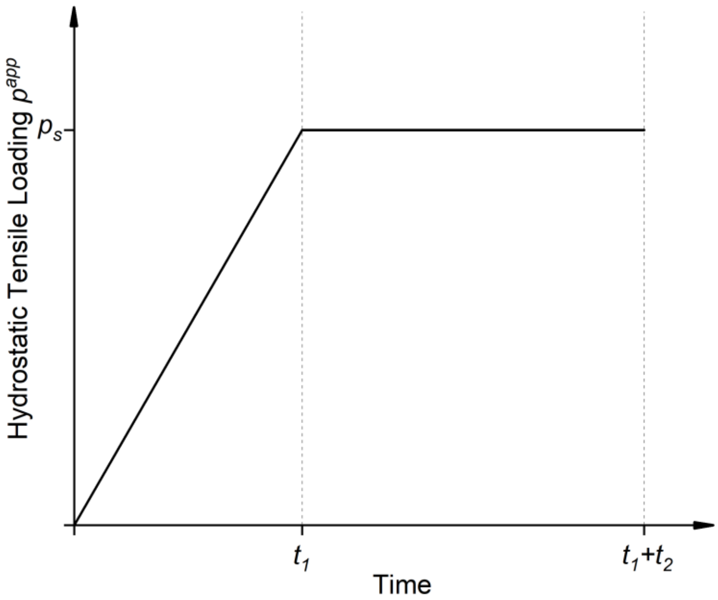

Figure 2: The path of hydrostatic tensile loading

3.2Size Effect

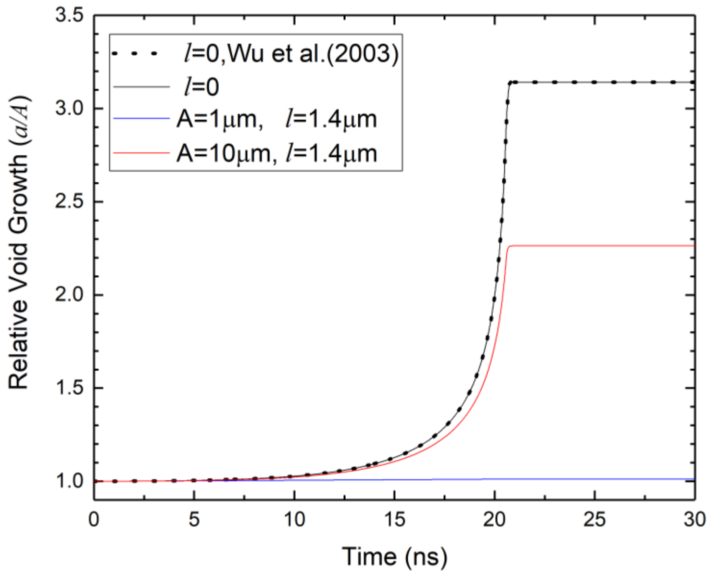

Figure 3: Numerical results on quasi-static void growth under hydrostatic tensile loading. The dotted line represents the simulations of Wu et al.[2],and the solid lines represent the simulated data based on strain gradient elasto-plasticity. The material parameters are shown in Table 1, and three cases are considered: (a) $l = 0$; (b) $A = 1\,\mu\text{m}, l = 1.4\,\mu\text{m}$; (c) $A = 10\,\mu\text{m}, l = 1.4\,\mu\text{m}$.

The growth of voids inside the material are shown in Fig.3. The hydrostatic tensile loading is $6.5\sigma_Y$ and the loading rate is $0.1\,\text{GPa / ns}$. The dotted line represents the simulated data given by Wu et al.[2],which is in good agreement with our simulation of the $l = 0$ case that size effect is not included.It shows that our void model based on strain gradient elasto-plasticity can well simulate the quasi-static growth of voids whenthe size effect is not considered.Referencingthe study of size effect[3, 17],scale parameter l is set to $1.4\,\mu\text{m}$ when considering size effect. For the $A=1\,\mu\text{m}, l=1.4\,\mu\text{m}$ case, the strength of the material especially around the void surface is greatly enhanced by the size effect, so that the deformation is small and not enough to make the void grow. For the $A=10\,\mu\text{m}, l=1.4\,\mu\text{m}$ case, the voidis able to grow and the maximum of relative void growth is 2.3 which is 0.9 less thanthe one of the $l=0$ case.This indicates that the size effect has an inhibitory effect on the growth of voidsespecially for smaller voids.

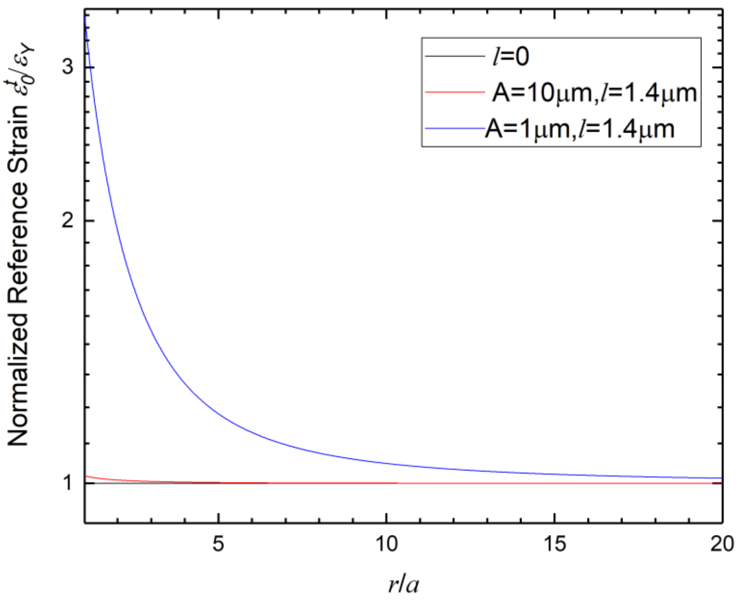

Figure 4: Normalized reference strain $\varepsilon_0^t / \varepsilon_Y$ varies with the distance from the surface of the void as the load time reaches 20 ns. Three cases are considered: (a) $l=0$; (b) $A=1\,\mu\text{m}, l=1.4\,\mu\text{m}$; (c) $A=10\,\mu\text{m}, l=1.4\,\mu\text{m}$.

In order to study the strengthening effect of size effect on material strength and the hindrance effect on void growth, we simulate the variations of reference strain with distancefrom the surface of the void as the load time reaches 20 ns, as shown in Fig. 4. For the $l=0$ case, the size effect does not exist and reference strain ε_0^t is always equal to the traditional yield strain ε_Y ,This is to say, the strength of the material is independent of the scale and is not enhanced.For the $A=1\,\mu\text{m}, l=1.4\,\mu\text{m}$ case, the reference strain even reaches $3.4\varepsilon_Y$ on the void surface and decreases with distance from the surface but never less than the traditional yield strain ε_Y , theelastic limit is raisedobviouslyand so that grow of the void requires greater loading.Otherwise, the growth of the void is hindered or even does not occur. For the $A=10\,\mu\text{m}, l=1.4\,\mu\text{m}$ case, the reference strain is slightly higher than the traditional yield strain ε_Y and slight size effect of elastic limit is shownon the void surface.This is a good reason why the void growth of the $A=10\,\mu\text{m}, l=1.4\,\mu\text{m}$ case is slightly lower than that of the $l=0$ case in Fig. 3.

3.3Scale Parameter

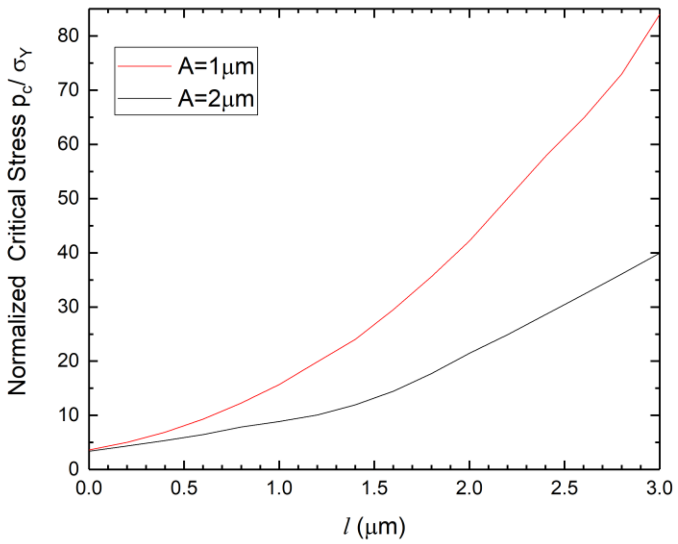

Figure 5: Normalized critical stress p_c / σ_Y varies with the scale parameter l, two cases are considered: (a) $A=1\ \mu m$; (b) $A=2\ \mu m$.

The size effect increases the strength of the material and delays the growth of voids, for scale parameter $l=1.4\ \mu m$,the $A=1\ \mu m$ case is not able to grow but the $A=10\ \mu m$ case can grow quicklyunder the loading of $6.5\sigma_Y$ .The critical stress required for void growth is different under various scale parameters and initial void

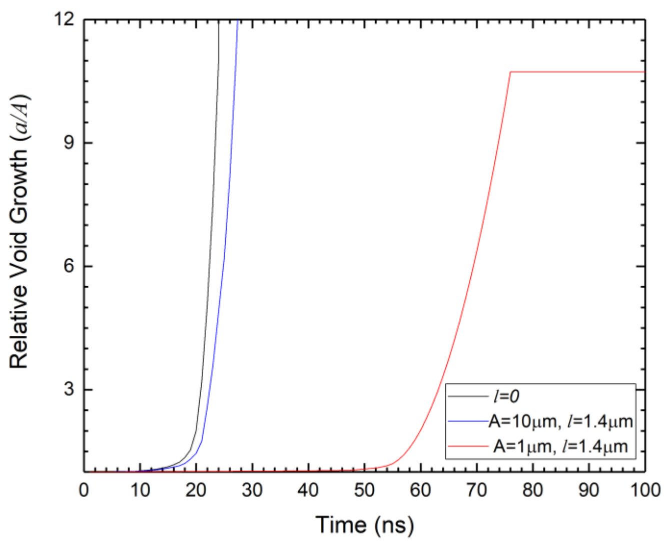

Figure 6: Relative void growth varies with the time. The loading is $30\sigma_Y$ and the loading rate is $0.14\ GPa/ns$. Three cases are considered: (a) $l=0$; (b) $A=1\ \mu m, l=1.4\ \mu m$; (c) $A=10\ \mu m, l=1.4\ \mu m$.

Radius, as shown in Fig.5. When the scale parameter $l=0$, the critical stress p_c is $3.6\sigma_Y$ for both the $A=1\ \mu m$ case and the $A=2\ \mu m$ case, in other words, the critical stress required for the void growth is independent of initial void radius as size effect is not included. As the scale parameter increases, the critical stress increases sharply when size effect is considered. Moreover, the critical stress is $84\sigma_Y$ for the $A=1\ \mu m$

case and is $40\sigma_Y$ for the $A = 2\mu\text{m}$ case. It can be seen that the magnitude of the critical stress is positively correlated with the scale parameter and negatively correlated with the initial void radius.

3.4Quasi-static Void Growth

Relative void growth varies with the time are shown in Fig. 6. The scale parameter is still set to $1.4\ \mu\text{m}$, and the critical stress is able to be found to be $24\sigma_Y$ in Fig. 5 so that the loading is set to $30\sigma_Y$. The $l = 0$ case that size effect is not included begin to grow as the time reaches $12\ \text{ns}$ while the $A = 10\ \mu\text{m}, l = 1.4\ \mu\text{m}$ case begin to grow as the time reaches $14\ \text{ns}$. Because of the size effect, the $A = 10\ \mu\text{m}, l = 1.4\ \mu\text{m}$ case is delayed for $2\ \text{ns}$. For the $A = 1\ \mu\text{m}, l = 1.4\ \mu\text{m}$ case, the void growth isdelayed for $42\ \text{ns}$. It can be found that the size effect strengthens the material and resists quasi-static void growth especially for smaller voids.

IV. CONCLUSION

In this paper, quasi-static void growth is analyzed based on the strain gradient elasto-plasticity. The strain gradientis found to show an extra hardening effect, increasing the strength of materialsand making voids inside materialsmore difficult to grow. For smaller voids, the size effect in elastic limit is moreprofound especially on void surface and the reference strain is much bigger than the traditional yield strain. Quasi-static void growth will be delayed or even prevented tooccur because of the size effect. Meanwhile, Larger scale parameter can further improve the material strength and delay thequasi-static void growth.

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